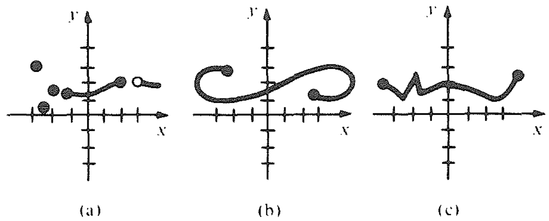

Figure R.6.10. Which curves are graphs of functions?

Solution (a) (i) yes; (ii) no; this is indicated by the white dot.
(b) (i) no; for example, the line $x = 3$ cuts the curve in two points.
(c) (i) yes; (ii) yes; (iii) 1. ▲

Example 10 Which of the curves $y^2 = x$ and $y^3 = x$ is the graph of a function?

Solution We begin with $y^2 = x$. Note that each value of y determines a unique value of x; we plot a few points in Fig. R.6.11.

We see immediately that the vertical line $x = 4$ meets $y^2 = x$ in two points, so $y^2 = x$ cannot be the graph of a function. (The curve $y^2 = x$ is a parabola whose axis of symmetry is horizontal.) Now look at $y^3 = x$. We begin by plotting a few points (Fig. R.6.12).

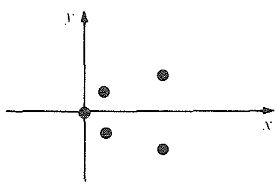

Figure R.6.11. Five points satisfying $y^2 = x$.

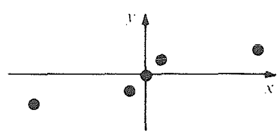

Figure R.6.12. Five points on $y^3 = x$.

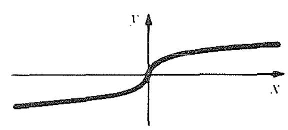

Figure R.6.13. The curve $y^3 = x$.

These points could all lie on the graph of a function. In fact, the full curve $y^3 = x$ appears as in Fig. R.6.13. We see by inspection that the curve intersects each vertical line exactly once, so there is a function f whose graph is the given curve. Since $f(x)$ is a number whose cube is x, f is called the *cube root* function. ▲

Exercises for Section R.6

Evaluate each of the functions in Exercises 1–6 at $x = -1$ and $x = 1$:

1. $f(x) = 5x^2 - 2x$
2. $f(x) = -x^2 + 3x - 5$
3. $f(x) = x^3 - 2x^2 + 1$
4. $f(x) = 4x^2 + x - 2$
5. $f(x) = -x^3 + x^2 - x + 1$
6. $f(x) = (x-1)^2 + (x+1)^2 + 2$

Find the domain of each of the functions in Exercises 7–12 and evaluate each function at $x = 10$.

7. $f(x) = \dfrac{x^2}{x-1}$
8. $f(x) = \dfrac{x^2}{x^2 + 2x - 1}$
9. $f(x) = 5x\sqrt{1 - x^2}$
10. $f(x) = \dfrac{x^2 - 1}{\sqrt{x - 4}}$
11. $f(x) = \dfrac{5x + 2}{x^2 - x - 6}$
12. $f(x) = \dfrac{x}{(x^2 - 2)^2}$

Draw the graphs of the indicated functions in Exercises 13–20.

13. f in Exercise 1
14. f in Exercise 2
15. f in Exercise 4
16. f in Exercise 6
17. $f(x) = (x-1)^2 + 3$
18. $f(x) = x^2 - 9$
19. $f(x) = 3x + 10$
20. $f(x) = x^2 + 4x + 2$

Draw the graphs of the functions in Exercises 21–24.

21. $f(x) = |x - 1|$
22. $f(x) = 3|x - 2|$
23. $f(x) = \sqrt{x - 1}$

24. $f(x) = 2\sqrt{x-2}$

Plot 10 points on the graphs of the functions in Exercises 25 and 26.

25. f in Exercise 7
26. f in Exercise 10

Plot the graphs of the functions in Exercises 27 and 28 using the given sets of values for x.

