EXERCISE SESSION XII

Exercise I
Prove that any continuous map from a connected space X to a space Y endowed with the discrete topology is constant.

Exercise II
Prove that a topological space X is connected if and only if every continuous function $f : X \to \{0, 1\}$ is constant, where $\{0, 1\}$ is endowed with the discrete topology.

Exercise III
Consider two topological spaces (X, τ_X) and (Y, τ_Y). Let A be a subspace of X and B a subspace of Y. Prove that the product topology on $A \times B$ coincides with the subspace topology ihnerited from $X \times Y$.

Exercise IV
Show that the product of two Hausdorff spaces is Hausdorff.