Out of Step Phase Shifting

Mariano Rivera[1], Rocky Bizuet[1], Amalia Martinez[2] and Juan A. Rayas[2]

[1]*Centro de Investigacion en Matematicas A.C.*
Apdo. Postal 402, Guanajuato, Gto., 36000, Mexico
[2]*Centro de Investigaciones en Optica A.C.*
Apdo. Postal 1-948, Leon, Gto., 37150, Mexico
mrivera@cimat.mx

http://www.cimat.mx/∼mrivera/

Abstract: We present a Phase Shifting robust method for irregular and unknown phase steps. The method is presented as a phase refinement strategy that uses as initial guess a coarsely computed phase corrupted with artifacts produce by an unprecise phase steps calibration. Then the algorithm, iteratively, refines the phase field and estimates the real phase steps by incorporating, effectively, redundant information in the fringe pattern set. The method performance is demonstrated by comparison with results computed using standard filtering and arbitrary phase steps detecting algorithm.

OCIS codes: (100.2650) Fringe analysis, (100.3020) Image reconstruction-restoration, (100.3190) Inverse problems, (120.2650) Fringe analysis, (120.5050) Phase measurement, (120.3180) Interferometry

References and links

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1. Introduction

Phase shifting (PS) is a popular interferometric analysis technique that consists of to acquire a set of fringe patterns each one with a known constant phase displacement, δ (phase step), with respect the previous one. In this work we proposed an algorithm for computing, simultaneously, a filtered phase map and the phase steps when the phase steps are unknown and irregular. First, we introduce our notation and the fringe set model. Let $g = \{g_1, g_2, \ldots, g_K\}$ the set of K phase shifted fringe patterns, then the k^{th} fringe pattern, g_k, can be modelled by

$$g_{kr} = a_{kr} + b_{kr} \cos(f_r + \Delta_k + \alpha_k) + \eta_{kr}, \tag{1}$$

where $r = [x, y]^T$ denotes the pixel position in the image lattice L, a_k is the background illumination component, b_k is the fringe contrast, f is the unknown phase, $\Delta_k = k\delta$ is the desired phase shift, α_k is the phase shift error and η_k represents additive independent noise. The k–dimension is commonly refereed as the time–dimension because it is assumed that the g set is a sequence of fringe patterns each one differing from the previous one in just a phase step equal to δ.

Recently there has been an intensive research on methods for phase recovering from a single closed fringe fringe pattern (see [1] and references therein). However PS methods are preferably used in stable acquisition condition (i.e. the temporal dependency of the illumination components, a and b, is eliminated) and the phase shifts can be introduced with high precision (α can be neglected). In such a case the wrapped phase $\hat{f}$, can be recovered by very simple algorithms; where $\hat{f} = W(f) \stackrel{def}{=} f + 2\pi n$ for an integer n such that $\hat{f} \in (-\pi, \pi]$ (where we denote by W the wrapping operator). For instance, for low level noise, $\hat{f}$ can be computed from 4 fringe pattern steps (by assuming $\delta = \pi/2$) with:

$$\hat{f}(r) = \tan^{-1}\left(\frac{g_{4r} - g_{2r}}{g_{1r} - g_{3r}}\right). \tag{2}$$

On the other hand, for high noise levels and unstable temporal condition, but again assuming precise phase steps (as in fringe projection) one can acquire a large pattern set (for instance for $K = 40$ and $\delta = \pi/4$) and to use the method reported in [2]:

$$\hat{f}(r) = \tan^{-1}\left(\frac{\sum_k g_{kr}\sin(\Delta_k)h(\Delta_k)}{\sum_k g_{kr}\cos(\Delta_k)h(\Delta_k)}\right), \tag{3}$$

where $h(\cdot)$ is a temporal window that reduces the effect of illumination instabilities.

The unwrapping process is an *ill posed* problem because the wrapped phase may correspond to multiple unwrapped phase, i.e. there exists many $\tilde{f} \neq f$ such that $\hat{f} = W(\tilde{f})$. Thus regularized solutions have been proposed in order to solve such an inverse problem, see for instance [3]. Where we denote by $\psi = W^{-1}(\hat{f})$ the unwrapped phase computed with the unwrapping operator W^{-1} that, in general, is implemented as the minimization of a regularized cost function [3].

