Figure 13

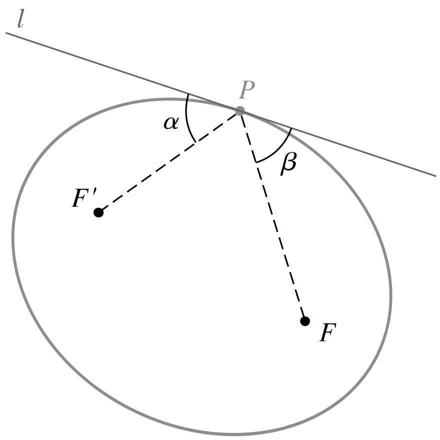
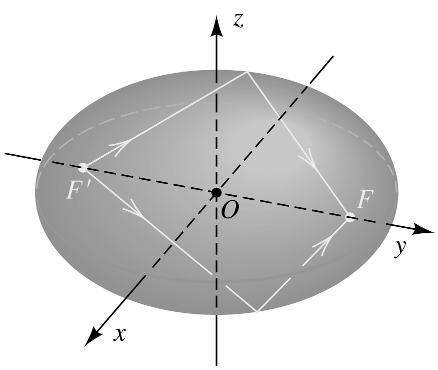



Figure 14




An ellipse has a *reflective property* analogous to that of the parabola discussed at the end of the previous section. To illustrate, let $l$ denote the tangent line at a point $P$ on an ellipse with foci $F$ and $F'$, as shown in Figure 13. If $\alpha$ is the acute angle between $F'P$ and $l$ and if $\beta$ is the acute angle between $FP$ and $l$, it can be shown that $\alpha = \beta$. Thus, if a ray of light or sound emanates from one focus, it is reflected to the other focus. This property is used in the design of certain types of optical equipment.

If the ellipse with center $O$ and foci $F'$ and $F$ on the $x$-axis is revolved about the $x$-axis, as illustrated in Figure 14, we obtain a three-dimensional surface called an **ellipsoid**. The upper half or lower half is a **hemi-ellipsoid**, as is the right half or left half. Sound waves or other impulses that are emitted from the focus $F'$ will be reflected off the ellipsoid into the focus $F$. This property is used in the design of *whispering galleries*—structures with ellipsoidal ceilings, in which a person who whispers at one focus can be heard at the other focus. Examples of whispering galleries may be found in the Rotunda of the Capitol Building in Washington, D.C., and in the Mormon Tabernacle in Salt Lake City.

The reflective property of ellipsoids (and hemi-ellipsoids) is used in modern medicine in a device called a *lithotripter,* which disintegrates kidney stones by means of high-energy underwater shock waves. After taking extremely accurate measurements, the operator positions the patient so that the stone is at a focus. Ultra–high frequency shock waves are then produced at the other focus, and reflected waves break up the kidney stone. Recovery time with this technique is usually 3–4 days, instead of the 2–3 weeks with conventional surgery. Moreover, the mortality rate is less than 0.01%, as compared to 2–3% for traditional surgery (see Exercises 51–52).

