1. Encontrar la forma polar de los siguientes números complejos

a) i b) $1+i$ c) $(1-i)^{10}$ d) $\dfrac{1}{1-i}$.

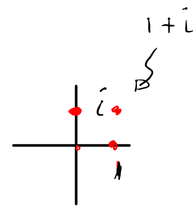
la forma rectangular → $\boxed{a+bi}$ ($z =$) notación

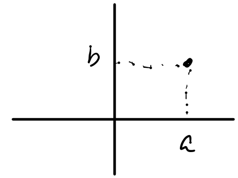
$a+bi \longleftrightarrow (a,b)$

$i \longleftrightarrow (0,1)$

$1 \longleftrightarrow (1,0)$

$1+i \longleftrightarrow (1,1)$





multiplicación: $(1+2i)(-3+\frac{1}{2}i) = -3+\frac{1}{2}i - 6i + (\cancel{2}i)(\frac{1}{\cancel{2}}i)$

$i^2=-1$

$= -4 - \left(\frac{11}{2}\right) i.$

la forma polar → $z = re^{i\theta}$

$e = ?$

$= 2.71\ldots$

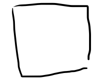
de

f

Def 1: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ acotada, creciente $\Rightarrow$ tiene límite

La definición bancaria

1 peso $\rightarrow$



100% interes anual

$1 \xrightarrow[\text{1 año}]{} 2$

$1 \xrightarrow[\text{6 meses}]{} 1.5 \xrightarrow[\text{6 meses}]{} 1.5 + .75 = 2.25 = \left(1+\frac{1}{2}\right)$

$1 \xrightarrow[\text{3 meses}]{} \left(1+\frac{1}{4}\right)^4$

4 veces

$1 \longrightarrow \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.71\ldots$

Def 2: $f(t) = e^t$, $e = f(1)$

f. de físico

La función $f = f'$, $f(0) = 1$

$(2^x)' =$

$(x^n)' = n x^{n-1}$

$\boxed{(e^x)' = e^x}$?

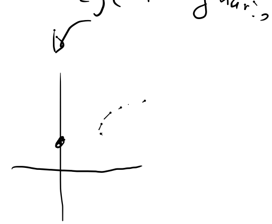
$y = 2^x \leftrightarrow x = \log_2 y$

Def 3 $e \overset{\text{def.}}{=} 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \ldots$

Taylor $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \ldots$

$e^{i\theta} = \sum_{k=0}^{\infty} \frac{1}{k!}(i\theta)^k = 1 + i\theta + \frac{\theta^2}{2} + \ldots$

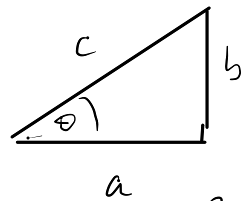
$\frac{d}{d\theta} e^{i\theta} = \sum \ldots = i e^{i\theta}$

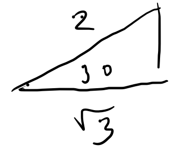


Defi $e^{i\theta} = \cos\theta + i\,\mathrm{sen}\,\theta$ (Euler).

$Re(z) = 0 \leftrightarrow z$ es imaginario

$\cos\theta = \frac{a}{c}$, $\mathrm{sen}\,\theta = \frac{b}{c}$





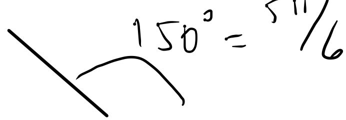
$\cos(-\pi/6) = ?$

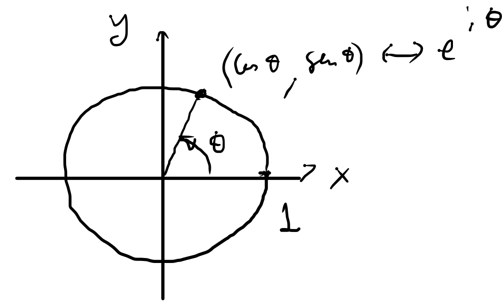
$\cos(\pi/6) = \sqrt{3}/2$

$\mathrm{sen}(\pi/6) = 1/2$

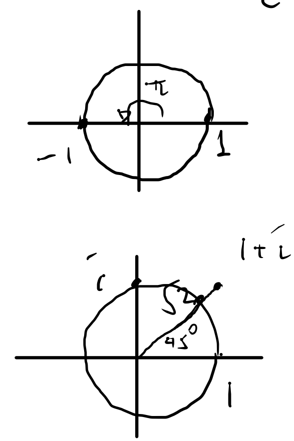
$\mathrm{sen}(-\pi/6) = ?$

$\cos(5\pi/6) = ?$





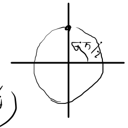
$e^{i\pi} = -1$



$1 + i = \sqrt{2}\, e^{i\pi/4}$

La forma polar.

