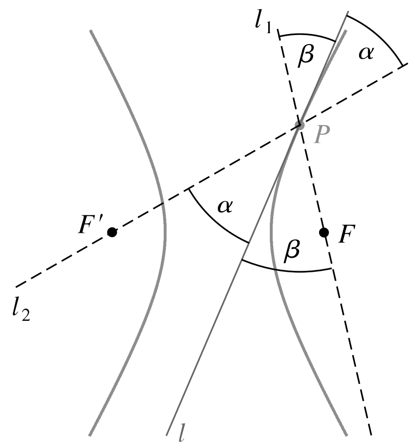
point P on a hyperbola with foci F and F', as shown in Figure 12. If α is the acute angle between $F'P$ and l and if β is the acute angle between FP and l, it can be shown that $\alpha = \beta$. If a ray of light is directed along the line l_1 toward F, it will be reflected back at P along the line l_2 toward F'. This property is used in the design of telescopes of the Cassegrain type (see Exercise 64).

Figure 12



11.3 Exercises

Exer. 1–16: Find the vertices, the foci, and the equations of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes and the foci.

1 $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$

2 $\dfrac{y^2}{49} - \dfrac{x^2}{16} = 1$

3 $\dfrac{y^2}{9} - \dfrac{x^2}{4} = 1$

4 $\dfrac{x^2}{49} - \dfrac{y^2}{16} = 1$

5 $x^2 - \dfrac{y^2}{24} = 1$

6 $y^2 - \dfrac{x^2}{15} = 1$

7 $y^2 - 4x^2 = 16$

8 $x^2 - 2y^2 = 8$

9 $16x^2 - 36y^2 = 1$

10 $y^2 - 16x^2 = 1$

11 $\dfrac{(y+2)^2}{9} - \dfrac{(x+2)^2}{4} = 1$

12 $\dfrac{(x-3)^2}{25} - \dfrac{(y-1)^2}{4} = 1$

13 $144x^2 - 25y^2 + 864x - 100y - 2404 = 0$

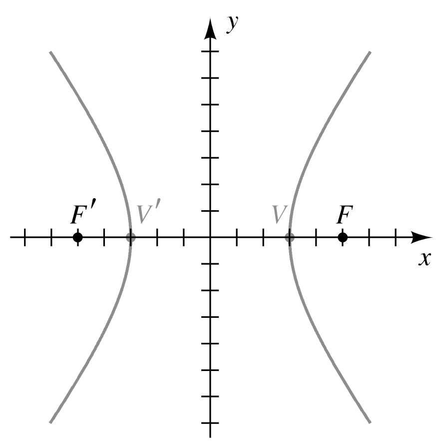
14 $y^2 - 4x^2 - 12y - 16x + 16 = 0$

15 $4y^2 - x^2 + 40y - 4x + 60 = 0$

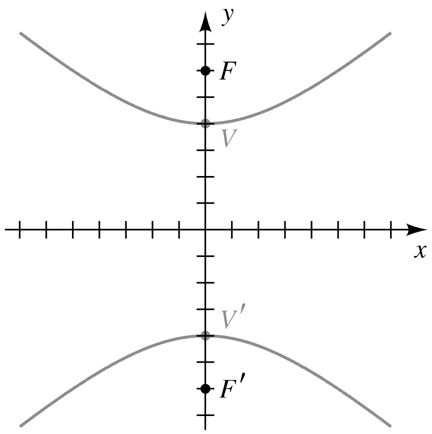
16 $25x^2 - 9y^2 + 100x - 54y + 10 = 0$

Exer. 17–20: Find an equation for the hyperbola shown in the figure.

17



18



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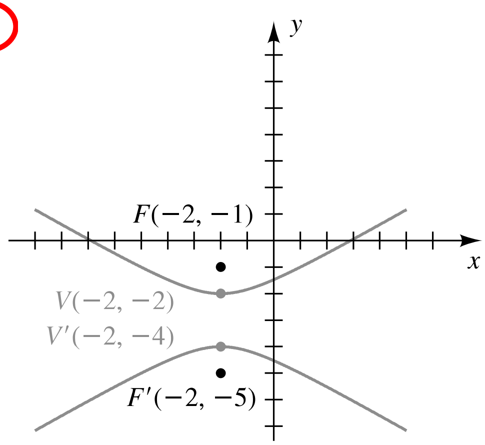

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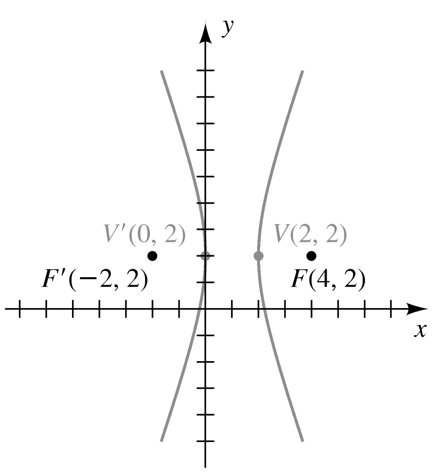


Exer. 21–32: Find an equation for the hyperbola that has its center at the origin and satisfies the given conditions.