27. $f(x) = 3x^3 - x^2 + 1$:
    (a) at $x = -2, -1, 0, 1, 2$;
    (b) at 0.2 intervals in $[-2, 2]$.
28. $f(x) = 2x^4 - x^3$:
    (a) at $x = -2, -1, 0, 1, 2$;
    (b) at 0.1 intervals in $[-2, 2]$.
29. Which of the curves in Fig. R.6.14 are graphs of functions?

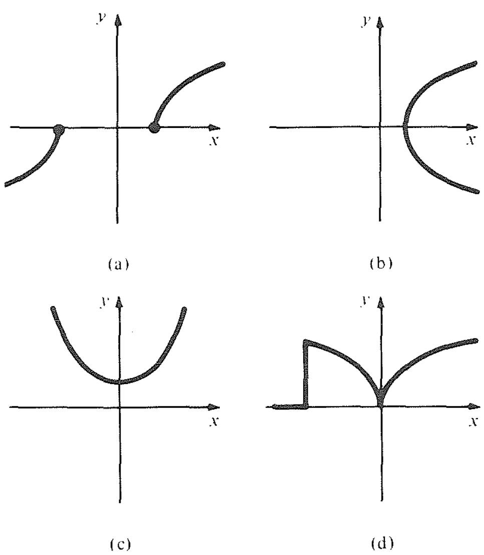

Figure R.6.14. Which curves are graphs of functions?

30. Match the following formulas with the sets in Fig. R.6.15:
    (a) $x + y \geqslant 1$
    (b) $x - y \leqslant 1$
    (c) $y = (x-1)^2$
    (d) $y = \left(\dfrac{x+|x|}{2}\right)^2$
    (e) Not the graph of a function
    (f) None of the above

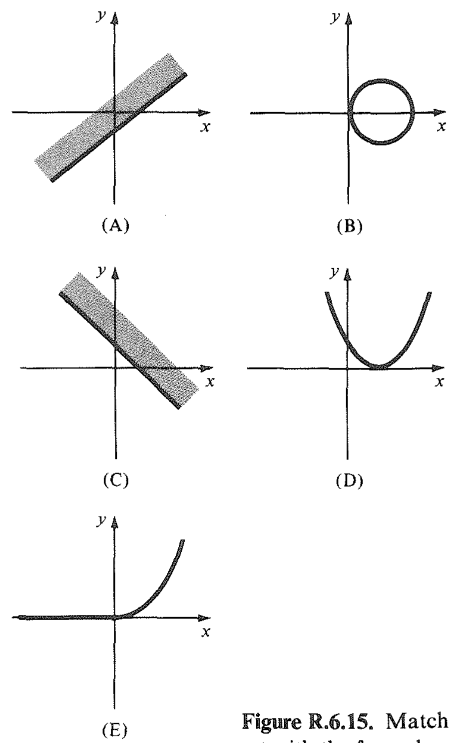

Figure R.6.15. Match the set with the formula.

Tell whether the curve defined by each of the equations in Exercises 31–36 is the graph of a function. If the answer is yes, find the domain of the function.

31. $xy = 1$
32. $x^2 - y^2 = 1$
33. $y = \sqrt{x^2 - 1}$
34. $x + y^2 = 3$
35. $y + x^2 = 3$
36. $x^2 - y^3 = 1$

Review Exercises for Chapter R

In Exercises 1–8, solve for x.

1. $3x + 2 = 0$
2. $x^2 + 2x + 1 = 0$
3. $x(x-1) = 1$
4. $(x-1)^2 - 9 = 0$
5. $(x+1)^2 - (x-1)^2 = 2$
6. $x^2 - 8(x-2) + 10 = 0$
7. $(x-1)^3 = 8$
8. $x^3 + 3x^2 - 3x = 9$

In Exercises 9–16, find all x satisfying the given inequality.

9. $8x + 2 > 0$
10. $10x - 6 < 5$
11. $x^2 - 5 < 0$
12. $x^2 - 5x + 6 < 0$
13. $x^2 - (x-1)^2 > 2$
14. $x^3 > 1$
15. $3x^2 - 7 < 0$
16. $x^2 - 2x + 1 > 0$