Spectral Analysis of Storm Waves using the SLEX algorithm and the Hilbert-Huang Transform

José B. Hernández Ch. *  J. Ortega †  George H. Smith ‡

Abstract

In this work we perform a comparative study of storm wave spectra using the Hilbert-Huang Transform and the Smooth Localized complex EXponential (SLEX) algorithm. This last algorithm divides the data set into approximately stationary sections by detecting changes in their spectra. We compare the spectra produced by both algorithms and also look at the behaviour of the Hilbert spectrum around the change points detected by SLEX. Finally, we look at large waves (higher than 6.5 m.) and their relation with the Hilbert spectrum. The data analyzed come from a North Sea storm. The results show that both methods can improve the information available for the understanding of time varying wave fields. The ability to produce stable, high resolution spectra for storm conditions is of considerable interest in the design of floating offshore structures, particularly wave energy converters, which may require tuning to provide greater efficiency of wave energy conversion.

Key words: SLEX, HHT, Hilbert spectrum, marginal Hilbert spectrum, Segmentation.

1 Introduction

Wave characteristics can change abruptly during a storm and also exhibit considerable non-linear effects, so a first approximation for building random wave models is to assume that the sea surface height, measured at a fixed point is a piecewise stationary processes, i.e. there exist instants at which the ‘state’ of the waves changes, but between two consecutive change-points sea surface height follows a stationary process. An advantage of this approach is that classical spectral analysis can be used for each stationary interval with the usual interpretation for the spectrum as the energy distribution in a range of frequencies. To implement this approach a method for detecting changes in the ‘state’ of the process is required and since the spectral characteristics and covariance structure of the process will change from one state to the next, it is reasonable to search for methods based on changes in the spectrum.

In this work we will use the SLEX (Smooth Localized complex EXponential) algorithm proposed by Ombao et al. [8] to detect changes in a storm data set that will be described in the next section. We will also use the Hilbert-Huang transform (HHT) [7] to analyze this data set and to obtain both the Hilbert and marginal Hilbert spectra for the intervals obtained with the SLEX procedure. The first method was originally employed for the analysis of non-stationary time series and the detection of their change points while the second method is useful for the analysis of non-linear signals, using a decomposition into a series of components of different frequencies for which the Hilbert transform is used to obtain he spectrum associated to the original series. Since sea waves are often non-stationary, non-linear time series we will use both algorithms. SLEX will be used for change-point detection and both algorithms will be used for the spectral analysis of the intervals obtained, so as to compare the results. These algorithms are independent of each other.

*Universidad Central de Venezuela, Caracas, Venezuela; email: jbhernandez@cantv.net
†Centro de Investigación en Matemáticas (CIMAT, A.C.), Guanajuato, Gto, México; email: jortega@cimat.mx
‡Exeter University, Cornwall Campus, UK; email: G.H.Smith@exeter.ac.uk
This work is structured as follows: In section 2 we describe the data considered in this work; in section 3 a brief description of the SLEX algorithm is given. In section 4 the Hilbert-Huang transform is described, introducing first the empirical mode decomposition (EMD) and then the Hilbert spectral analysis. Section 5 is devoted to the analysis of the storm data set using first the SLEX algorithm to obtain a segmentation of the storm and then both the SLEX and HHT algorithms for the spectral analysis. Afterwards an analysis of important events (Waves higher than 6.5 m.) is carried out. Finally the conclusions from this work are presented.

2 Data

Data was recorded from the North Alwyn platform situated in the northern North Sea, about 100 miles east of the Shetland Islands (60°48.5' North and 1°44.17' East) in a water depth of approximately 130 metres. There are three Thorn EMI Infra-red wave height meters mounted on the platform and their heights are between 25 and 35 metres above the water. The data are recorded continuously and simultaneously at 5Hz and then divided into 20 minute records for which the summary statistics of $H_s$, $T_p$ and the spectral moments are calculated. For data with $H_s > 3$ m all the surface elevation records are kept. Further details are available in [11]. Only data from the North East altimeter are used here.

