Ejemplo:

$\mathbb{B}^n = \{ z \in \mathbb{C}^n \mid |z| < 1 \}$

Kernel de Bergman

$$K(z, w) = (1 - z \cdot \overline{w})^{-(n+1)}$$

Entonces:

$$K(z, z) = \Delta(z)^{-(n+1)}, \quad \Delta(z) = (1 - |z|^2)$$

Por tanto:

$$\log K(z, z) = -(n+1) \log \Delta(z).$$

Luego:

$$\frac{\partial}{\partial z_j} \log K(z, z) = -(n+1) \Delta(z)^{-1} \frac{\partial \Delta}{\partial z_j}(z)$$

$$\frac{\partial^2}{\partial z_j \partial \overline{z}_h} \log K(z, z) =$$

$$= -(n+1) \frac{\partial}{\partial \overline{z}_h} \left(\Delta(z)^{-1} \frac{\partial \Delta}{\partial z_j}(z) \right)$$

$$= -(n+1) \Delta(z)^{-1} \frac{\partial^2 \Delta}{\partial z_j \partial \overline{z}_h}(z)$$

$$+ (n+1) \Delta(z)^{-2} \frac{\partial \Delta}{\partial z_j}(z) \frac{\partial \Delta}{\partial \overline{z}_h}(z)$$

$$= (n+1)\Big(-(1-|z|^2)^{-1}(-\delta_{jh})$$

$$\left(\frac{\partial(1-|z|^2)}{\partial z_j} = -\bar{z}_j, \quad \frac{\partial^2(1-|z|^2)}{\partial z_j \partial \bar{z}_h} = -\delta_{jh} \right)$$

$$+ (1-|z|^2)^{-2}(-\bar{z}_j)(-z_h) \Big)$$

$$= (n+1)\left((1-|z|^2)^{-1}\delta_{jh} + (1-|z|^2)^{-2}\bar{z}_j z_h \right)$$

$$= (n+1)(1-|z|^2)^{-2}\left((1-|z|^2)\delta_{jh} + \bar{z}_j z_h \right)$$

Tomamos:

$$g = \frac{1}{n+1}\sum_{j,h=1}^{n} \frac{\partial^2 \log K(z,z)}{\partial z_j \partial \bar{z}_h} dz_j \otimes d\bar{z}_h$$

$$= (1-|z|^2)^{-2}\sum_{j,h=1}^{n} \left((1-|z|^2)\delta_{jh} + \bar{z}_j z_h \right) dz_j \otimes d\bar{z}_h$$

$\therefore\ \omega =$

$$= i(1-|z|^2)^{-2}\sum_{j,h=1}^{n} \left((1-|z|^2)\delta_{jh} + \bar{z}_j z_h \right) dz_j \wedge d\bar{z}_h$$

Deseamos calcular el mapeo de momento para la acción de $\mathbb{T}^n$:

$$\mathbb{T}^n \times \mathbb{B}^n \longrightarrow \mathbb{B}^n$$
$$t \cdot z = (t_1 z_1, \dots, t_n z_n).$$

Si $X = (s_1, \dots, s_n) \in \mathbb{R}^n$, entonces:

$$X^\#_z = (i s_1 z_1, \dots, i s_n z_n)$$
$$= \sum_{j=1}^{n} \left(i s_j z_j \frac{\partial}{\partial z_j} - i s_j \bar{z}_j \frac{\partial}{\partial \bar{z}_j} \right)$$

Usamos la base canónica $X_1, \dots, X_n$, donde $X_j = (0, \dots, \underset{j}{1}, \dots, 0)$.

$$\therefore \quad X_j^\#(z) = i z_j \frac{\partial}{\partial z_j} - i \bar{z}_j \frac{\partial}{\partial \bar{z}_j}.$$

Evaluamos:

$$\omega(X_j^\#, \cdot) =$$
$$= i(1-|z|^2)^{-2} \sum_{r,s=1}^{n} \left((1-|z|^2)\delta_{rs} + \bar{z}_r z_s \right) \overbrace{dz_r \wedge d\bar{z}_s}^{dz_r \otimes d\bar{z}_s - d\bar{z}_s \otimes dz_r}(X_j^\#, \cdot)$$

$$= i(1-|z|^2)^{-2} \Bigg(\sum_{r,s=1}^{n} \left((1-|z|^2)\delta_{rs} + \bar{z}_r z_s \right) i z_j \delta_{rj} \, d\bar{z}_s$$

$$+ \sum_{r,s=1}^{n} \left((1-|z|^2)\delta_{rs} + \bar{z}_r z_s \right) i \bar{z}_j \delta_{js} \, dz_r \Bigg)$$

$$= -(1-|z|^2)^{-2} \Bigg(\sum_{r=1}^{n} \underbrace{\left((1-|z|^2)\delta_{rj} + \bar{z}_r z_j \right) \bar{z}_j}_{(1)} dz_r$$

$$+\sum_{s=1}^{n} \underbrace{\left((1-|z|^2)\delta_{js}+\bar{z}_j z_s\right) z_j}_{②} \, d\bar{z}_s \Bigg) \quad ⊛$$

Buscamos $f_j \ni df_j = \omega(X_j^{\#}, \cdot)$

Por tanto:

$$\frac{\partial f_j}{\partial z_r} = -(1-|z|^2)^{-2} ①, \quad \frac{\partial f_j}{\partial \bar{z}_s} = -(1-|z|^2)^{-2} ②.$$

Consideramos f_j dada por:

$$f_j(z) = -\frac{|z_j|^2}{1-|z|^2}$$

que es $\mathbb{T}^n$-invariante.

Calculamos:

$$\frac{\partial f_j}{\partial z_r}(z) = -(1-|z|^2)^{-2}\Big(\delta_{jr}\bar{z}_j(1-|z|^2) - |z_j|^2(-\bar{z}_r)\Big)$$

$$= -(1-|z|^2)^{-2}\left((1-|z|^2)\delta_{jr}+\bar{z}_r z_j\right)\bar{z}_j$$

$$\frac{\partial f_j}{\partial \bar{z}_s}(z) = -(1-|z|^2)^{-2}\Big(\delta_{js} z_j - |z_j|^2(-z_s)\Big)$$

$$= -(1-|z|^2)^{-2}\left((1-|z|^2)\delta_{js}+\bar{z}_j z_s\right) z_j$$

Por tanto, el mapeo de momento es:

$$u : \mathbb{B}^n \longrightarrow \mathbb{R}^n$$

$$u(z) = \sum_{j=1}^{n} f_j(z)\, X_j$$

$$= - \frac{(|z_1|^2, \dots, |z_n|^2)}{1 - |z|^2}$$