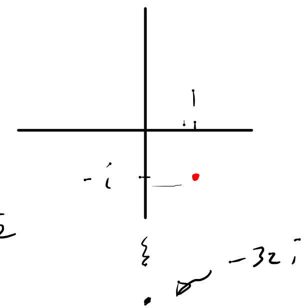
1c. forma polar de $(1-i)^{10}$ -

$$(1-i)^{10} = \left[\sqrt{2}\, e^{-i\pi/4}\right]^{10} = 2^5 e^{-i10\pi/4} = 32 e^{-i\pi/2} = 32\overbrace{e^{i\frac{3\pi}{2}}}^{-i}$$

$$= -32\, i$$

$$1-i = \sqrt{2}\, e^{-i\pi/4}$$

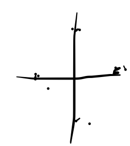
$$-\frac{10\pi}{4} = -\frac{5\pi}{2} \underset{\text{mod } 2\pi}{\equiv} -\frac{5+4}{2}\pi = -\frac{\pi}{2}$$



$2+3i$, $e^{i\pi} = e^{\pi i}$ (todo igual)

$\|$

$2+i3$



$$(1-i)^{10} = \sum_{k=0}^{10} \binom{10}{k} 1^k\, i^{10-k} = \sum_{k=0}^{10} \binom{10}{k} i^k$$

10 45 . . . .

=

$$= \underbrace{1+i-1-i}+\underbrace{1+i-1-i}+1+i = -32i$$

2 d $\frac{z-1}{z+1} = \frac{z+1-2}{z+1} = 1 - \frac{2}{z+1}$

$$\frac{1}{z+1} = \frac{1}{x+iy+1} = \frac{x+1-iy}{[(x+1)-iy][(x+1)+iy]} = \frac{x+1-iy}{(x+1)^2+y^2}$$

$$= \frac{x+1}{(x+1)^2+y^2} - i\frac{y}{(x+1)^2+y^2}$$

$$\frac{1}{z+1} = \frac{\bar{z}+1}{(z+1)(\bar{z}+1)} = \frac{\bar{z}+1}{z\bar{z}+z+\bar{z}+2} =$$

$$= \frac{1+\bar{z}}{|z|^2+2+2\,\mathrm{Re}(z)} \cdot = \frac{1+x}{2+2x+|z|^2} + i\frac{y}{---}.$$

$z+\bar{z} = 2\,\mathrm{Re}(z) = 2x.$

f) $e^{i2021\pi/3} = \cos\frac{2021\pi}{3} + i\,\text{sen}\,\frac{2021\pi}{3} = $ con calculadora

otro modo:

$$
\begin{array}{r|l}
12 & 2021 \\
 & 168 \\
\hline
-12 & \\
82 & \\
-72 & \\
101 & \\
-96 & \\
5 &
\end{array}
$$

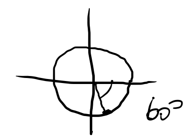
$\Rightarrow \boxed{2021 = 12\cdot(168) + 5}$

$\Rightarrow \frac{2021\pi}{3} = \frac{(12\cdot 168 + 5)\pi}{3} = 4\pi\cdot 168 + \frac{5\pi}{3}$

$\Rightarrow e^{i\frac{2021\pi}{3}} = \underbrace{\left(e^{i4\pi}\right)^{168}}_{1}\cdot e^{i5\pi/3} = e^{i\frac{5\pi}{3}}$

$$
\begin{array}{r|l}
6 & 2021 \\
 & 336 \\
\hline
-18 & \\
22 & \\
-18 & \\
41 & \\
-36 & \\
5 &
\end{array}
$$

$\frac{5\pi}{3} = \pi + \frac{2\pi}{3} \underset{2\pi}{\equiv} \frac{2\pi}{3} - \pi = -\frac{\pi}{3}$



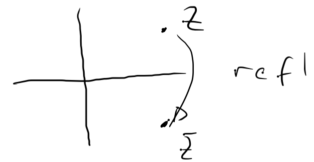
$e^{i\theta} = e^{i\theta_1}$ $\qquad \theta_1 \in [-\pi, \pi]$

$\theta \equiv \theta_1 \mod 2\pi$.

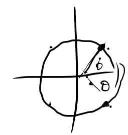
Resp. $e^{-i\pi/3} = \frac{1}{2} - i\frac{\sqrt{3}}{2}$. $\qquad \cos 60° = 1/2$,

$z = re^{i\theta}$

$\Rightarrow \bar{z} = r\overline{e^{i\theta}} = re^{-i\theta}$



dem. visual

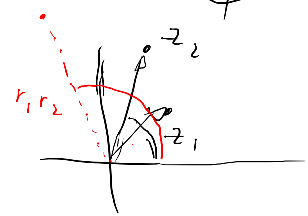


Interpretación geométrica de mult. de núm. complejos

en el "argumento".

$z_1 = r_1 e^{i\theta_1}$

$z_2 = r_2 e^{i\theta_2}$



$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

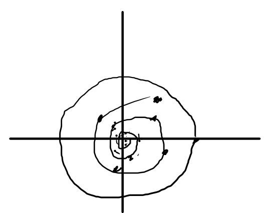
$z_1 z_2$ tiene valor abs. el prod. de los val. abs.,

y argumento que es la suma de los argumentos.