The present data consists of a series of records of 20 minutes duration, sampled by the altimeter at a rate of 5 Hz,. The measurements were recorded between midnight on December 23rd and 9.00 a.m. on December 26th 1999 and consisted of 244, 20 minute, records. Figure 1 shows the evolution of significant wave height and peak period during the storm. This data starts at a high level with a significant wave height of between 6.5 and 7 m and then drops down to about 3.5 m before increasing back up to around 7 m for around 20 hours. It then reduces again, before increasing to about 6.5 m, dropping to about 5 and then increasing again to around 5.5 m before finally dropping to less than 3.5 m at the end of the dataset. As such this data includes two relative large increases in $H_s$ and a section in which two peaks occur within a relatively short time period.

![Figure 1: Significant wave height during the storm.](image)

Since there were some short intervals missing in the data, it was divided into five sets that cover the storm. In Table 1 we give a list of the five sets along with some basic characteristics of the wave records: Significant wave height $H_s$, mean wave period $T_{m01}$, spectral peak period $T_p$ and spectral bandwidth parameter $\nu$.

3 The SLEX Algorithm

The auto-SLEX algorithm is a statistical procedure that automatically divides time series in segments that are approximately stationary and automatically chooses a smoothing parameter for the estimation
of the spectrum that changes with time. The method is based on the SLEX (Smooth Localized complex EXponential) transform, which uses the SLEX vectors which are closely related to the classical Fourier transform. The method is presented in [8] and we follow here their presentation. The algorithms have been implemented in Matlab and are available in the web-page www.stat.uiuc.edu/~ombao.

As is well-known, Fourier functions are adequate for representing stationary random processes, since they are localized in frequency and the spectral properties of stationary processes are time-invariant, but they cannot represent, accurately, processes with time-evolving spectral properties. To tackle the time localization problem, smooth compactly supported windows have been applied, but the functions resulting are no longer orthogonal. It is well-known that there does not exist a smooth window such that the windowed Fourier basis vectors are both orthogonal and localized in time and frequency. The SLEX functions avoid this problem using a projection operator, instead of a window, on the complex exponentials. The action of the projection operator on a periodic function is equivalent to applying two especially constructed smooth windows to the Fourier basis functions.

The functions on the SLEX basis \( \phi_{\omega}(u) \) are of the form

\[
\phi_{\omega}(u) = \Psi_{+}(u) \exp(i2\pi \omega u) + \Psi_{-}(u) \exp(-i2\pi \omega u),
\]

where \( \omega \in [-\frac{1}{2}, \frac{1}{2}] \) and \( \Psi_{+}(u), \Psi_{-}(u) \) are specific smooth real valued functions that will be defined later. The SLEX basis functions have support on \([-\delta, 1 + \delta]\), where \( 0 < \delta < 0.5 \). Thus SLEX functions at different dyadic blocks overlap but they remain orthogonal (see figure 2 top).

The SLEX basis functions generalize directly to an orthogonal SLEX basis vectors for representing time series. Let \( a_0 < a_1 \) be two integer time points, \(|S| = a_1 - a_0\) and the overlap \( \epsilon = |\delta|S| \), where \([\cdot]\) denotes the integer part. The support \( S \) of SLEX vectors on block \( S \) consists of time points defined on \( S \) and the overlap: \( S = \{a_0 - \epsilon, \cdots, a_0, \cdots, a_1 - a_0, a_1 - 1 + \epsilon, \epsilon\} \). A SLEX basis vector defined on block \( S \) has elements \( \{\phi_{S,\omega_k}(t)\} \) with

\[
\phi_{S,\omega_k}(t) = \phi_{\omega_k}((t - a_0)/|S|)
\]

\[
= \Psi_{S,+}((t - a_0)/|S|) \exp(i2\pi \omega_k(t - a_0))
\]

\[
+ \Psi_{S,-}((t - a_0)/|S|) \exp(-i2\pi \omega_k(t - a_0))
\]

where \( \omega_k = k/|S|, k = -\lfloor |S|/2 \rfloor, \cdots, \lfloor |S|/2 \rfloor \). The windows can be represented in terms of a rising cut-off function \( r \):

\[
\Psi_{S,+}(t) = r^2 \left( \frac{t - a_0}{\epsilon} \right) \exp \left( \frac{a_1 - t}{\epsilon} \right)
\]

\[
\Psi_{S,-}(t) = r \left( \frac{t - a_0}{\epsilon} \right) \exp \left( \frac{a_0 - t}{\epsilon} \right) - r \left( \frac{t - a_1}{\epsilon} \right) \exp \left( \frac{a_1 - t}{\epsilon} \right)
\]