In this paper we focus on to reduce the effect of miss-calibrated phase steps. We assume that the illumination conditions are stable or the fringe pattern set is normalized so the illumination temporal dependency are eliminated by a preprocess, see [4]. Such a normalized fringe pattern set is denoted by

$$\hat{g}_{kr} = \cos(f_r + \Delta_k + \alpha_k). \tag{4}$$

2. Adaptive Phase Stepping

First, we assume that a fringe pattern set is acquired with a desirable phase step of $\Delta = \Delta_1, \Delta_2, \ldots, \Delta_K$ but with small unknown residual phase step $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_K\}$ produced by a miss-calibration of the stepping device. Now, let ψ a coarse unwrapped phase computed with a standard algorithm by neglecting the residual steps, for instance with (2) in the case of $K = 4$ and $\Delta = 2\pi/K$. Then the relationship between a normalized fringe pattern, $\hat{g}_k$, and the coarsely computed phase, ψ, is given by:

$$\hat{g}_{kr} = \cos(\psi_r + \Delta_k + \phi_r + \alpha_k). \tag{5}$$

where ϕ is a unknown residual phase field that corrects artifacts produced by residual steps, α. Then the task here is to compute such a residual phase, ϕ_r, and therefore to compute the real phase $f_r = \phi_r + \psi_r$. For such a purpose we need to estimate the phase steps residuals, α's.

In Ref. [5] there was proposed a method for computing unknown phase steps, therein is shown that the steps are easily computed, with a closed formula, if the quadrature fringe pattern set is known. Thus, the method in [5] proposed a complexus nonlinear optimization procedure for, simultaneously, computing: the quadrature fringe pattern set, the local frequency and the corresponding phase steps. More recently was reported an efficient method for computing the residual phase from a coarse one for a single fringe pattern: Algorithm 1 in [1]. Such method transforms a, originally, no-linear optimization problem in a sequence of quadratic optimization problems. Here we extend such a formulation for computing the real phase, $\psi+\phi$, and the true phase steps, $\Delta_k+\alpha_k$, in the case of phase shifted pattern sets. Following [1], we assume that $|\phi_r+\alpha_k|$ is relatively small such that the first order Taylor series can be used to define the residual error:

$$E_{kr}(\phi,\alpha) \stackrel{def}{=} \hat{g}_{kr} - \cos(\psi_r+\Delta_k) + (\phi_r+\alpha_k)\sin(\psi_r+\Delta_k) \approx 0. \tag{6}$$

Therefore, we propose to compute the phase correction field, ϕ, the phase step corrections, α, and an outliers detection field, ω, by alternating quadratic minimization of the cost function:

$$\begin{aligned} U(\phi,\alpha,\omega) \quad = \quad & \sum_{k=1}^{K}\sum_{r\in L}\left[\omega_r^2 E_{kr}^2(\phi,\alpha) + \mu\,(1-\omega_r)^2\right] + \gamma\left[\sum_{k=1}^{K}\alpha_k^2 + \sum_{r\in L}\phi_r^2\right] \\ & +\lambda \sum_{\langle q,r,s\rangle\in R}\left[\psi_q+\phi_q - 2\,(\psi_r+\phi_r) + \psi_s+\phi_s\right]^2, \end{aligned} \tag{7}$$

Cost function (7) extends the proposed in [1] to deal with a set of fringe pattern as data. Moreover, we have included two terms (weighted by the parameter γ) that enforce small values for the step correction vector, α, and the residual phase field, ϕ. Such terms improved significantly the convergence stability of the minimization process. The details of the phase refinement procedure are formalized in Algorithm 1. It is important to note in Algorithm 1 that once a residual (ϕ or α) is computed the corresponding base variable (ψ or Δ) is updated. Such a strategy reduces iteratively the value of the unknown residual and the fitness of the first order Taylor approach; consequently the performance of the algorithm.

