NON-MANDATORY EXERCISES FOR WEEK VII

Exercise I
Let X and Y be topological spaces and $x \in X$. Prove that the map $i_x : Y \to X \times Y$ defined as $i_x(y) = (x, y)$ is an homeomorphism from X to $\{x\} \times Y$.

Exercise II
Let $f : [0, 1]^2 \in [1, e]^2$ be the map defined by $f(x, y) = (e^x, e^y)$ for every $x, y \in [0, 1]$. Prove that f is an homeomorphism.

Exercise III
Let A, B be two subspaces of a topological space X. Prove that the following identities hold.

(i) $(A \times B)^- = \overline{A} \times \overline{B}$
(ii) $(A \times B)^\bullet = (A^\bullet \times B) \cup (A \times B^\bullet)$.