In the specific implementation we use \( r \) is

\[
r(u) = \text{sen}(\frac{\pi}{4}(1 + u))
\]

where \( u \in [-1, 1] \).

<table>
<thead>
<tr>
<th></th>
<th>Duration</th>
<th>( H_m )</th>
<th>( T_{m01} )</th>
<th>( T_p )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storm1999a</td>
<td>8h, 40m</td>
<td>5.34</td>
<td>9.31</td>
<td>11.87</td>
<td>0.509</td>
</tr>
<tr>
<td>Storm1999b</td>
<td>6h</td>
<td>3.72</td>
<td>8.48</td>
<td>11.22</td>
<td>0.514</td>
</tr>
<tr>
<td>Storm1999c</td>
<td>18h</td>
<td>5.07</td>
<td>8.25</td>
<td>10.50</td>
<td>0.510</td>
</tr>
<tr>
<td>Storm1999d</td>
<td>24h</td>
<td>5.87</td>
<td>8.99</td>
<td>11.70</td>
<td>0.492</td>
</tr>
<tr>
<td>Storm1999e</td>
<td>24h</td>
<td>5.10</td>
<td>8.75</td>
<td>11.70</td>
<td>0.506</td>
</tr>
</tbody>
</table>

Table 1: Basic characteristics of the 5 data intervals.
Auto-SLEX also uses the Best Basis Algorithm (BBA) of Coifman and Wickerhauser [2] to choose the best segmentation using a cost function defined in terms of logarithms of the SLEX periodograms. The cost of a given configuration is determined using a complexity penalized Kullback-Leibler criteria

\[
\text{Cost}(j,b) = \sum_{k=-M_{j}+1}^{M_{j}} \log \det(I_{j,b,k}) + \beta(j,b)\sqrt{M_{j}}
\]

where \(M_{j}\) is the block length at level \(j\), \(M_{j} = T/2^{j}\), \(T\) is the data vector length and \(b\) is the block index \(b = 0, 1, \ldots, 2^{j} - 1\).

First, the spectrum using the SLEX basis for the whole set is calculated, then the set is divided in two and the SLEX spectrum calculated for each half. The cost of each configuration is calculated and the algorithm chooses the lower cost configuration. This procedure goes on until one arrives at the best configuration in terms of lower cost or the minimum size for the intervals is reached. The SLEX spectrum is calculated using the FFT algorithm and the set of data must have length a power of 2. Since the subintervals are obtained by successive divisions in two of the initial set, the length of all intervals obtained is also a power of 2, and their endpoints are sums of powers of 2. Since there is a minimum size for the intervals, related to the smallest set of data required to have a good estimation of the spectra, there is a limit to the precision with which the algorithm can detect the change-points of a time series. This is a shortcoming of the SLEX algorithm. More details can be seen in Ombao et al. (2002).

Figure 2 (bottom) shows a subdivision of an interval for \(j = 2\), where the light-colored blocks have lower cost and the dark ones are not kept in the final configuration.

Figure 2: Top: SLEX functions. Bottom: Division of the data in blocks.

4 HHT Analysis

The Hilbert-Huang transform (HHT) was proposed by Huang et al. [7, 5, 6] as an adequate method for the spectral analysis of non-stationary, non-linear processes. We give a brief description of HHT. A detailed presentation can be found in the original articles of Huang et al. [7, 5] as well as in Huang [4, 3].