$i = e^{i\pi/2}$



$$\frac{1}{1-i} = \frac{1+i}{2} = \frac{1}{2}(1+i) = \frac{1}{2}\left(\sqrt{2}\, e^{i\pi/4}\right)$$
$$= \frac{1}{\sqrt{2}} e^{i\pi/4}.$$

$w/z = w \cdot \frac{1}{z}$

$$\frac{1}{z} = \frac{\bar z}{z \cdot \bar z} = \frac{\bar z}{a^2+b^2} \qquad z \cdot \frac{1}{z} = z\frac{\bar z}{a^2+b^2} = \frac{z\bar z}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2} = 1$$

$z = a+ib, \quad \bar z := a - ib$

$z\bar z = (a+ib)(a-ib) = a^2 - (ib)^2 = a^2+b^2 = |z|^2$

Teo: $e^{i\theta_1} e^{i\theta_2} = e^{i\theta_1 + i\theta_2}$ $\qquad \boxed{a^x a^y = a^{x+y}.}$

$$\Updownarrow$$

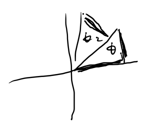
$$(\cos\theta_1 + i\,\mathrm{sen}\,\theta_1)(\cos\theta_2 + i\,\mathrm{sen}\,\theta_2) = \cos(\theta_1+\theta_2) + i\,\mathrm{sen}(\theta_1+\theta_2)$$

$$(\cos\theta_1\cos\theta_2 - \mathrm{sen}\,\theta_1\,\mathrm{sen}\,\theta_2) + i(\cos\theta_1\,\mathrm{sen}\,\theta_2 + \mathrm{sen}\,\theta_1\cos\theta_2)$$

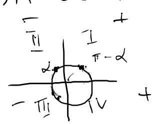
$$\Updownarrow$$

$$\begin{cases} \cos(\theta_1+\theta_2) = \cos\theta_1\cos\theta_2 - \mathrm{sen}\,\theta_1\,\mathrm{sen}\,\theta_2 \\ \mathrm{sen}(\theta_1+\theta_2) = \mathrm{sen}\,\theta_1\cos\theta_2 + \cos\theta_1\,\mathrm{sen}\,\theta_2. \end{cases}$$

$$0 < \theta_1, \theta_2, \theta_1+\theta_2 < \pi/2$$



demost. elemental.



	II
$\cos\alpha$	$-\cos(\pi-\alpha)$
	$-\cos(\alpha-\pi)$
	$\cos(2\pi-\alpha)$

3. Expresar los siguientes números complejos en la forma $a + ib$:

a) $(1+2i)^3$ b) $\dfrac{5}{-3+4i}$ c) $e^{i\pi}$ d) $e^{i\pi/2}$

e) $e^{i\pi/3}$ f) $e^{i2021\pi/3}$ g) $(3+4i)^{10}(3-4i)^{10}$

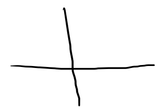
h) $(1+i)^{10}$

(En el último: conviene escribir primero $1+i$ en su forma polar).

h) ¿ sin sufrir usando el teo $\boxed{e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1+\theta_2)}}$

$$(1+i)^{10} = \left(\sqrt{2}\, e^{i\pi/4}\right)^{10} = (\sqrt{2})^{10}\, e^{i\pi/4 \cdot 10} =$$

$$= 32 \cdot \underbrace{e^{i5\pi/2}}_{i} = 32i$$



$$\frac{5\pi}{2} = \underline{\underline{2\pi + \pi/2}}$$

$$e^{i\frac{5\pi}{2}} = e^{i2\pi + i\pi/2} = \underbrace{e^{2\pi i}}_{1} \cdot \underbrace{e^{i\pi/2}}_{i} = i$$

$$(1+i)^{10} = \underset{1}{\underset{\shortparallel}{1^{10}}} + 10\cdot 1\cdot i^9 + \binom{10}{2} 1^2\, i^8 + \dots + i^{10}$$

g) $z\bar{z} = (a+ib)(a-ib) = a^2+b^2 = |z|^2$

$$(3+4i)^{10}(3-4i)^{10} = \left[(3+4i)(3-4i)\right]^{10} = (5^2)^{10} = 5^{20}.$$

2. Sea $z = x + iy \in \mathbb{C}$. Expresar la parte real e imaginaria de los siguientes números complejos en términos de x y y:

a) z^2 b) z^3 c) $\dfrac{1}{z}$ d) $\dfrac{z-1}{z+1}$

a) $(x+iy)^2 = x^2 - y^2 + i2xy$.

$\Rightarrow \operatorname{Im}(z^2) = 2xy$

$\operatorname{Re}(z^2) = x^2 - y^2$.

b) $(x+iy)^3 = x^3 + 3i x^2 y - 3xy^2 - i y^3$

$= (x^3 - 3xy^2) + 3i(x^2 y - y^3)$.

$i^3 = i^2 i = -i$

c) $\dfrac{1}{z} = \dfrac{\bar{z}}{z\bar{z}} = \dfrac{x-iy}{x^2+y^2} = \underbrace{\dfrac{x}{x^2+y^2}}_{Re} - i\underbrace{\dfrac{y}{x^2+y^2}}_{Im}$.