# EXERCISES FOR WEEK VIII

**Exercise I**
Let $X$ be the space $\{(x, y) \in \mathbb{R}^2 \mid x \in \mathbb{Q} \text{ or } y \in \mathbb{Q}\}$ considered as a subspace of $\mathbb{R}^2$. Prove that $X$ is connected.

**Exercise II**
Let $X := \mathbb{R}$ be endowed with the topology $\tau_{low}$ generated by the basis $\mathcal{B}_{low} := \{[a, \infty) \ ; \ a \in \mathbb{R}\}$. Prove or disprove the claim "$X$ is connected".

**Exercise III**
Let $X$ be a set and let $\tau$ and $\tau'$ be two topologies such that $\tau \leq \tau'$. If $(X, \tau)$ is connected, can we deduce that $(X, \tau')$ is also connected? what about the converse implication?