The Hilbert Huang Transform is based on an empirical algorithm called the Empirical Mode Decomposition (EMD), used to decompose a time series into individual characteristic oscillations known as the Intrinsic Mode Functions (IMF). This technique is based on the assumption that any signal consists of different modes of oscillation based on different time scales, so that each IMF represents one of these embedded oscillatory modes. Each IMF has to satisfy two criteria: 1) The number of local
extreme points and of zero-crossings must either be equal or differ at most by one, 2) At any instant, the mean of the envelope defined by the local maxima and the envelope corresponding to the local minima must be zero. These two conditions are required to avoid inconsistencies in the definition of the instantaneous frequency.

An Intrinsic Mode Function represents a simple oscillatory mode analogous to a simple harmonic function, but more general: whereas the latter has fixed amplitude and frequency, an IMF can have time-dependent amplitude and frequency.

Once the signal is decomposed, the Hilbert Transform is applied to each IMF. The Hilbert transform \( y(t) \) of a function \( x(t) \) is defined as \( \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{t-s} x(s) \, ds \) (7) where the integral is taken as the Cauchy principal value. Then, if \( z(t) \) is the analytical signal associated to \( x(t) \), we have for all \( t \),

\[
z(t) = x(t) + iy(t) = A(t) \exp(i\theta(t))
\]

with \( A(t) = \sqrt{x^2(t) + y^2(t)} \) and \( \theta(t) = \arctan(y(t)/x(t)) \). The instantaneous frequency is defined now as the derivative of the phase function of the analytical signal \( z(t) \):

\[
\omega(t) = \frac{d\theta(t)}{dt}
\]

(9)

Once the signal has been decomposed into IMFs and the Hilbert transform for each has been obtained, the signal \( x(t) \) can be represented as

\[
x(t) = \text{Re} \left[ \sum_{j=1}^{n} A_j(t) \exp \left( i \int_{0}^{t} \omega_j(t) \, dt \right) \right]
\]

(10)

where \( \text{Re} \) denotes the real part, which is a generalized form of the Fourier expansion for \( x(t) \) in which both amplitude and frequency are functions of time.

The time-frequency distribution of the amplitude or the amplitude squared is defined as the Hilbert amplitude spectrum or the Hilbert energy spectrum, respectively. For these spectra the time resolution can be as precise as the sampling rate of the data (Huang et al. [9]). The lowest frequency that can be obtained is \( 1/T \), where \( T \) is the duration of the record, and the highest is \( 1/n\Delta t \), where \( n \) is the minimal number of data points needed to obtain the frequency accurately (usually \( n = 5 \)) and \( \Delta t \) is the sampling rate (Huang et al. [1]). The corresponding marginal Hilbert spectrum is defined as

\[
h(\omega) = \int_{0}^{T} H(\omega, t) \, dt.
\]

(11)

5 Data Analysis

5.1 Change-point detection using SLEX

The SLEX algorithm was used on each data set to determine the change points for the SLEX spectra. Several parameter values, starting with the values set by default and based on previous experience with the algorithm, were used until the best segmentation was obtained for each data set. The values considered for the penalization parameter were 2; 1.5; 1; 0.5; 0.25; 0.125; 0.1; 0.05 and 0.025 and for the smoothing parameter 0.1; 0.02; 0.015; 0.01; 0.005 and 0. The values finally chosen were 0.25 and 0, respectively, since they gave the division that seemed most adequate in terms of the number of segments and their mean length. The intervals obtained had a mean duration between 16 and 20 minutes, as can be seen in Table 3, while for other test values the number of intervals resulted too large, with a mean duration of around 3.5 min. which was deemed too small even for a storm.
Since the SLEX algorithm works with data sets with length a power of 2, two situations were considered: 1) Each data set was truncated to the largest power of two less than or equal to the total data length and 2) each data set was completed with enough 0’s at the end to reach the smallest power of two that is larger than the total data length. In Table 2 the lengths of both data sets are given as well as the powers of two corresponding to the truncated and completed data sets.

