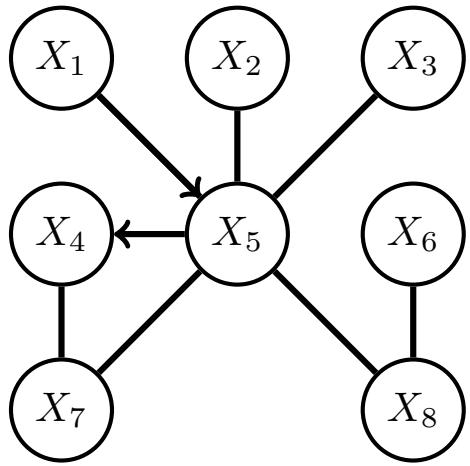
Some basic graph definitions[1]

Salvador Ruiz Correa

Centro de Investigación en Matemáticas (CIMAT)

[1] These slides are *adapted* from those that accompany the book *Bayesian Reasoning and Machine Learning*. The book and demos can be downloaded from `www.cs.ucl.ac.uk/staff/D.Barber/brml`. We acknowledge David Barber for providing the original slides.

Graphs



Definition

A graph consists of nodes (vertices) and undirected or directed links (edges) between nodes.

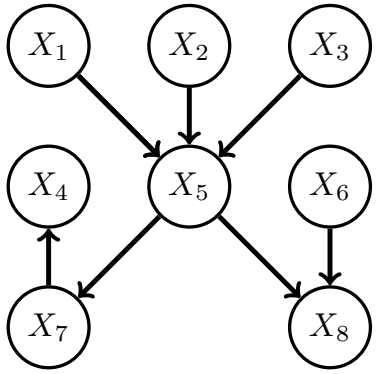
Path

A path from X_i to X_j is a sequence of connected nodes starting at X_i and ending at X_j.

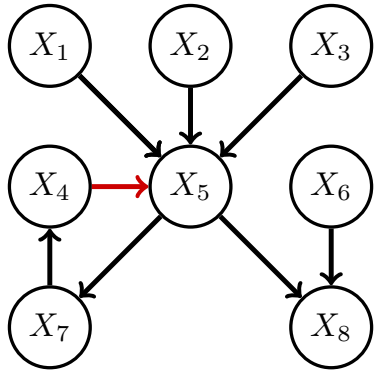
Directed Graphs

All the edges are directed:

Directed Acyclic Graph



Directed Cyclic Graph



DAG

Directed Acyclic Graph: Graph in which by following the direction of the arrows a node will never be visited more than once.

Parents and Children:

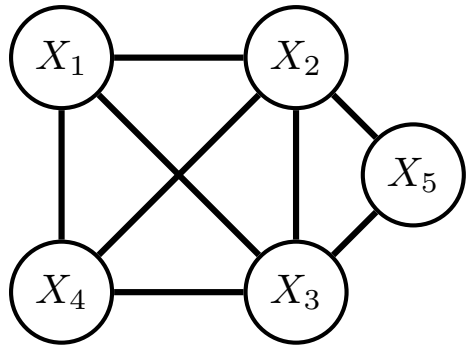
X_i is a parent of X_j if there is a link from X_i to X_j. X_i is a child of X_j if there is a link from X_j to X_i.

Ancestors and Descendants:

The ancestors of a node X_i are the nodes with a directed path ending at X_i. The descendants of X_i are the nodes with a directed path beginning at X_i.

Undirected Graph

All the edges are undirected:



Clique

A clique is a fully connected subset of nodes. (X_1, X_2, X_4) forms a (non-maximal) clique.

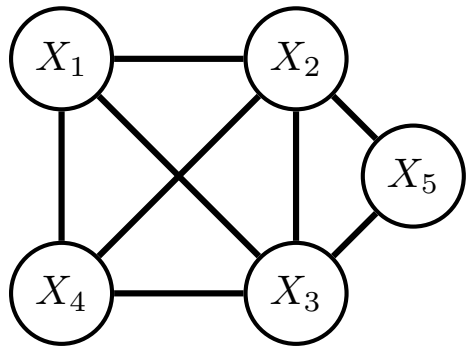
Maximal Clique

Clique which is not a subset of a larger clique. (X_1, X_2, X_3, X_4) and (X_2, X_3, X_5) are both maximal cliques.

Connectivity

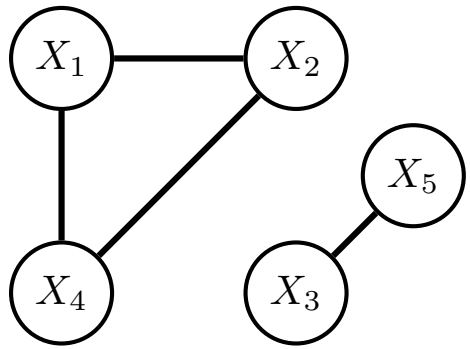
Connected graph

There is a path between every pair of vertices:



Connected components

In a non-connected graph, the connected components are the connected-subgraphs:

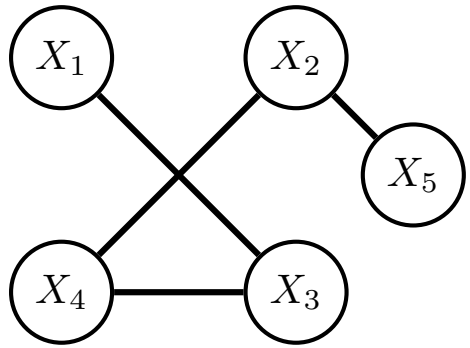


(X_1, X_2, X_4) and (X_3, X_5) are the two connected components.

Connectedness

Singly-connected

There is only one path from any node a to another other node b



Multiply-connected

A graph is multiply-connected if it is not singly-connected: