EXERCISE SESSION IV

Exercise I
Prove that $A^{\circ-} = A^{\circ-\circ-}$ and $A^{-\circ} = A^{-\circ-\circ}$.

Exercise II
Prove that $\bar{A} \cup B^{\circ} = \bar{A} \cup (A \cup B)^{\circ}$.

Exercise III
Let A, B be two subspaces of a topological space X. Show that if A and B are two disjoint closed subspaces of $A \cup B$, then $\bar{A} \cap B = A \cap \bar{B}$.

Exercise IV
Let X be a nonempty set and $Cl : 2^X \to 2^X$ be a map with the following properties

(1) $Cl(\emptyset) = \emptyset$,
(2) $A \subset Cl(A)$,
(3) $Cl(A \cup B) = Cl(A) \cup Cl(B)$,
(4) $Cl(Cl(A)) = Cl(A)$,

for every $A, B \in 2^X$. Prove the following

(1) If $A \subset B$, prove thet $Cl(A) \subset Cl(B)$
(2) Prove that $\tau = \{X \backslash F | Cl(F) = F\}$ is a topology on X.
(3) Prove that the closure $\bar{A}$ of a subspace A of (X, τ) is equal to Cl(A).