<table>
<thead>
<tr>
<th>Set</th>
<th>Duration</th>
<th>Length (Nr. points)</th>
<th>Truncated Set</th>
<th>Completed Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>storm1999a</td>
<td>8h.40m</td>
<td>156000</td>
<td>$2^{17} = 131072$</td>
<td>$2^{18} = 262144$</td>
</tr>
<tr>
<td>storm1999b</td>
<td>6h</td>
<td>108000</td>
<td>$2^{16} = 65536$</td>
<td>$2^{17} = 131072$</td>
</tr>
<tr>
<td>storm1999c</td>
<td>18h</td>
<td>324000</td>
<td>$2^{18} = 262144$</td>
<td>$2^{19} = 524288$</td>
</tr>
<tr>
<td>storm1999d</td>
<td>24h</td>
<td>432000</td>
<td>$2^{18} = 262144$</td>
<td>$2^{19} = 524288$</td>
</tr>
<tr>
<td>storm1999e</td>
<td>24h</td>
<td>432000</td>
<td>$2^{18} = 262144$</td>
<td>$2^{19} = 524288$</td>
</tr>
</tbody>
</table>

Table 2: Length of data sets.

After using both alternatives it was observed that in the intersection of both sets the change-points coincide. Figure 3 shows the change points obtained with SLEX; the upper half shows the change-points for the completed set while the lower part corresponds to the truncated set for the data in Storm1999c. Since the truncated sets cover only part of the data we always use the results for the completed sets.

The maximum number of dyadic divisions for the different data sets was 7 for storm1999a and storm1999b and 9 for the other three. The reason for this is that higher values produce intervals with less than 1024 data points, for which the spectral estimation is very imprecise.

Figure 3: SLEX change-points for Storm1999c.

At the end of the original data set, where the completion with 0’s begins, SLEX produces spurious change-points that reflect the change from data to a constant, as can be seen in Figure 4, hence the last change-point considered is the one previous to the end of the data set.

For the first data set Storm1999a 30 change-points were obtained; for the second Storm1999b, 24; for the third Storm1999c, 60, for the fourth Storm1999d and the fifth Storm1999e, 72 for each. Table 3 gives some basic statistics for the length of the intervals obtained for each data set.

Figure 5 shows boxplots for interval length for each data set.

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Figure 4: Change-points at the end of the storm1999a data set.

<table>
<thead>
<tr>
<th>Data set</th>
<th>storm1999a</th>
<th>storm1999b</th>
<th>storm1999c</th>
<th>storm1999d</th>
<th>storm1999e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>00:03:04.04</td>
<td>00:03:04.04</td>
<td>00:03:04.04</td>
<td>00:03:04.04</td>
<td>00:03:04.04</td>
</tr>
<tr>
<td>1st. quartile</td>
<td>00:06:08.08</td>
<td>00:06:08.08</td>
<td>00:03:04.04</td>
<td>00:06:08.08</td>
<td>00:06:08.08</td>
</tr>
<tr>
<td>Mean</td>
<td>00:18:17.17</td>
<td>00:15:52.52</td>
<td>00:18:09.09</td>
<td>00:20:09.09</td>
<td>00:20:10.10</td>
</tr>
<tr>
<td>3rd. quartile</td>
<td>00:27:32.32</td>
<td>00:13:16.16</td>
<td>00:13:16.16</td>
<td>00:27:32.32</td>
<td>00:27:32.32</td>
</tr>
<tr>
<td>Maximum</td>
<td>00:54:04.04</td>
<td>00:54:04.04</td>
<td>01:49:08.08</td>
<td>01:49:08.08</td>
<td>01:49:08.08</td>
</tr>
<tr>
<td>Variance</td>
<td>00:00:08.08</td>
<td>00:00:08.08</td>
<td>00:00:29.29</td>
<td>00:00:25.25</td>
<td>00:00:26.26</td>
</tr>
</tbody>
</table>

Table 3: Basic statistics for interval length (min).

Figure 5: Boxplot for interval length obtained with the SLEX algorithm.

