

Exercise session I
(problems taken from the exercise sheet from Bruno Teheux)

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Exercise I

Give all possible topologies on a three element set.

Exercise II

Let τ be the collection of sets

$$\tau := \{\emptyset, \mathbb{R}\} \cup \{(a, \infty) ; a \in \mathbb{R}\}.$$

Prove that τ is a topology over the set X .

Exercise III

Let X be a set and $A \subset X$. Prove that

$$\tau := \{\emptyset\} \cup \{\Omega \subset X ; A \subset \Omega\}$$

is a topology over X .

Exercise IV

State the definition of topology in terms only of closed sets (justify your answer).

Exercise V

Let τ be the collection of subsets of \mathbb{R} of the form

$$\tau = \{\emptyset, \mathbb{R}\} \cup \{(q, \infty) ; q \in \mathbb{Q}\}.$$

Prove or disprove: τ is a topology over \mathbb{R} .