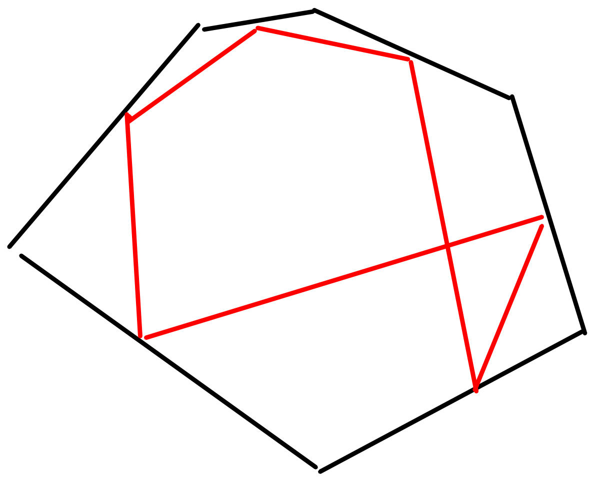
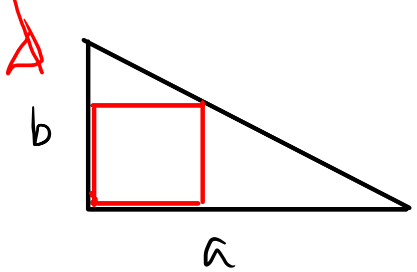
358. Into a right triangle with legs a and b units long, a square is inscribed in such a way that one of its angles is the right angle of the triangle, and the vertices of the square lie on the sides of the triangle. Find the perimeter of the square.



un polígono inscrito en otro.



P.C. el perímetro del $\square$ en términos de los catetos a, b

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$$p = 2^{2^n} + 1$$

Mersenne

$2^1 + 1 = 3$

$2^2 + 1 = 5$] Griegos.

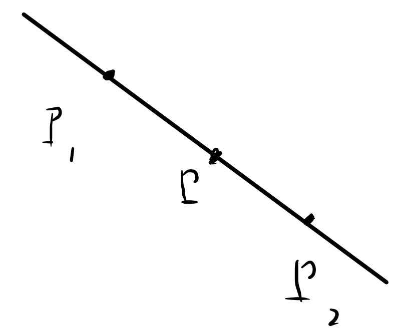
$2^4 + 1 = 17$

$2^8 + 1 = 257$ } Gauss.

Rectas

Teo: dados $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2) \in \mathbb{R}^2$, $P_1 \neq P_2$,

$\exists !$ recta que pasa por P_1, P_2.



Dem: sea $P = (x, y)$ en la recta que buscamos.

pend: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y - y_1}{x - x_1}$

$$(y_2 - y_1)(x - x_1) = (y - y_1)(x_2 - x_1)$$

$$\underbrace{(y_2 - y_1)}_{A} x \underbrace{- (x_2 - x_1)}_{B} y = \underbrace{x_1 (y_2 - y_1) - y_1 (x_2 - x_1)}_{C}$$

- $A \neq 0$ ó $B \neq 0$
- (x_1, y_1) satisface: $(y_2 - y_1) x_1 - (x_2 - x_1) y_1 = \text{- - -}$ ✓
- (x_2, y_2) — " —

