En coordenadas:

$$\varphi = (z_1, \dots, z_n)$$

en U, tenemos la base

$$dz_j, \ d\bar{z}_j \qquad \text{de } T_z^{\mathbb{C}*} M$$

$\forall z \in U$. Por tanto, las 1-formas complejas se escriben en U como:

$$\alpha = \underbrace{\sum_{j=1}^{n} f_j \, dz_j}_{(1,0)\text{-forma}} + \underbrace{\sum_{j=1}^{n} g_j \, d\bar{z}_j}_{(0,1)\text{-forma}}$$

$f_j, g_j : U \longrightarrow \mathbb{C}$.

f_j, g_j suaves cuando α es suave.

Si $\alpha, \beta \in T_z^* M$, entonces:

$$\alpha \otimes \beta : T_z^* M \times T_z^* M \longrightarrow \mathbb{R}$$

$$(\alpha \otimes \beta)(u, v) = \alpha(u)\beta(v)$$

Si $\alpha, \beta \in T_z^{\mathbb{C}*} M$, entonces:

$$\alpha \otimes \beta : T_z^{\mathbb{C}*} M \times T_z^{\mathbb{C}*} M \longrightarrow \mathbb{C}$$

$$(\alpha \otimes \beta)(u, v) = \alpha(u)\beta(v)$$

Con coordenadas:

$$\varphi = (z_1, \dots, z_n), \quad z_j = x_j + i y_j$$

*) $\theta_z : T_z M \times T_z M \longrightarrow \mathbb{R}$ bilineal

$\forall z \in \mathbb{R}$

$$\theta_z = \sum_{j,h=1}^{n} \theta_z\left(\frac{\partial}{\partial x_j}, \frac{\partial}{\partial x_h}\right)(dx_j \otimes dx_h)\Big|_z$$

$$+ \sum_{j,h=1}^{n} \theta_z\left(\frac{\partial}{\partial y_j}, \frac{\partial}{\partial y_h}\right)(dy_j \otimes dy_h)\Big|_z$$

$$+ \dots$$

*) Si $\theta_z : T_z^{\mathbb{C}} M \times T_z^{\mathbb{C}} M \longrightarrow \mathbb{C}$ es $\mathbb{C}$-bilineal $\forall z \in U$:

$$\theta = \sum_{j,h=1}^{n} \theta\left(\frac{\partial}{\partial z_j}, \frac{\partial}{\partial z_h}\right) dz_j \otimes dz_h \quad (2,0) \leftarrow \text{tipo}$$

$$+ \sum_{j,h=1}^{n} \theta\left(\frac{\partial}{\partial \bar{z}_j}, \frac{\partial}{\partial \bar{z}_h}\right) d\bar{z}_j \otimes d\bar{z}_h \quad (0,2)$$

$$+ \sum_{j,h=1}^{n} \theta\left(\frac{\partial}{\partial z_j}, \frac{\partial}{\partial \bar{z}_h}\right) dz_j \otimes d\bar{z}_h \quad (1,1)$$

$$+ \sum_{j,h=1}^{n} \theta\left(\frac{\partial}{\partial \bar{z}_j}, \frac{\partial}{\partial z_h}\right) d\bar{z}_j \otimes dz_h \quad (1,1)$$

Podemos tomar $\theta = g$.

——— o ———

Tenemos el diagrama conmutativo:

$$\begin{tikzcd}
 & T_z^{1,0}M \arrow[dd] & w \arrow[dd, mapsto] \\
T_zM \arrow[ur, "u \mapsto \frac{1}{2}(u - iJu)"] \arrow[dr, "u \mapsto \frac{1}{2}(u + iJu)"'] & & \\
 & T_z^{0,1}M & \bar{w}
\end{tikzcd}$$

Respecto del tensor:

$$h = \sum_{j,k=1}^{n} g_{jk}\, dz_j \otimes d\bar{z}_k$$

tenemos las siguientes interpretaciones:

*) $h_z : T_z^{\mathbb{C}} M \times T_z^{\mathbb{C}} M \longrightarrow \mathbb{C}$

$$h_z(u, v) = \sum_{j,k=1}^{n} g_{jk}\, dz_j(u)\, d\bar{z}_k(v)$$

$$u = \sum_{j=1}^{n} \left(a_j \frac{\partial}{\partial z_j} + b_j \frac{\partial}{\partial \bar{z}_j} \right), \quad v = \sum_{j=1}^{n} \left(c_j \frac{\partial}{\partial z_j} + d_j \frac{\partial}{\partial \bar{z}_j} \right)$$

$$\therefore \quad h_z(u, v) = \sum_{j,k=1}^{n} a_j\, g_{jk}\, d_k$$

*) $h_z : T_z^{1,0} M \times T_z^{1,0} M \longrightarrow \mathbb{C}$

$$h_z(u, v) = \sum_{j,k=1}^{n} g_{jk}\, dz_j(u)\, d\bar{z}_k(\bar{v})$$

$$u = \sum_{j=1}^{n} a_j \frac{\partial}{\partial z_j}, \quad v = \sum_{j=1}^{n} b_j \frac{\partial}{\partial z_j}$$

$$\therefore \quad h_z(u, v) = \sum_{j,k=1}^{n} a_j\, g_{jk}\, \bar{b}_k$$