

## EXERCISE SESSION IX

### Exercise I

Let  $X$  be a set and  $A \subset X$ . Recall that  $\tau := \{Y \subset X ; A \subset Y\}$  is a topology on  $X$ . Characterize the compact subspaces of  $X$  (hint: finite subspaces of  $X \setminus A$  play a key role).

### Exercise II

Let  $X$  be an infinite set with cofinite topology. Prove that  $X$  is compact.

### Exercise III

Let  $(X, \tau)$  be a topological space. Prove the following conditions are equivalent:

- (1)  $X$  is compact.
- (2) Any collection  $H$  of subsets of  $X$  with the finite intersection property has non-empty intersection.