5.2 SLEX and HHT Spectral Analysis

In this section we look at the energy spectra obtained with both algorithms with the purpose of comparing the results and seeing what conclusions can be drawn from the application of each to the analysis of the data. The HHT method was used on all 5 data sets using the HHT-DPS software developed by NASA. Further details on this implementation of HHT and the settings for its use can be seen in [10]. The number of IMFs obtained varies for the different data sets and seems to increase with the length. As was remarked in [10] this may be due to the fact that a longer time interval allows for the detection of lower frequencies.
<table>
<thead>
<tr>
<th>Data set</th>
<th>Duration</th>
<th>Num. IMF’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storm1999a</td>
<td>8h. 40m.</td>
<td>17</td>
</tr>
<tr>
<td>Storm1999b</td>
<td>6h.</td>
<td>15</td>
</tr>
<tr>
<td>Storm1999c</td>
<td>18h.</td>
<td>19</td>
</tr>
<tr>
<td>Storm1999d</td>
<td>24h.</td>
<td>21</td>
</tr>
<tr>
<td>Storm1999e</td>
<td>24h.</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 4: Number of IMFs for each data set.

For each of the five data sets the marginal Hilbert spectrum was calculated. Further, for each segment obtained with the SLEX algorithm, both the SLEX spectrum and the classical Fourier spectrum were calculated. The Fourier spectra were obtained using the WAFO software. Figure 7 shows the SLEX spectrogram for the Storm1999a data set and figure 6 gives the boxplots for the energy maxima obtained with the HHT algorithm for each SLEX segment. The SLEX spectrogram has a similar interpretation to the usual Fourier spectrogram.

Figure 6: Boxplots for maximal HHT energy in each segment for the five data sets.

We look now at the behaviour of the Hilbert spectrum around the change points detected using the SLEX algorithm. We looked both at the Hilbert spectrum and the marginal Hilbert spectrum in a neighbourhood of the change points, taking 300 data points (equivalent to 60 secs.) on each side of the change-point, for a total time of 2 min. We comment, using the results for the first data set Storm1999a. For most change-points there are noticeable changes in the energy. For 20 of the 28 change-points the changes are pronounced; in 8 of them the energy increases and in the remaining 12 it decreases. Figures 8 and 9 shows examples of both situations. For the other 8 change-points the change is not as pronounced, although changes in the total energy can be observed. An example of this is given in Figure 10

5.3 An analysis of results for 'Large' Waves

In this section we analyze the energy spectra and their evolution for large waves, i.e. for waves higher than 6.5 m. Table 5 gives the location of these events, their height and the total energy in a neighbourhood of the event comprising 150 data points, corresponding to 30 seconds, with the maximal point at the centre. The mean wave period is about 11 s.
Figures 11 to 15 give the Hilbert and marginal Hilbert spectra for some large waves. Figure 11 (top) corresponds to the Hilbert spectrum for the seventh SLEX interval of Storm1999a, which shows the energy evolution for the interval. One can see large amounts of energy near the endpoints of the interval located approximately at 82 and 95 min. and with energy reaching $0.6m^2/s/rad$ and $0.75m^2/s/rad$, respectively. These two events correspond to large waves 1 and 2 and are singled-out.
Table 5: Large waves

