Multi-Tensor Field Spectral Segmentation for White Matter Fiber Bundle Classification
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1. Abstract
We present an algorithm for segmenting the white-matter axon fiber bundles from HARDI images. We formulate the segmentation problem as a Multi-Tensor Field segmentation problem in which the compartments of each Multi-Tensor may belong to different classes, allowing the algorithm to handle fiber crossings.

2. DW-MRI data
The DW-MRI data may be regarded as a function that assigns an input signal to each voxel of the volume of interest.

3. Multi-Tensor Model
The Multi-Tensor Model assumes that the input signal at each voxel is a linear combination of a few diffusion tensor functions.

\[ S_{v,i} = S_0(v) \sum_{j=1}^{K} \beta_j^{(v)} \exp(-\tau q_i^T D_v^{(j)} q_i) \]

4. MTF Spectral Segmentation
Assigning a Multi-Tensor to each voxel, may also be thought as assigning a tensor to each compartment. The set of compartments is

\[ C = \Omega \times I \]

Where K is the maximum number of compartments that each voxel can have, and

\[ I = \{1, 2, \ldots, K\} \]

Since the Diffusivity profile is constant, each tensor is characterized by its Principal Diffusion Direction (a 3D vector). Therefore, the Multi-Tensor Field can be codified as a function that assigns a vector to each compartment.

\[ \mathcal{T} : (v, j) \in C \mapsto p_j \in \mathbb{R}^3 \]

Instead of segmenting the set of voxels, we aim to segment the set of compartments (which is well defined as long as each compartment corresponds to a fiber).

5.1 Compartment Similarity
We define the "distance" between two compartments located at voxels \( v, w \) with Principal Diffusion Directions \( r, s \):

\[ D((v, r), (w, s)) = \frac{|v-w|^2}{d_0^2} + \frac{\Delta(r,s)}{\theta_0} \]

6.2 Fibercup Phantom
MTF obtained using DBF
Segmentation at fiber crossings
Compartment embedding
4. The DBF model

The DBF model uses a fixed set of possible orientations for the diffusion tensor functions and a constant diffusivity profile. Together, the profile and the orientations define a dictionary called Diffusion Basis Functions (DBF).

Constant Diffusivity Profile

\[ (\lambda_L, \lambda_T, \lambda_T) \]

\[ \{d_1, d_2, ..., d_M\} \]

We apply the Meila and Shi embedding algorithm [7] to map the compartments to points in \( \mathbb{R}^d \) such that tensors that are close to each other according to the above distance \( d \) are mapped to nearby points.

\[ C \]

\[ \mathbb{R}^d \]

\[ \beta^* = \min_{\beta \in \mathbb{R}^N} \| \Phi \beta - S_v \|_2^2, \text{ s.t. } \beta \geq 0 \]

After fitting the Multi-Tensor Model to all voxels of the volume, we obtain a Multi-Tensor Field representation of the geometric structure of the volume: each voxel is assigned a multi-tensor.

\[ v \in \Omega \mapsto \{p_1, p_2, ..., p_K\} \]

We apply Entropy-Controlled Quadratic Markov Measure Field (ECQMMF)[9] to obtain the final segmentation.

6. Experiments

We performed experiments at three levels of difficulty:
1) 2012 HARDI Reconstruction Challenge dataset (MTF is known)
2) Fibercup phantom (MTF is unknown)
3) Real data

6.1 Synthetic data

\[ \text{Input MTF} \]

\[ \text{Segmentation} \]

Volume segments obtained

6.2 Embedding and Segmentation

\[ \text{Input} \]

\[ \text{Output} \]

\[ \text{Segmentation (MTF obtained using DBF)} \]

\[ \text{Compartment Embedding} \]

Iso-surfaces of the resulting segments

6.3 Real Data

7. References


http://www.cimat.mx/~omar