

$$C: \mathfrak{sl}(n, \mathbb{C}) \times \mathfrak{sl}(n, \mathbb{C}) \longrightarrow \mathbb{C}$$

$$C(X, Y) = \operatorname{tr}(XY)$$

es no degenerada. Para verlo probar que:

$$\mathfrak{sl}(n, \mathbb{C}) = \mathfrak{sym}_0(n, \mathbb{C}) \oplus \mathfrak{so}(n, \mathbb{C})$$

$$X = X^T$$

$$X = -X^T$$

$\mathfrak{sym}(n, \mathbb{C}) \perp \mathfrak{so}(n, \mathbb{C})$ para C

$$C|_{\mathfrak{sym}(n, \mathbb{C})}(X, Y) = \operatorname{tr}(XY^T)$$

$$= \sum_{j, k=1}^n X_{jk} Y_{jk}$$

$$C|_{\mathfrak{so}(n, \mathbb{C})}(X, Y) = -\operatorname{tr}(XY^T)$$

$$= -\sum_{j, k=1}^n X_{jk} Y_{jk}$$

y ambas restricciones son no degeneradas.

En el Teorema 6.3 página 181:

Si $\mathfrak{g} = \mathfrak{sl}(n, \mathbb{C})$, $h \in \mathfrak{sl}(n, \mathbb{C})$
matrices diagonales, entonces la construcción de la
demostración da $\mathfrak{su}(n)$.

Observamos que:

$$\mathfrak{sl}(n, \mathbb{C}) = \mathfrak{su}(n) \oplus i\mathfrak{su}(n)$$



$$z^* = -z$$

$$\text{tr}(z) = 0$$

$$z^* = z$$

$$\text{tr}(z) = 0$$

Otra forma real:

$$\mathfrak{sl}(n, \mathbb{C}) = \mathfrak{sl}(n, \mathbb{R}) \oplus i\mathfrak{sl}(n, \mathbb{R})$$

$$\mathfrak{su}(n) \longleftrightarrow \text{SU}(n) \quad \text{compacto}$$

$$\mathfrak{sl}(n, \mathbb{R}) \longleftrightarrow \text{SL}(n, \mathbb{R}) \quad \text{no compacto.}$$