

En coordenadas:

$$\varphi = (z_1, \dots, z_n)$$

en  $U$ , tenemos la base

$$dz_j, d\bar{z}_j \quad \text{de } T_z^{\mathbb{C} \times} M$$

$\forall z \in U$ . Por tanto, las  $L$ -formas complejas se escriben en  $U$  como:

$$\alpha = \sum_{j=1}^n f_j dz_j + \sum_{j=1}^n g_j d\bar{z}_j$$

$$f_j, g_j : U \longrightarrow \mathbb{C}.$$

$f_j, g_j$  suaves cuando  $\alpha$  es suave.

$(1,0)$ -forma

$(0,1)$ -forma

Si  $\alpha, \beta \in T_z^* M$ , entonces:

$$\alpha \otimes \beta: T_z^* M \times T_z^* M \longrightarrow \mathbb{R}$$

$$(\alpha \otimes \beta)(u, v) = \alpha(u) \beta(v)$$

Si  $\alpha, \beta \in T_z^{\mathbb{C}*} M$ , entonces:

$$\alpha \otimes \beta: T_z^{\mathbb{C}*} M \times T_z^{\mathbb{C}*} M \longrightarrow \mathbb{C}$$

$$(\alpha \otimes \beta)(u, v) = \alpha(u) \beta(v)$$

Con coordenadas:

$$q = (z_1, \dots, z_n), \quad z_j = x_j + iy_j$$

\*  $\theta_z: T_z M \times T_z M \longrightarrow \mathbb{R}$  bilinear

$$\forall z \in \mathbb{R}$$

$$\theta_z = \sum_{j, k=1}^n \theta_z \left( \frac{\partial}{\partial x_j}, \frac{\partial}{\partial x_k} \right) (dx_j \otimes dx_k) \Big|_z$$

$$+ \sum_{j, k=1}^n \theta_z \left( \frac{\partial}{\partial y_j}, \frac{\partial}{\partial y_k} \right) (dy_j \otimes dy_k) \Big|_z$$

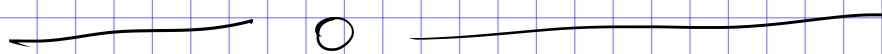
+ ...

\* Si  $\Theta_z: T_z^c M \times T_z^c M \rightarrow \mathbb{C}$   
 es  $\mathbb{C}$ -bilinear  $\forall z \in U$ :

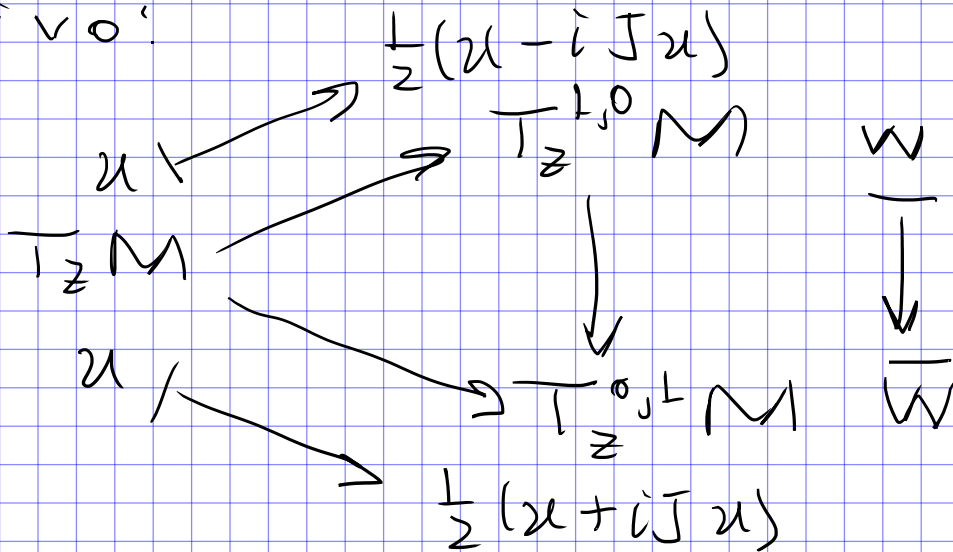
tipo  
 $\downarrow$

$$\begin{aligned} \Theta &= \sum_{j,k=1}^n \Theta \left( \frac{\partial}{\partial z_j}, \frac{\partial}{\partial z_k} \right) dz_j \otimes dz_k \quad (2,0) \\ &+ \sum_{j,k=1}^n \Theta \left( \frac{\partial}{\partial \bar{z}_j}, \frac{\partial}{\partial \bar{z}_k} \right) d\bar{z}_j \otimes d\bar{z}_k \quad (0,2) \\ &+ \sum_{j,k=1}^n \Theta \left( \frac{\partial}{\partial z_j}, \frac{\partial}{\partial \bar{z}_k} \right) dz_j \otimes d\bar{z}_k \quad (1,1) \\ &+ \sum_{j,k=1}^n \Theta \left( \frac{\partial}{\partial \bar{z}_j}, \frac{\partial}{\partial z_k} \right) d\bar{z}_j \otimes dz_k \quad (1,1) \end{aligned}$$

Podemos tomar  $\Theta = g$ .



Tenemos el diagrama conmutativo:



Respecto del tensor:

$$h = \sum_{j,k=1}^n g_{jk} dz_j \otimes d\bar{z}_k$$

tenemos las siguientes interpretaciones:

$$*) h_z: T_z^{\mathbb{C}} M \times T_z^{\mathbb{C}} M \longrightarrow \mathbb{C}$$

$$h_z(u, v) = \sum_{j,k=1}^n g_{jk} dz_j(u) d\bar{z}_k(v)$$

$$u = \sum_{j=1}^n (a_j \frac{\partial}{\partial z_j} + b_j \frac{\partial}{\partial \bar{z}_j}), \quad v = \sum_{j=1}^n (c_j \frac{\partial}{\partial z_j} + d_j \frac{\partial}{\partial \bar{z}_j})$$

$$\therefore h_z(u, v) = \sum_{j,k=1}^n a_j g_{jk} d_k$$

$$*) h_z: T_z^{L,0} M \times T_z^{L,0} M \longrightarrow \mathbb{C}$$

$$h_z(u, v) = \sum_{j,k=1}^n g_{jk} dz_j(u) d\bar{z}_k(v)$$

$$u = \sum_{j=1}^n a_j \frac{\partial}{\partial z_j}, \quad v = \sum_{j=1}^n b_j \frac{\partial}{\partial z_j}$$

$$\therefore h_z(u, v) = \sum_{j,k=1}^n a_j g_{jk} b_k$$