indicated a few of their applications. Before we can usefully discuss other applications of derivatives, we need to develop efficient techniques for calculating them. The next section begins that task.

Exercises for Section 1.1

In Exercises 1–4, y represents the distance a bus has travelled after x seconds. Find Δy and the average velocity during the time interval Δx for the following situations.

(a) $x_0 = 2$, $\Delta x = 0.5$ (b) $x_0 = 2$, $\Delta x = 0.01$
(c) $x_0 = 4$, $\Delta x = 0.1$ (d) $x_0 = 4$, $\Delta x = 0.01$

1. $y = x^2 + 3x$
2. $y = 3x^2 + x$ c) d)
3. $y = x^2 + 10x$
4. $y = 2x$ a) b)

In Exercises 5–8, $f(x)$ is the number of meters a bus has gone at a time x (in seconds). Find the instantaneous velocity at the given time x_0.

5. $x^2 + 3x$; $x_0 = 2$
6. $x^2 + 3x$; $x_0 = 4$
7. $3x^2 + x$; $x_0 = 2$
8. $3x^2 + x$; $x_0 = 4$

In Exercises 9–12, y is the position (measured in meters) of a bus at time x (in seconds). (a) Find the instantaneous velocity at an arbitrary (positive) time x_0. (b) At what time is the instantaneous velocity 10 meters per second?

9. $y = x^2 + 3x$
10. $y = 3x^2 + x$
11. $y = x^2 + 10x$
12. $y = 2x$

In Exercises 13–16, use the $\Delta y/\Delta x$ method of Example 4 to find the slope of the tangent line to the graph of the given function at the given point. Sketch.

13. $y = x^2$; $x_0 = 1$
14. $y = -x^2$; $x_0 = 2$
15. $y = 5x^2 - 3x + 1$; $x_0 = 0$
16. $y = x + 1 - x^2$; $x_0 = 2$

In Exercises 17–20 use the $\Delta y/\Delta x$ method of Example 5 to compute the derivative of $f(x)$ at x_0; a is a constant in each case.

17. $f(x) = ax + 2$; $x_0 = 0$
18. $f(x) = 2x + a$; $x_0 = 0$
19. $f(x) = ax^2$; $x_0 = 1$
20. $f(x) = 8x^2 + a$; $x_0 = 2$

In Exercises 21–24, use the quadratic function rule to find the derivative of the given function at the indicated point.

21. $f(x) = x^2 + x - 1$; $x_0 = 1$
22. $f(x) = x^2 - x$; $x_0 = 2$
23. $f(x) = 3x^2 + x - 2$; $x_0 = -2$
24. $f(x) = -3x^2 - x + 1$; $x_0 = -1$

In Exercises 25–28, find the vertex of the given parabola using (a) derivatives and (b) algebra.

25. $y = x^2 - 16x + 2$
26. $y = x^2 + 8x + 2$
27. $y = -2x^2 - 8x - 1$
28. $y = -2x^2 - 3x + 5$

Differentiate the functions in Exercises 29–36 using the quadratic function rule.

29. $f(x) = x^2 + 3x - 1$
30. $f(x) = -3x + 4$
31. $f(x) = (x - 1)(x + 1)$
32. $f(x) = (9 - x)(1 - x)$
33. $g(t) = -4t^2 + 3t + 6$
34. $g(r) = \pi r^2 + 3$
35. $g(s) = 1 - s^2$
36. $h(t) = 3t^2 - 5t + 9$
37. Inspector Clumseaux is on a moving passenger train. His distance function is $2t^2 + 3t$. On the adjacent track is a long moving freight train; the distance function for the center of the freight train is $3t^2 + t$. What is the best time for him to jump to the freight train?
38. Two trains, A and B, are moving on adjacent tracks with positions given by the functions $A(t) = t^2 + t + 5$ and $B(t) = 3t + 4$. What is the best time for a hobo on train B to make a moving transfer to train A?
39. An apple falls from a tall tree toward the earth. After t seconds, it has fallen $4.9t^2$ meters. What is the velocity of the apple when $t = 3$?
40. A rock thrown down from a bridge has fallen $4t + 4.9t^2$ meters after t seconds. Find its velocity at $t = 3$.
41. $f(x) = x^2 - 2$; find $f'(3)$
42. $f(x) = -13x^2 - 9x + 5$; find $f'(1)$
43. $f(x) = 1$; find $f'(7)$
44. $g(s) = 0$; find $g'(3)$
45. $k(y) = (y + 4)(y - 7)$; find $k'(-1)$
46. $x(f) = 1 - f^2$; find $x'(0)$
47. $f(x) = -x + 2$; find $f'(3.752764)$
48. $g(a) = 10a - 8$; find $g'(3.1415)$

In Exercises 49–54, find the derivative of each of the given functions by finding the value approximated by $\Delta y/\Delta x$ for Δx small:

49. $4x^2 + 3x + 2$
50. $(x - 3)(x + 1)$
51. $1 - x^2$
52. $-x^2$
53. $-2x^2 + 5x$
54. $1 - x$
55. Let $f(x) = 2x^2 + 3x + 1$. (a) For which values of x is $f'(x)$ negative, positive, and zero? (b) Identify these points on a graph of f.
56. Show that two quadratic functions which have the same derivative must differ by a constant.
57. Let $A(x)$ be the area of a square of side length x. Show that $A'(x)$ is half the perimeter of the square.
58. Let $A(r)$ be the area of a circle of radius r. Show that $A'(r)$ is the circumference.
59. Where does the line tangent to the graph of $y = x^2$ at $x_0 = 2$ intersect the x axis?
60. Where does the line tangent to the graph of $y = 2x^2 - 8x + 1$ at $x_0 = 1$ intersect the y axis?
61. Find the equation of the line tangent to the graph of $f(x) = 3x^2 + 4x + 2$ at the point where $x_0 = 1$. Sketch.
62. Find the tangent line to the parabola $y = x^2 - 3x + 1$ when $x_0 = 2$. Sketch.

★63. Find the lines through the point $(4, 7)$ which are tangent to the graph of $y = x^2$. Sketch. (*Hint*: Find and solve an equation for the x coordinate of the point of tangency.)

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