Bicritical rational functions on positive residue characteristic

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Febrero 2020

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In this work, we focus our attention in a family of rational functions with exactly two critical points acting on the Berkovich projective line associated to a complete and algebraic closed non-archimedean field. These functions are called bicritical functions.

Bicritical rational functions over non-archimedean fields have been studied by several authors. In 2014 Kiwi gave a complete description of dynamics of quadratic rational functions over the field of Laurent series with coefficients in the complex plane. Later, in 2022 Nie and Pilgrim extended Kiwi's description to bicritical rational functions of any degree over the field of Laurent series. The following year, Nopal Coello made a similar analysis for quadratic functions over any non-archimedean field, and later on he proved the existence of wandering domains for quadratic rational functions for certain fields such as $\mathbb{C}_p$, with $p = 2$. Finally, Lee, Mane and Troung proved in 2025, among other results, that the Julia set for a unicritical polynomial is contained in a local field if the finite fixed points are repelling and the polynomial is defined over a local field. This thesis aim to continue the study of bicritical rational functions over the fields of complex p-adic numbers and Laurent series with coefficients in $\mathbb{F}_p$, which we denote by $\mathbb{C}_p$ and $\mathbb{L}_p$ respectively. A description of each of the four main results in this thesis is the following.

- It is known that bicritical rational functions over the complex numbers do not have Herman rings. The counterpart of Herman rings in non-archimedean settings are the Rivera domains which are not discs. Along this line we obtained the following result.

  Theorem A. A bicritical rational function over $\mathbb{C}_p$ or $\mathbb{L}_p$ has at most one Rivera domain that is not a disc.

- The trichotomy presented by Kiwi holds in our fields of interest as long as the ramification locus coincides with the total ramification locus. However, for the fields $\mathbb{C}_p$ and $\mathbb{L}_p$, Kiwi's trichotomy changes to a dichotomy.

  Theorem B. Let f be a non-simple bicritical function of degree n with $n = p$ or $(n, p) = 1$. Then one of the following holds:

(a) f has an attracting fixed point and its Berkovich Fatou set consist of the immediate basin of attraction, or

(b) f has a unique Rivera domain which is not a disc.

- We provide a complete description of the dynamics of bicritical rational functions over $\mathbb{C}_p$ or $\mathbb{L}_p$. For the case of unicritical polynomials over $\mathbb{C}_p$, we introduced a new notion (to our knowledge) of a space of symbols, denoted by Σ, that lies between a complete shift in $mp^{\kappa-1}$ symbols and the complete shift in mp^{κ} symbols and prove the following theorem.

  Theorem C. Let $f \in \mathbb{C}_p(z)$ be a unicritical polynomial of degree $n = mp^{\kappa}$ with a fixed point at $1 \in \mathbb{C}_p$ and $\zeta = \zeta(1, r)$ the closest Julia point to 1. If $\deg_{\zeta(f)} = p^s$ for some $0 \leq s \leq \kappa$, then there is $q \geq 0$ such that (J_f, f) is topologically conjugate to (Σ, σ_q). Furthermore, all Fatou components that do not contain infinity (if any) are discs of radius r.

  Its counterpart in positive characteristic is as follows.

  Theorem D. Let $f \in \mathbb{L}_p(z)$ be a non-simple unicritical polynomial of degree $n = mp^{\kappa}$ with $m > 1$. Then (J_f, f) is topologically conjugate to (Σ_m, σ) and all bounded Fatou components (if any) are discs of the same radii.

Theorems C and D provide a description for a large variety of bicritical rational functions. However, there are bicritical rational functions which do not have these behaviours as shown through examples.