Proposición 1.6

$$\begin{array}{ccc} \mathfrak{g} \ni 0 & & 0 \in N_0 \\ \exp \downarrow & & \downarrow \exp|_{N_0} \text{ es difeom} \\ G \ni e & & e \in N_e \end{array}$$

Dem.

$d(\exp)_0 = ?$

$$d(\exp)_0(X) = d(\exp)_0(\alpha'(0))$$

$$(\alpha(t) = tX)$$

$$= \left.\frac{d}{dt}\right|_{t=0} \exp(\alpha(t))$$

$$= \left.\frac{d}{dt}\right|_{t=0} \exp(tX) = X$$

$$\therefore \quad d(\exp)_0 : T_0\mathfrak{g} = \mathfrak{g} \longrightarrow T_eG = \mathfrak{g}$$

$$X \longmapsto X$$

Se aplica TFI. //

Ejemplos de mapeo exponencial.

1) $G = \mathbb{R}^n$, $\mathfrak{g} = T_0 \mathbb{R}^n = \mathbb{R}^n$

$$\exp : \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

$$\exp(X) = X$$

porque

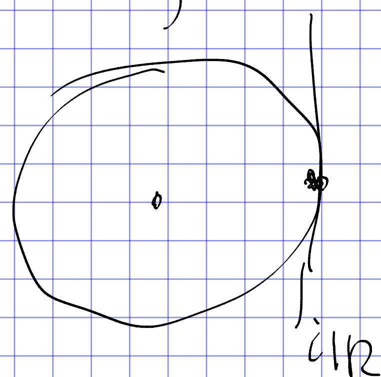
$$t \longmapsto tX$$

es subgrupo uniparamétrico

2) $G = S^1 = \{ z \in \mathbb{C} \mid |z| = 1 \}$

$$T_e G = T_1 S^1 = i\mathbb{R}$$

$$\cong \mathbb{R}$$



$$\exp : i\mathbb{R} \longrightarrow S^1$$

$$\exp(ix) = e^{ix}$$

puesto que

$$e^{i(x+y)} = e^{ix} e^{iy}.$$

En general, para cualquier G:

$$\exp(tX+sX) = \exp(tX)\exp(sX)$$

pero:

$$\exp(tX+sY) \overset{?}{=} $$

$$\neq \exp(tX)\exp(sY)$$

3) $G = GL(n, \mathbb{R})$

$$= \{A \in M_{n\times n}(\mathbb{R}) \mid \det(A) \neq 0\}$$

$$\mathfrak{gl}(n, \mathbb{R}) = T_{I_n} GL(n, \mathbb{R})$$

$$= M_{n\times n}(\mathbb{R})$$

$$\exp : M_{n\times n}(\mathbb{R}) \longrightarrow GL(n, \mathbb{R})$$

$$\exp(A) = e^A := \sum_{k=0}^{+\infty} \frac{A^k}{k!}$$

puesto que $t \longmapsto e^{tA}$ es subgrupo uniparamétrico.

En la página 105:

$$f(g\exp(X)) = p(x_1, \ldots, x_n)$$

$$= \sum_{m \in \mathbb{N}^n} a_m \underbrace{x_1^{m_1} \cdots x_n^{m_n}}_{x^m}$$

Para el Teorema 1.7:

$$\frac{1-e^{-ad(X)}}{ad(X)} \text{ significa:}$$

$$e^{-ad(X)} = \sum_{k=0}^{+\infty} \frac{(-1)^k}{k!} ad(X)^k$$

$$1 - e^{-ad(X)} = -\sum_{k=1}^{+\infty} \frac{(-1)^k}{k!} ad(X)^k$$

$$\frac{1-e^{-ad(X)}}{ad(X)} = -\sum_{k=1}^{+\infty} \frac{(-1)^k}{k!} ad(X)^{k-1}$$

El "Remark" de la página 107 implica (con algo de trabajo) que

$$R(X,Y) = -\frac{1}{4}\,\mathrm{ad}([X,Y])$$

(curvatura) para ciertas conexiones en G.