$\Rightarrow$ existencia

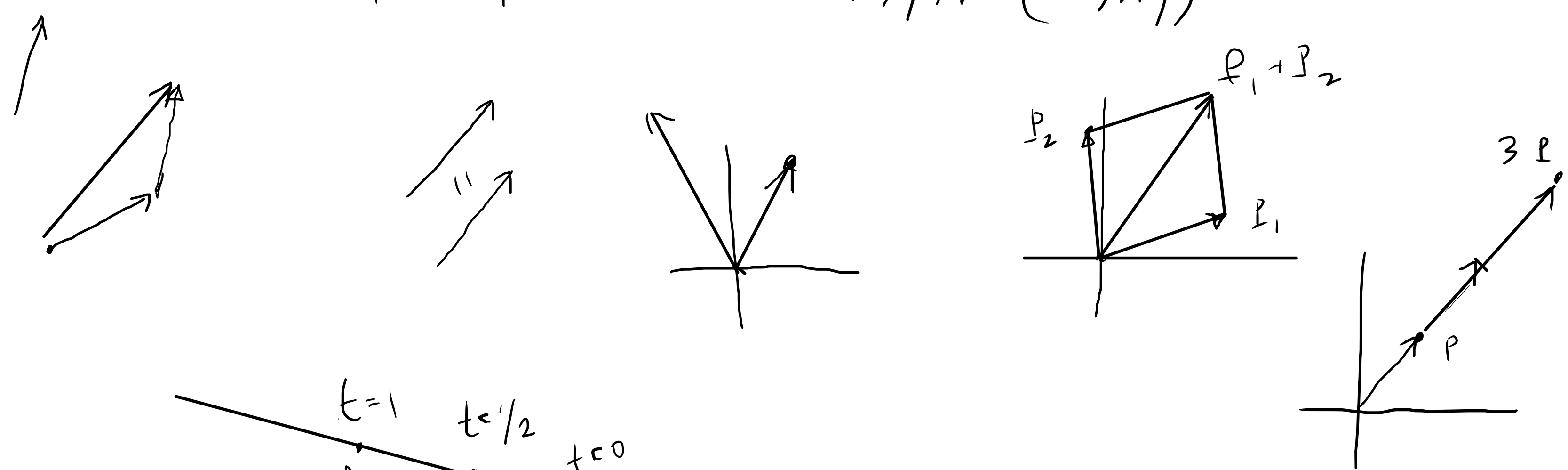
Unicidad (ej).

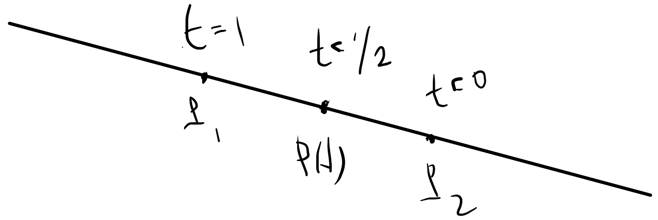
Parametrización de la recta que pasa P_1, P_2:

<u>Teo</u>: (1) la recta está parametrizada por $P(t) = P_1\, t + P_2\,(1-t)$, $t \in \mathbb{R}$.

<u>Explicación</u>: "suma" de puntos $\in \mathbb{R}^2$: $(x_1, y_1) + (x_2, y_2) := (x_1 + x_2, y_1 + y_2)$.

mult. punt. por núm : $\lambda(x, y) := (\lambda x, \lambda y)$



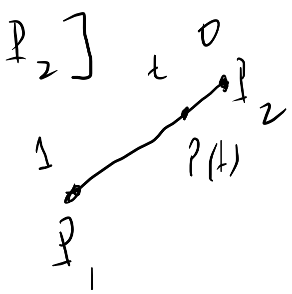


es decir, $P(t)$ está en la recta $\overline{P_1P_2}$ y todo $P \in \overline{P_1P_2}$ es de la forma $\boxed{tP_1 + (1-t)P_2}$

$P(t)$, para único $t \in \mathbb{R}$.

Es más, (2) $t \in (0,1) \Leftrightarrow P(t)$ está entre P_1 y P_2.

(3) $P(t)$, $0<t<1$, divide el segmento $[P_1, P_2]$ en una prop. t: $\dfrac{|P(t)P_2|}{|P_1P_2|} = t$.



<u>Dem 3</u> $\dfrac{\|tP_1 + (1-t)P_2 - P_2\|}{\|P_1 - P_2\|} = \ldots$

completar $\|(a, b)\| = \sqrt{a^2 + b^2}$

$|P_1P_2| = \|P_1 - P_2\|$