Algorithm 1 Out of Step Phase Shifts.

Let $g=\{g_1,g_2,\ldots,g_K\}$ a fringe pattern set with expected phase steps equal to Δ and ψ an initial coarse phase.

Set, initially $\phi=0$, $\omega=1$ and given $\varepsilon>0$;
For all the pixels $r\in R$:
while $\left\|\hat{g}-\hat{b}.*\cos\psi\right\|>\varepsilon$ **do**
    Compute $\psi_r=\psi_r+\phi_r$ and then set $\phi_r=0$;
    Compute $\alpha=\arg\min_{\alpha} U(\phi=0,\omega,\alpha)$; {use (8)}
    Update $\Delta=\Delta+\alpha$ and then set $\alpha=0$;
    Compute $\omega=\arg\min_{\omega} U(\phi=0,\omega,\alpha=0)$; {use (9)}
    Compute $\phi=\arg\min_{\phi} U(\phi,\omega,\alpha=0)$; {see Appendix A in [1]}
end while

Now we discuss how to compute an effective initial guess for Algorithm 1. The initial coarse phase, ψ, can be computed with standard algorithms by assuming correct phase steps, i.e. by neglecting the residual steps, α. Such a wrapped phase is corrupted with artifacts introduced by the residual steps. Then it recommended to use a robust algorithm for unwrapping the coarse wrapped phase. In particular, we use the half-quadratic convex unwrapping algorithm reported in [3]. The resultant unwrapped phase, $\hat{\psi}$, may have a constant residual step, Δ_{dc}, that can be coarsely estimated by a reduced search, i.e:

$$\Delta_{dc} = \arg\min_{d\in D} \|g - \hat{b}.*\cos(\hat{\psi}+d)\|_2^2,$$

where $D = \{d_i = 2\pi i/N\}$, for $i = 0,1,2,\ldots,N-1$, is a N steps set (we use $N = 20$ in our experiments) and $.*$ denotes the componentwise product of vectors. Alternatively, the method reported by Cai et al. [6] can be used for the same purpose. However, the accuracy of the method in [6] is reduced if the fringe patterns are corrupted by additive independent noise, see experiments in Fig. 4. Nevertheless the Δ_{dc} is estimated, we initialize the coarse phase with: $\psi = \hat{\psi} - \Delta_{dc}$. On the other hand, the steps Δ_k's can be initialized with the ideal values or, to avoid large α residuals, can be estimated (as Δ_{dc}).

In the following we present details of the partial minimizations (steps 5, 7 and 8) in Algorithm 1. First, we note that, for $\phi = 0$ and ω fixed, cost function (7) can be written as:

$$U(\phi=0,\omega,\alpha) = \sum_r \left[\omega_r^2 \sum_k (\hat{g}_{kr} - \cos\hat{\psi}_{kr} + \alpha_k \sin\hat{\psi}_{kr})^2 + \gamma\alpha_k^2\right] + Q(\phi=0,\omega),$$

where $\hat{\psi}_{kr} \overset{def}{=} \psi_r + \Delta_k$ and the potential $Q(\cdot)$ contains the α–independent terms. Equating to zero the partial gradient with respect to (w.r.t.) α and solving for α_k, we obtain a closed formula for computing the α's optimum coefficients:

$$\alpha_k = \frac{\sum_r \omega_r^2 \sin\hat{\psi}_{kr}\,(\cos\hat{\psi}_{kr} - \hat{g}_{kr})}{\gamma + \sum_r \omega_r^2 \sin^2\hat{\psi}_{kr}}. \tag{8}$$

In a similar way, for $\phi = 0$ and $\alpha = 0$, we obtain a closed formula for computing the ω field:

$$\omega_r = \frac{\mu}{\mu + \sum_k (\hat{g}_{kr} - \cos\hat{\psi}_{kr})^2}. \tag{9}$$

Finally, ϕ is obtained by solving the linear system that results of equaling to zero the partial gradient w.r.t. ϕ of $U(\phi,\alpha=0,\omega)$, keeping $\alpha = 0$ and ω fixed. In particular, we use a Gauss-Seidel scheme similar to the one proposed in the Appendix A of [1].

