# NON-MANDATORY EXERCISES FOR WEEK VI

**Exercise I**
Let $(X, \leq)$ be a totally ordered set.

(i) Prove that the family made of the finite intersections of the sets

$$(a, \infty) := \{x \in X | a < x\}$$

and

$$(-\infty, b) := \{x \in X | x > b\},$$

where $a, b \in X$ is a basis of topology (which is called the order topology).

(ii) What is the order topology on $\mathbb{N}$? What is the order topology on $\mathbb{R}$?

(iii) What are the isolated points of $\{0,1\} \times \mathbb{N}$, equipped with the order topology relatively to the lexicographic order?

(iv) Compare the Euclidean topology on $\mathbb{R}^2$ with the order topology relatively to the lexicographic order on $\mathbb{R}^2$ (hint: give a basis of topology).

(v) Let $Y := [0, 4) \cup \{5\}$. Is 5 an isolated point in $Y$ if $Y$ is considered as a subspace of $\mathbb{R}$ equipped with the Euclidean topology? Same question if $Y$ is equipped with the order topology relatively to the restriction on $Y$ of the natural order on $\mathbb{R}$?

(vi) Let $Y := [0, 1 \times [0, 1]]$. Is $A = \{1/2\} \times (1/2, 1]$ open in $Y$ if $Y$ is considered as a subspace of $\mathbb{R}^2$ equipped with the euclidean topology? Same question if $Y$ is equipped with the order topology relatively to the lexicographic order on $Y$. Same question if $Y$ is considered as a subspace of $\mathbb{R}^2$ equipped with the order topology relatively to the lexicographic order on $\mathbb{R}^2$.

**Exercise II**
Let $\lfloor \cdot \rfloor : \mathbb{R}_+ \to \mathbb{N}$ be the floor function, that is, the function that maps $x$ to its integer part. Assume that $\mathbb{N}$ is equipped with the cofinite topology. What is the smallest topology on $\mathbb{R}_+$ that turns $\lfloor \cdot \rfloor$ to a continuous function (this topology is called the initial topology relatively to $\lfloor \cdot \rfloor$)?