Seminario de geometría diferencial y sistemas dinámicos, CIMAT

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Título: The Geometry of the Fibers of an Analytic Function

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Abstract. Given a simple space, like a ball B in $\mathbb{R}^n$, and an analytic function $f : B \to \mathbb{R}$, the fibers $F_t := f^{-1}(t)$ define a partition $B := \bigcup_t F_t$. The general fibre (over a regular = non-singular value) is an $(n-1)$-dimensional manifold. The fibers F_t have non-trivial topology and geometry. Over intervals $(a, b) \subset \mathbb{R}$ formed by regular values, the fibers have (locally) a topological product structure (but not differential geometric). It is interesting to understand what happens to this product structure when we approach a singular value, say F_b. Part of the topology is going to contract to the singular points of F_b. We measure the topology of F_t by its homology groups $H_j(F_t, \mathbb{Z})$, in particular, its ranks, the Betti numbers $\beta_j(t)$. The homology which gets contracted is called the *vanishing homology* $H^{van}(F_t, \mathbb{Z})$.

It is a remarkable fact that the vanishing homology is filtered naturally by the 'order' of vanishing, i.e. the homology vanishes at different speeds as $t \to b$:

$$H^{van}(F_t, \mathbb{Z}) \supset H^{van,r_1}(F_t, \mathbb{Z}) \supset H^{van,r_2}(F_t, \mathbb{Z}) \supset \cdots \supset H^{van,r_s}(F_t, \mathbb{Z}).$$

This gives rise to vanishing Betti numbers with speed:

$$\beta_i(t) > \beta_i(t)^{r_1} > \beta_i(t)^{r_2} > \cdots > \beta_i(t)^{r_s}.$$

As s increases from b into another interval of regularity, (b, c), we get a similar filtration

$$H^{van}(F_s, \mathbb{Z}) \supset H^{van,r_1}(F_s, \mathbb{Z}) \supset H^{van,r_2}(F_s, \mathbb{Z}) \supset \cdots \supset H^{van,r_s}(F_s, \mathbb{Z})$$

and vanishing Betti numbers with speed:

$$\beta_i(s) > \beta_i(s)^{r_1} > \beta_i(s)^{r_2} > \cdots > \beta_i(s)^{r_s}.$$

In this talk I will introduce these topics, and, of course, restricted to the case of an isolated singularity, in a small neighborhood of the singular point and extending to the complex numbers, with the real case being the fixed points of the involution given by complex conjugation.