3. Experiments

First experiment demonstrates the method performance in synthetic noisy test data. First row in Fig. 1 shows the noisy fringe pattern set generated from a synthetic phase that has low and high frequencies product of a slight tilt and sharp gaussian peaks. Then a coarse phase map in Fig. 2 is computed with (2) by assuming regular phase steps of $\pi/2$ [Fig 4(a)] while real random phase shifts are plotted in Fig. 4(b). As one can note in the Fig. 2 phase artifacts, correlated with the fringe pattern, corrupt the computed phase. Such a coarse phase is used as initial guess for the proposed method. Note that the proposed method recovers the wide bandwidth phase, by smoothing spurious artifacts and preserving real high frequencies (see first row in Fig. 3). Moreover the method recovers effectively the phase steps [Fig. 4(c)]. We performed experiments for different size of the fringe pattern set, K. As it is expected, the method performance is improved as K grows given that cost function (7) effectively incorporates redundant information in fringe pattern set. On the other hand, a simple low–pass filtering of the coarse phase despite redundant information. Thus, the spurious artifacts are not eliminated and real high frequencies are over–smoothed, see second row in Fig. 3. Fig. 4. show obtained results from electronic speckle pattern interferometry (ESPI) set. The fringe pattern corresponds to a steel plate under mechanical stress.

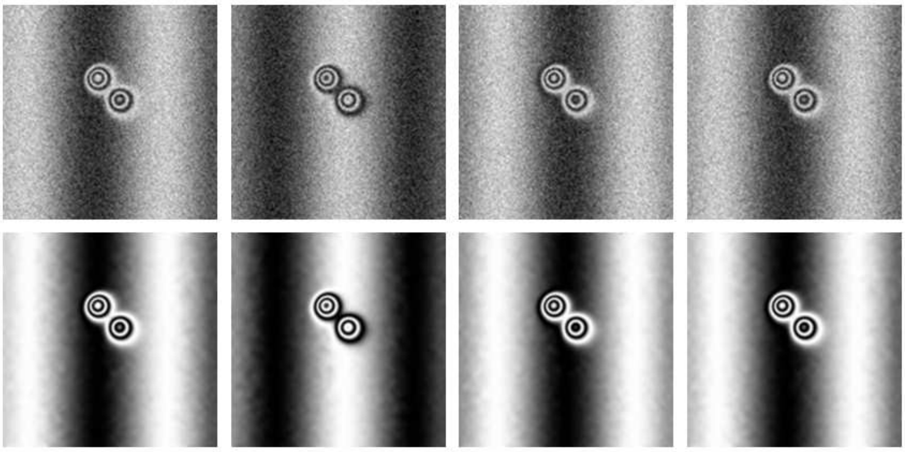

Fig. 1. Synthetic fringe pattern set (first Row) and the respective reconstruction with computed phase map and the phase steps

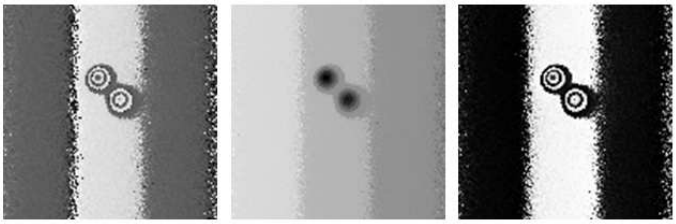

Fig. 2. Computed coarse solution assuming ideal steps ($\pi/2$) with Eq. (2). From left to right: Wrapped phase, phase unwrapped with convex algorithm in [3] and its cosine (reconstructed fringe pattern), respectively.

