EXERCISE SESSION X

Exercise I
Let $f, g : X \to Y$ be two continuous functions between two topological spaces, such that Y is Hausdorff. Let D be a dense set in X (namely, $\overline{D} = X$). Prove that the following conditions are equivalent

- The functions f and g coincide in X.
- The functions f and g coincide in D.

Exercise II
Let (X, τ) be a topological space and denote by Δ_X denote the diagonal set

$$\Delta_X := \{(x, x) \ ; \ x \in X\}.$$

Prove that X is Hausdorff if and only if Δ_X is a closed subset of $X \times X$.

Exercise III
Prove that if $f : \mathbb{S}^1 \to \mathbb{R}$ is a continuous map (where $\mathbb{S}^1$ denotes the unit circle in $\mathbb{R}^2$), then there exists a point $p \in \mathbb{S}^1$ such that $f(p) = f(-p)$.