Proposition 2.20:
If $f \in L^+(X,\mu)$ and $\int f < +\infty$, then

$$\{x \in X \mid f(x) = +\infty\}$$

is null and:

$$\{x \in X \mid f(x) > 0\}$$

is σ-finite.

Proof:
For every $n \in \mathbb{N}$:

$$n\chi_{f^{-1}(\infty)} \leq f$$

and so:

$$n\mu(f^{-1}(\infty)) \leq \int f < +\infty$$

$$\forall n \in \mathbb{N}$$

This implies $\mu(f^{-1}(\infty)) = 0$, and proves the first part.

We can write:

$$\{x \in X \mid f(x) > 0\} = \bigcup_{n=1}^{+\infty} f^{-1}\left(\left[\tfrac{1}{n}, +\infty\right)\right) \cup f^{-1}(\infty)$$

we already now that $f^{-1}(\infty)$ is null.

And we have:

$$\frac{1}{n}\mu\left(f^{-1}\left(\left[\tfrac{1}{n}, +\infty\right)\right)\right) = \int_{f^{-1}\left(\left[\frac{1}{n}, +\infty\right)\right)} \frac{1}{n} \leq$$

$$\leq \int_{f^{-1}\left(\left[\frac{1}{n}, +\infty\right)\right)} f \leq \int f < +\infty$$

Hence:

$$\mu\left(f^{-1}\left(\left[\tfrac{1}{n}, +\infty\right)\right)\right) \leq n \int f < +\infty$$

which proves that

$$\{x \in X \mid f(x) > 0\}$$

is a countable union of finite measure subsets. //