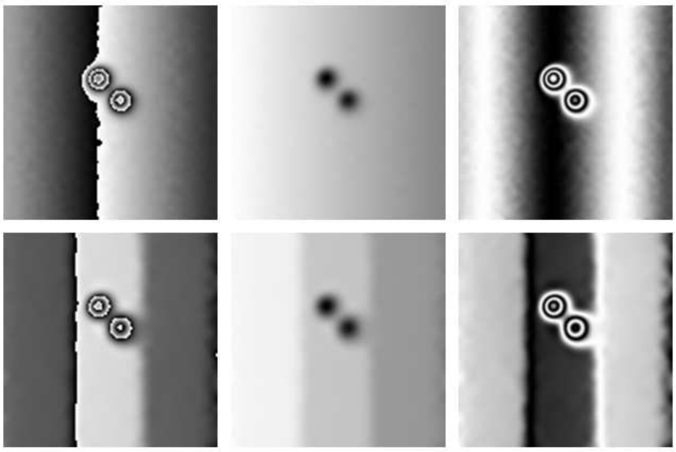

Fig. 3. Rewrapped phase, unwrapped phase and its cosine for the results computed with the proposed method (first row) and the result of a Gaussian filter on the coarse computed phase (in Fig. 1)

4. Conclusion

We have presented a PS robust method for irregular unknown phase stepping based on a phase refinement strategy. The method takes advantage of the redundant information in the phase shifted fringe pattern set for smoothing out artifacts produced by miss–calibrated phase steps and for preserving real high frequencies. The method, implemented as successive quadratic minimizations of a nonlinear cost function, guarantees convergence to a local minima and is computationally efficient. The method performance was demonstrated by experiments.

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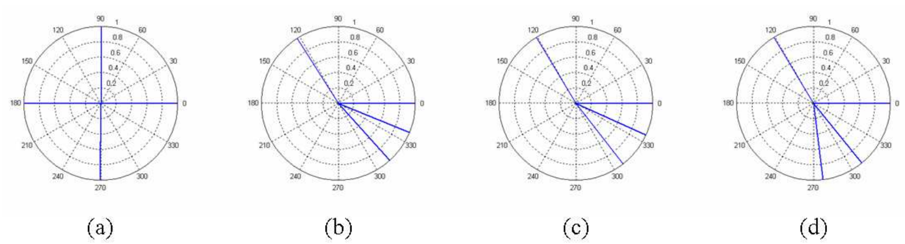

Fig. 4. Phaser plots: (a) Ideal phase shifts ($k\pi/4$), (b) real phase shifts, (c) phase shifts computed with the proposed method and (d) phase shift computed with the method in [6].

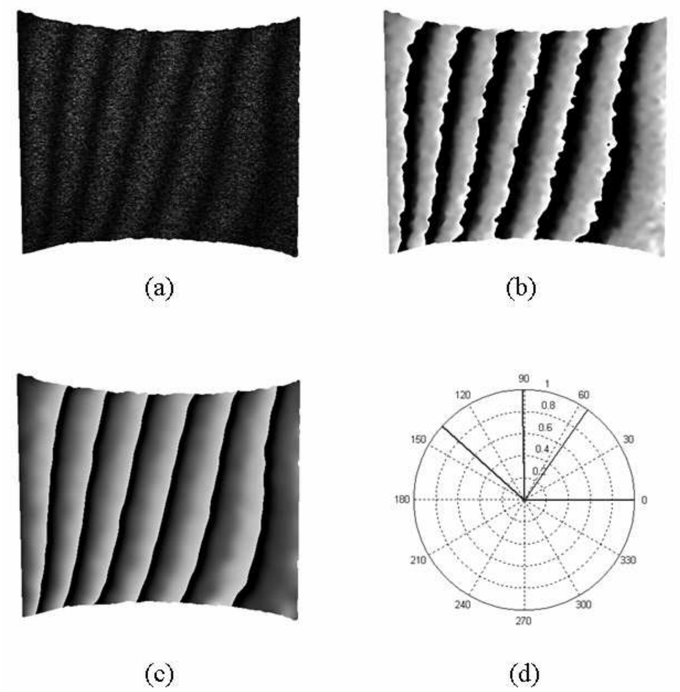

Fig. 5. Real data experiment: (a) An original ESPI fringe pattern, (b) computed wrapped phase with a standard four steps (assuming $\delta = \pi/2$), (c) refined phase computed with the proposed algorithm (rewrapped for illustration purposes) and (d) computed phase shifts.