<table>
<thead>
<tr>
<th>Data set</th>
<th>Data point</th>
<th>Time (min)</th>
<th>Height (m)</th>
<th>Energy (HHT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999a</td>
<td>Wave 1</td>
<td>24705</td>
<td>82.3</td>
<td>6.93</td>
</tr>
<tr>
<td></td>
<td>Wave 2</td>
<td>25856</td>
<td></td>
<td>6.61</td>
</tr>
<tr>
<td>1999c</td>
<td>Wave 1</td>
<td>260365</td>
<td>867.9</td>
<td>6.62</td>
</tr>
<tr>
<td></td>
<td>Wave 2</td>
<td>273146</td>
<td>910.5</td>
<td>6.57</td>
</tr>
<tr>
<td></td>
<td>Wave 3</td>
<td>279587</td>
<td>932.0</td>
<td>6.94</td>
</tr>
<tr>
<td></td>
<td>Wave 4</td>
<td>316510</td>
<td>1055.0</td>
<td>7.66</td>
</tr>
<tr>
<td>1999d</td>
<td>Wave 1</td>
<td>22885</td>
<td>76.3</td>
<td>6.91</td>
</tr>
<tr>
<td></td>
<td>Wave 2</td>
<td>23765</td>
<td>79.2</td>
<td>6.88</td>
</tr>
<tr>
<td></td>
<td>Wave 3</td>
<td>24653</td>
<td>82.2</td>
<td>7.27</td>
</tr>
<tr>
<td></td>
<td>Wave 4</td>
<td>70298</td>
<td>234.3</td>
<td>6.69</td>
</tr>
<tr>
<td></td>
<td>Wave 5</td>
<td>74329</td>
<td>247.8</td>
<td>7.20</td>
</tr>
<tr>
<td></td>
<td>Wave 6</td>
<td>77015</td>
<td>256.7</td>
<td>8.59</td>
</tr>
<tr>
<td></td>
<td>Wave 7</td>
<td>89891</td>
<td>299.6</td>
<td>6.89</td>
</tr>
<tr>
<td></td>
<td>Wave 8</td>
<td>92022</td>
<td>306.7</td>
<td>6.99</td>
</tr>
<tr>
<td></td>
<td>Wave 9</td>
<td>171602</td>
<td>572.0</td>
<td>6.72</td>
</tr>
<tr>
<td>1999e</td>
<td>Wave 1</td>
<td>51695</td>
<td>172.3</td>
<td>6.69</td>
</tr>
</tbody>
</table>

in the bottom graphs. The total energy accumulated in a 30 second neighbourhood of large wave 1 is 45.39 m²/s/rad while for large wave 2 it is 33.58 m²/s/rad.

Figure 12 shows a colour-coded contour graph for the Hilbert spectra for large waves 1 and 2 of Storm1999a at the top and the actual waves at the bottom with the same time scale. The figure for large wave 1 shows that there is a large amount of energy around this event. In contrast, the graph for large wave 2 shows a large amount of energy after the wave, in the time interval between 5720 and 5725 secs.

Figure 13 (top) shows the Hilbert spectrum for the second SLEX interval for Storm1999d and (bottom) the corresponding spectra for large waves 1 and 2 of that interval. The top graph gives the evolution of the energy during the whole interval, which includes large waves 1 and 2 roughly at 76 and 79 minutes, towards the right-hand end of the interval. The bottom graphs show the energy around these events. The total energy accumulated in a 30 second neighbourhood of large wave 1 is 17.7316 m²/s/rad while for large wave 2 it is 33.54 m²/s/rad.

Figure 14 shows a contour graph for the Hilbert spectrum (top) for interval 2 of the SLEX segmentation of Storm1999d, covering a total of 27.3 min. On the bottom there is a graph of wave height for the same interval using the same time scale. There are two large waves in this interval, the first
Figure 11: (top) Hilbert spectrum for interval 7 of Storm1999a. (bot.) Hilbert spectra for large waves 1 and 2.

Figure 12: Contour graph for the Hilbert spectra for large waves 1 and 2 of Storm1999a. The actual waves are shown below.

one around 76.3 min. and the second around 79.2 min. For the first wave frequencies are distributed between 0.05 and 0.2 Hz. in a 30 second neighbourhood of the maxima. For the second the energy is more concentrated around 0.05 Hz. and for the 30 second neighbourhood of this wave the energy varies between 0.05 and 0.3 Hz. Both waves are the result of the superposition of several Intrinsic Mode Functions, none of which has a large amount of energy associated (see Figure 15). This phenomenon was also observed and commented in [9].

6 Conclusions

We have used the SLEX and HHT algorithms for the segmentation and spectral analysis of wave data recorded during a storm that took place in the North Sea in 1999. The data was divided into 5 sets lasting between 6 and 24 hours, as shown in Table 1. Each data set was segmented into stationary intervals using SLEX and the corresponding SLEX spectra were calculated. Using the software HHT-
Figure 13: (top) Hilbert spectrum for interval 2 of Storm1999d. (bot.) Hilbert spectra for large waves 1 and 2.