$1, z, z^2, z^3, \ldots$

si $|z|<1$

$z = re^{i\theta}$, $0<r<1$.



Teoremas de trigonometria

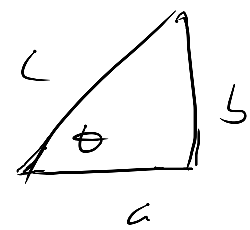
$\operatorname{sen}^2\theta + \cos^2\theta = 1$ (pitagoras)

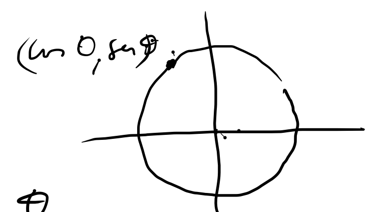
$\Vert$ $\Updownarrow$

$\left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2 = 1$

$\Updownarrow$

$a^2 + b^2 = c^2.$





Es cierto en general.

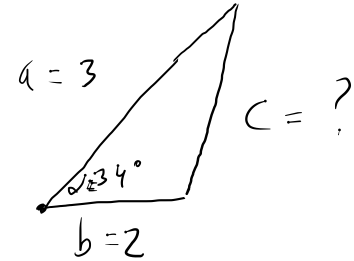
Conseq. de Euler $e^{i\theta} = \cos\theta + i\operatorname{sen}\theta$

Ley de cosenos

si $\alpha = 90^\circ$, $c^2 = a^2 + b^2$,

en general,

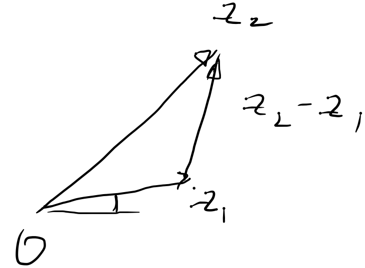
$$c^2 = a^2 + b^2 - 2ab\cos\alpha$$



$$|z_2 - z_1|^2 = (z_2 - z_1)(\bar{z}_2 - \bar{z}_1) =$$

$$= |z_1|^2 + |z_2|^2 - z_1\bar{z}_2 - z_2\bar{z}_1$$

$$= |z_1|^2 + |z_2|^2 - (\underbrace{z_1\bar{z}_2}_{w} + \underbrace{\bar{z}_1 z_2}_{\bar{w}})$$



$z_1 = |z_1| e^{i\theta_1}$, $z_2 = |z_2| e^{i\theta_2}$

$w = a + ib$
$\bar{w} = a - ib$

$$z_1\bar{z}_2 = |z_1||z_2| e^{i(\theta_1 - \theta_2)} =$$

$w + \bar{w} = 2\,\mathrm{Re}(w)$

$$= |z_1||z_2|\, 2\,\mathrm{Re}\left[e^{i(\theta_1-\theta_2)}\right] =$$

$$= 2|z_1||z_2| \cos(\underbrace{\theta_1 - \theta_2}_{\text{el ángulo entre } z_1,\, z_2})$$

• Otro ejemplo cómo usar núm. complejos en trigonometría.

$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = 1 - 2\operatorname{sen}^2\alpha$.   conocidas

$\operatorname{sen} 2\alpha = 2 \operatorname{sen}\alpha \cos\alpha$.

• Ej: expresar $\cos 5\alpha$ en términos de $\operatorname{sen}\alpha, \cos\alpha$

tediosa! $\cos(4\alpha + \alpha) = (\cos 4\alpha)(\cos\alpha) - (\operatorname{sen} 4\alpha)(\operatorname{sen}\alpha)$; $\cos 4\alpha = \cos(3\alpha+\alpha) = \ldots$ etc.

$$\cos 5\alpha = \operatorname{Re} e^{i5\alpha}$$

$$e^{i5\alpha} = (e^{i\alpha})^5 = (\underset{\overset{\shortparallel}{c}}{\cos\alpha} + i\,\underset{\overset{\shortparallel}{s}}{\operatorname{sen}\alpha})^5 = (c+is)^5 = c^5 + \binom{5}{2} c^3 s^2 i^2 + \binom{5}{4} c\,(i^4)\,s^4$$

$$= c^5 - 10c^3 s^2 + 5cs^4 =$$

$$= c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2 = \ldots$$

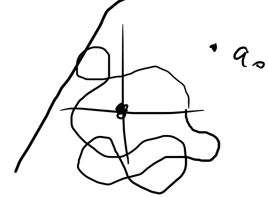
El teorema más importante acerca de los números complejos:

$\boxed{\text{"el teorema fundamental álgebra":}}$

Con núm. complejos se puede resolver $z^2+1=0$ $\quad (z=\pm i)$.

Toda ecuación polinomial $p(z)=a_n z^n + a_{n-1} z^{n-1} + \dots + a_0 = 0$ con coef. complejos $a_0, a_1, \dots, a_n \in \mathbb{C}$, tiene raíz (=solución) compleja.

Es un teorema de topología. $z = re^{i\theta}$



cuando $r >> 1$ $\quad \frac{p(z)}{a_n} \approx z^n = r\underline{\underline{e^{ni\theta}}}$ $\sim$ deformar $r \to 0$

Gauss, ~1820.

Topología: se estudia propiedades de figuras y funciones que no cambian bajo "cambios continuos".