21 Foci $F(0, \pm 4)$, vertices $V(0, \pm 1)$

22 Foci $F(\pm 8, 0)$, vertices $V(\pm 5, 0)$

23 Foci $F(\pm 5, 0)$, vertices $V(\pm 3, 0)$

24 Foci $F(0, \pm 3)$, vertices $V(0, \pm 2)$

25 Foci $F(0, \pm 5)$, conjugate axis of length 4

26 Vertices $V(\pm 4, 0)$, passing through $(8, 2)$

27 Vertices $V(\pm 3, 0)$, asymptotes $y = \pm 2x$

28 Foci $F(0, \pm 10)$, asymptotes $y = \pm \frac{1}{3}x$

29 x-intercepts ± 5, asymptotes $y = \pm 2x$

30 y-intercepts ± 2, asymptotes $y = \pm \frac{1}{4}x$

31 Vertical transverse axis of length 10, conjugate axis of length 14

32 Horizontal transverse axis of length 6, conjugate axis of length 2

Exer. 33–42: Identify the graph of the equation as a parabola (with vertical or horizontal axis), circle, ellipse, or hyperbola.

33 $\frac{1}{3}(x + 2) = y^2$

34 $y^2 = \frac{14}{3} - x^2$

35 $x^2 + 6x - y^2 = 7$

36 $x^2 + 4x + 4y^2 - 24y = -36$

37 $-x^2 = y^2 - 25$

38 $x = 2x^2 - y + 4$

39 $4x^2 - 16x + 9y^2 + 36y = -16$

40 $x + 4 = y^2 + y$

41 $x^2 + 3x = 3y - 6$

42 $9x^2 - y^2 = 10 - 2y$

Exer. 43–44: Find the points of intersection of the graphs of the equations. Sketch both graphs on the same coordinate plane, and show the points of intersection.

43 $\begin{cases} y^2 - 4x^2 = 16 \\ y - x = 4 \end{cases}$

44 $\begin{cases} x^2 - y^2 = 4 \\ y^2 - 3x = 0 \end{cases}$

Exer. 45–48: Find an equation for the set of points in an *xy*-plane such that the difference of the distances from *F* and *F′* is *k*.

45 $F(13, 0)$, $F'(-13, 0)$; $k = 24$

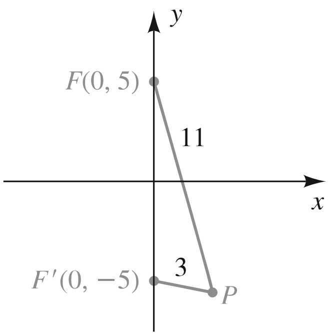
46 $F(5, 0)$, $F'(-5, 0)$; $k = 8$

47 $F(0, 10)$, $F'(0, -10)$; $k = 16$

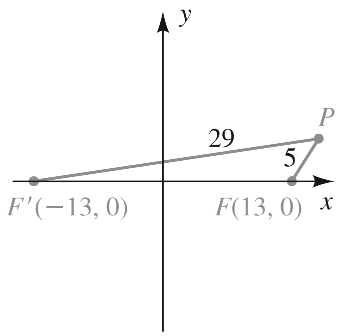
48 $F(0, 17)$, $F'(0, -17)$; $k = 30$

Exer. 49–50: Find an equation for the hyperbola with foci *F* and *F′* that passes through *P*. Sketch the hyperbola.

49



50



Exer. 51–58: Describe the part of a hyperbola given by the equation.

51 $x = \frac{5}{4}\sqrt{y^2 + 16}$

52 $x = -\frac{5}{4}\sqrt{y^2 + 16}$

53 $y = \frac{3}{7}\sqrt{x^2 + 49}$

54 $y = -\frac{3}{7}\sqrt{x^2 + 49}$

55 $y = -\frac{9}{4}\sqrt{x^2 - 16}$

56 $y = \frac{9}{4}\sqrt{x^2 - 16}$

57 $x = -\frac{2}{3}\sqrt{y^2 - 36}$

58 $x = \frac{2}{3}\sqrt{y^2 - 36}$

59 The graphs of the equations

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{and} \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

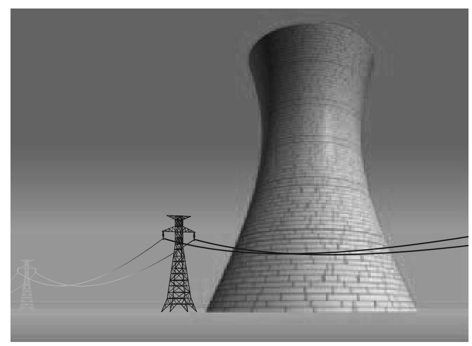
are called *conjugate hyperbolas.* Sketch the graphs of both equations on the same coordinate plane, with $a = 5$ and $b = 3$, and describe the relationship between the two graphs.

