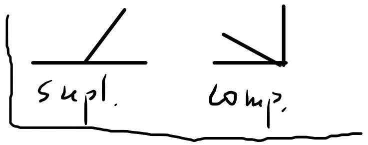
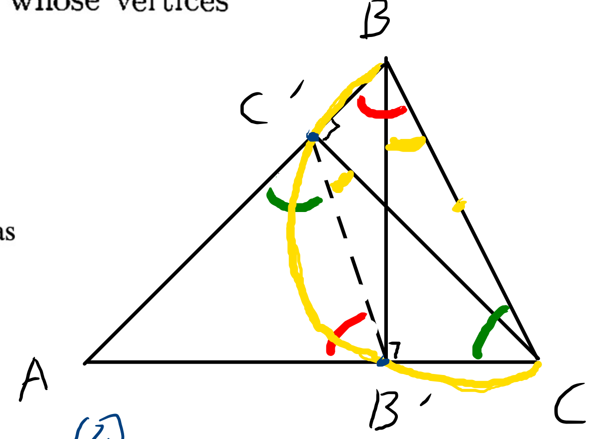
354. The line connecting the feet of two altitudes of any triangle cuts off a triangle similar to it. Derive from this that altitudes of any triangle are angle bisectors in another triangle, whose vertices are the feet of these altitudes.

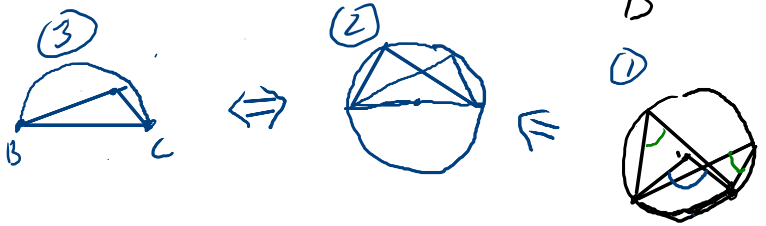




P.D. $\Delta AB'C' \sim \Delta ABC$

Sugerencia para 354. Sea ABC un triángulo y B',C' los pies de las alturas desde B,C (resp.). Demuestra que el círculo con diámetro BC pasa por B',C'. De aquí concluye que $\angle CC'B' = \angle CBB'$, por lo que $\angle AC'B' = \angle ACB$.

Lemma: Sea γ un círculo con diam. BC. Ent $P \in \gamma \iff \measuredangle BPC = 90^\circ$. ($P \neq B, C$.)

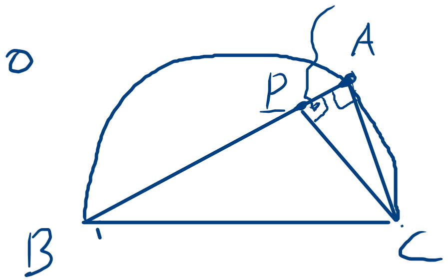


$\Rightarrow$: el teo 2.

$\Leftarrow$:

- Si P está adentro de γ, $P \notin BC$, $0 < \measuredangle APC < 90$

$\Rightarrow 90 < \measuredangle BPC = 180 - \measuredangle APC < 180$

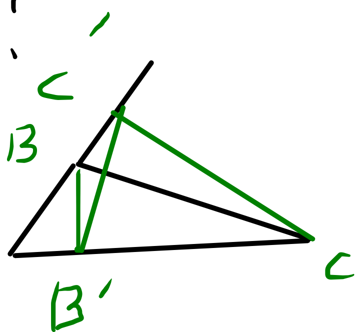


- Si P está afuera de γ,

.... (completar).

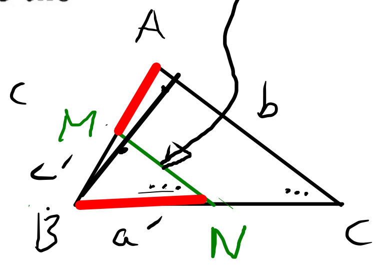
OJO: qué sucede con Δ obtuso?

Checar!



357. In a triangle ABC with sides a, b, and c units long, a line MN parallel to the side AC is drawn, cutting on the other two sides the segments $AM = BN$. Find the length of MN.

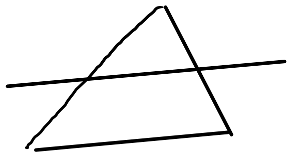
$$b' = \frac{bc}{a+c}$$



$MN = ?$

$\Delta BMN \sim \Delta BAC$

$$\Rightarrow \begin{cases} \dfrac{BM}{c} = \dfrac{BN}{a} = \dfrac{MN}{b} \quad (\text{tales}) \\ c - BM = BN \end{cases}$$



$$\Rightarrow \frac{BM}{c} = \frac{c - BM}{a} = \frac{MN}{b}$$

$$\Rightarrow BM \cdot a = (c - BM)c = c^2 - BM \cdot c$$

$$BM(a+c) = c^2$$

$$BM = \frac{c^2}{a+c}$$

$$MN = \frac{b}{a}\left(c - \frac{c^2}{a+c}\right) = \frac{bc}{a}\left(1 - \frac{c}{a+c}\right) = \frac{bc(a+\cancel{c}-\cancel{c})}{\cancel{a}(a+c)} =$$

$$= \frac{bc}{a+c}$$

Más elegante?

$$\text{area}(ABC) = \frac{b \cdot h}{2}$$

$$\text{area}(BMN) \leftarrow \frac{b' \cdot h'}{2}$$

$$+\text{area}(AMNC) \leftarrow \frac{b+b'}{2} \cdot (h - h')$$

$$c - c' = a'$$

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = \frac{h}{h'}$$

5 eqn's
5 incog.