Figure 14: Contour graph for the Hilbert spectra for interval 2 of Storm1999d. Wave heights for the whole interval are shown below.

DPS each data set was decomposed into Intrinsic Mode Functions and the corresponding Hilbert spectra were obtained. For each interval obtained in the SLEX segmentation the marginal Hilbert spectra were calculated.

The SLEX algorithm is very useful for finding change points of a large set of data, as in this work, looking at changes in the spectral distribution of energy. SLEX divides the data set into segments that are approximately stationary. One disadvantage of this method is that the length of the data set must be a power of two ($L = 2^N$). However, this can solved either by completing the data set with zeros up to the next power of two or truncating the data set at the largest power of two less than the length of the data set. Both methods were used in this work and common data subset the change points coincided, so that the first method, which covers the whole data set, was preferred. It is important to observe that the completion with zeros may produce false change points near the end of the data set. Another limitation of SLEX is that, since it needs to calculate the spectra of subsets of the data set in order to compare the energy distribution in each one, the subsets must have enough data points.
to allow for a reasonably accurate estimation of the SLEX spectra. This implies a limitation in the resolution obtained with this algorithm. Finally, the fact that the algorithm proceeds by halving the successive data sets and comparing the spectra for each new data set, means that the change points cannot be located anywhere, but only at the endpoints of such sets. On the other hand SLEX allows for the calculation of the SLEX spectrum of each interval obtained in the segmentation, as well as a spectrogram for the whole data set (see Figure 7), which are very useful tools for the spectral analysis of a time series.

The HHT algorithm was also used for the spectral analysis of the storm. This algorithm is meant for the analysis of nonlinear non-stationary signals, which is clearly the case for this data set. The HHT-DPS software automatically performs the signal decomposition into Intrinsic Mode Functions (IMFs), giving as output two $N \times k$ matrices, where $N$ is the length of the data set and $k$ is the number of IMFs. One matrix gathers the values for the IMFs while the other stores the frequencies associated to each IMF.

A key outcome to consider is the fact that the temporal resolution for the HHT algorithm is very sharp, and this allows a very detailed spectral analysis for each interval in the SLEX segmentation of the data sets. This can be seen with the analysis of the large waves (higher than 6.5 m.). A 30 second interval around the maxima for each large wave was analyzed and interesting features such a different types of IMF decompositions were observed. HHT allows a very detailed and flexible spectral analysis, allowing the consideration of subintervals or small segments through the marginal Hilbert spectra.

We also look at the behaviour of the Hilbert spectrum around change points detected by SLEX, by considering an 60 seconds interval centred at the change points. In the large majority of cases there are noticeable changes of energy in these intervals, indicating that SLEX accurately detected points where the spectrum changes.

This work examines two methods that may be of considerable importance in ocean wave analysis where the understanding of the change in spectrum during changes to the statistical structure in the time series is an important consideration. Applications will include the analysis of the structural response of compliant floating structures or energy production from wave energy converters in time varying storm conditions. For example there is considerable interest in how one might ‘tune’ a wave energy converter to up coming wave fields. This work will provide information in the types of change that can be expected in stormy conditions and for which control algorithms can then be developed.
7 Acknowledgements

The authors would like to thank Total E&P UK for the wave data from the Alwyn North platform. The Hilbert-Huang transform Data Processing System whose copyright is with the United States Government as represented by the Administrator of the National Aeronautics and Space Administration, was used with permission. The software WAFO [1] developed by the Wafo group at Lund University of Technology, Sweden was used for the calculation of all Fourier spectra and associated spectral characteristics. This software is available at http://www.maths.lth.se/matstat/wafo. This work was partially supported by CONACYT, Mexico, Proyecto Análisis Estadístico de Olas Marinas. This work was partially done while the first author visited CIMAT. The support of CIMAT and the Universidad Central de Venezuela are gratefully acknowledge. George Smith would like to gratefully acknowledge the support offered by Scottish and Southern Energy PLC.

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