60 Find an equation of the hyperbola with foci $(h \pm c, k)$ and vertices $(h \pm a, k)$, where

$$0 < a < c \quad \text{and} \quad c^2 = a^2 + b^2.$$

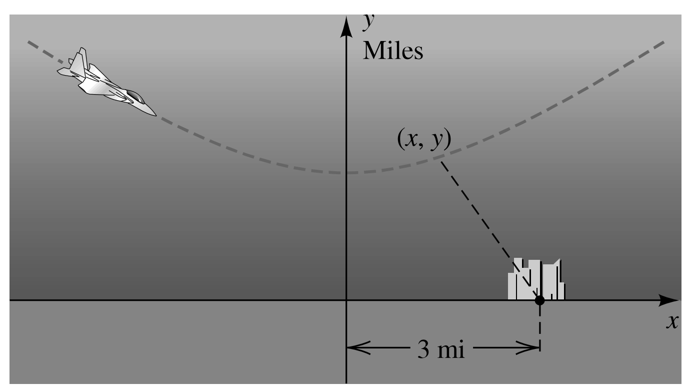
61 **Cooling tower** A cooling tower, such as the one shown in the figure, is a hyperbolic structure. Suppose its base diameter is 100 meters and its smallest diameter of 48 meters occurs 84 meters from the base. If the tower is 120 meters high, approximate its diameter at the top.

Exercise 61



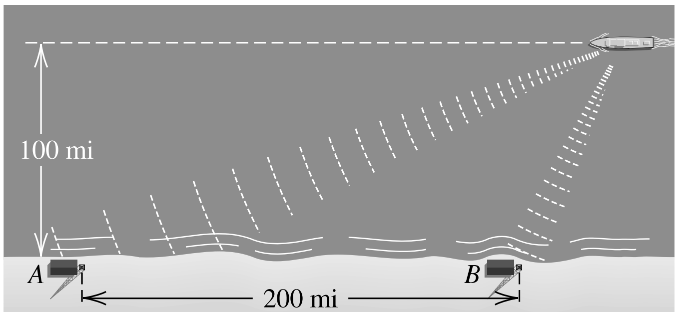
62 **Airplane maneuver** An airplane is flying along the hyperbolic path illustrated in the figure. If an equation of the path is $2y^2 - x^2 = 8$, determine how close the airplane comes to a town located at $(3, 0)$. (*Hint:* Let S denote the square of the distance from a point (x, y) on the path to $(3, 0)$, and find the minimum value of S.)

Exercise 62



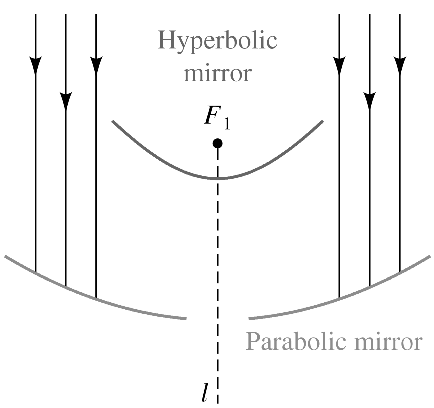
63 **Locating a ship** A ship is traveling a course that is 100 miles from, and parallel to, a straight shoreline. The ship sends out a distress signal that is received by two Coast Guard stations A and B, located 200 miles apart, as shown in the figure. By measuring the difference in signal reception times, it is determined that the ship is 160 miles closer to B than to A. Where is the ship?

Exercise 63



64 **Design of a telescope** The Cassegrain telescope design (dating back to 1672) makes use of the reflective properties of both the parabola and the hyperbola. Shown in the figure is a (split) parabolic mirror, with focus at F_1 and axis along the line l, and a hyperbolic mirror, with one focus also at F_1 and transverse axis along l. Where do incoming light waves parallel to the common axis finally collect?

Exercise 64



65 **Comet's path** Comets can travel in elliptical, parabolic, or hyperbolic paths around the sun. If a comet travels in a parabolic or hyperbolic path, it will pass by the sun once and never return. Suppose that a comet's coordinates in miles can be described by the equation

$$\frac{x^2}{26 \times 10^{14}} - \frac{y^2}{18 \times 10^{14}} = 1 \quad \text{for} \quad x > 0,$$

where the sun is located at a focus, as shown in the figure.

(a) Approximate the coordinates of the sun.

(b) For the comet to maintain a hyperbolic trajectory, the minimum velocity v of the comet, in meters per second, must satisfy $v > \sqrt{2k/r}$, where r is the distance between the comet and the center of the sun in meters and $k = 1.325 \times 10^{20}$ is a constant. Determine v when r is minimum.

Exercise 65