## 11.2 Exercises

**Exer. 1–14: Find the vertices and foci of the ellipse. Sketch its graph, showing the foci.**

1 $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

2 $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

3 $\dfrac{x^2}{15} + \dfrac{y^2}{16} = 1$

4 $\dfrac{x^2}{45} + \dfrac{y^2}{49} = 1$

5 $4x^2 + y^2 = 16$

6 $y^2 + 9x^2 = 9$

7 $4x^2 + 25y^2 = 1$

8 $10y^2 + x^2 = 5$

9 $\dfrac{(x-3)^2}{16} + \dfrac{(y+4)^2}{9} = 1$

10 $\dfrac{(x+2)^2}{25} + \dfrac{(y-3)^2}{4} = 1$

11 $4x^2 + 9y^2 - 32x - 36y + 64 = 0$

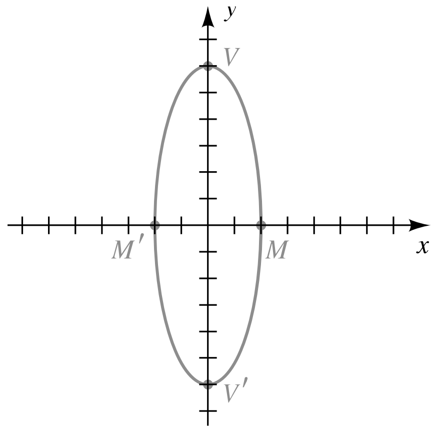
12 $x^2 + 2y^2 + 2x - 20y + 43 = 0$

13 $25x^2 + 4y^2 - 250x - 16y + 541 = 0$

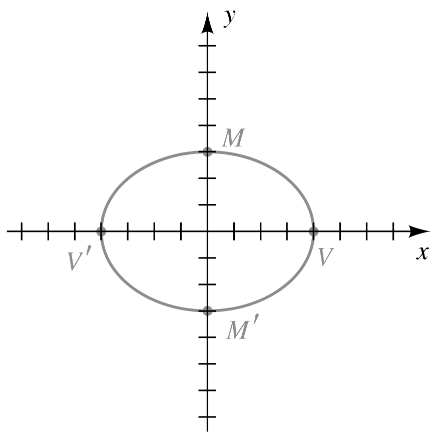
14 $4x^2 + y^2 = 2y$

**Exer. 15–18: Find an equation for the ellipse shown in the figure.**

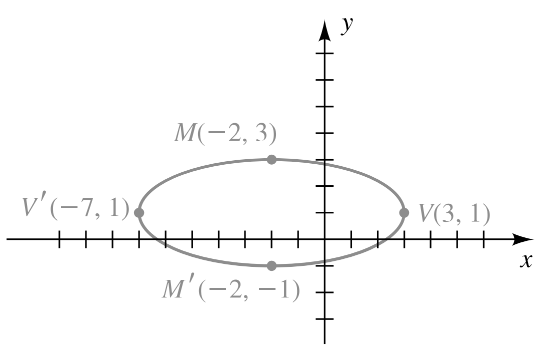
15



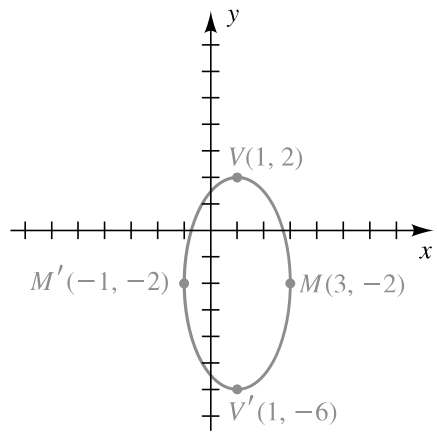

16




17




18




**Exer. 19–30: Find an equation for the ellipse that has its center at the origin and satisfies the given conditions.**

19 Vertices $V(\pm 8, 0)$, foci $F(\pm 5, 0)$

20 Vertices $V(0, \pm 7)$, foci $F(0, \pm 2)$

21 Vertices $V(0, \pm 5)$, minor axis of length 3

22 Foci $F(\pm 3, 0)$, minor axis of length 2

23 Vertices $V(0, \pm 6)$, passing through (3, 2)

24 Passing through (2, 3) and (6, 1)

25 Eccentricity $\frac{3}{4}$, vertices $V(0, \pm 4)$

26 Eccentricity $\frac{1}{2}$, vertices on the $x$-axis, passing through (1, 3)

27 $x$-intercepts $\pm 2$, $y$-intercepts $\pm \frac{1}{3}$

28 $x$-intercepts $\pm \frac{1}{2}$, $y$-intercepts $\pm 4$

29 Horizontal major axis of length 8, minor axis of length 5

30 Vertical major axis of length 7, minor axis of length 6

**Exer. 31–32: Find the points of intersection of the graphs of the equations. Sketch both graphs on the same coordinate plane, and show the points of intersection.**

31 $\begin{cases} x^2 + 4y^2 = 20 \\ x + 2y = 6 \end{cases}$

32 $\begin{cases} x^2 + 4y^2 = 36 \\ x^2 + y^2 = 12 \end{cases}$

**Exer. 33–36: Find an equation for the set of points in an *xy*-plane such that the sum of the distances from *F* and *F′* is *k*.**

33 $F(3, 0)$, $F'(-3, 0)$; $k = 10$

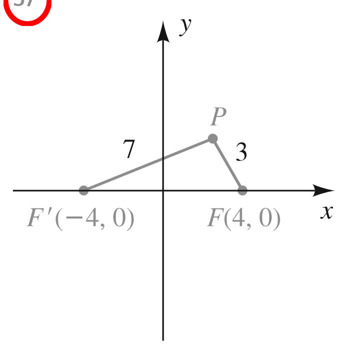
34 $F(12, 0)$, $F'(-12, 0)$; $k = 26$

35 $F(0, 15)$, $F'(0, -15)$; $k = 34$

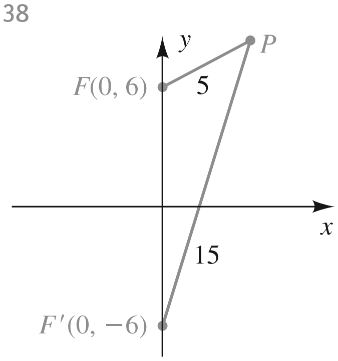
36 $F(0, 8)$, $F'(0, -8)$; $k = 20$

**Exer. 37–38: Find an equation for the ellipse with foci *F* and *F′* that passes through *P*. Sketch the ellipse.**

37




38




**Exer. 39–46: Determine whether the graph of the equation is the upper, lower, left, or right half of an ellipse, and find an equation for the ellipse.**

39 $y = 11\sqrt{1 - \dfrac{x^2}{49}}$

40 $y = -6\sqrt{1 - \dfrac{x^2}{25}}$

41 $x = -\frac{1}{3}\sqrt{9 - y^2}$

42 $x = \frac{4}{5}\sqrt{25 - y^2}$

43 $x = 1 + 2\sqrt{1 - \dfrac{(y + 2)^2}{9}}$

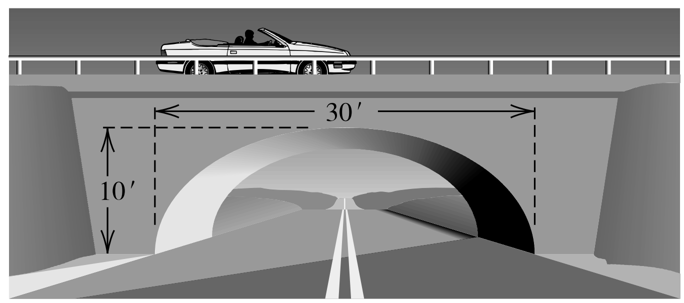
44 $x = -2 - 5\sqrt{1 - \dfrac{(y - 1)^2}{16}}$

45 $y = 2 - 7\sqrt{1 - \dfrac{(x + 1)^2}{9}}$

46 $y = -1 + \sqrt{1 - \dfrac{(x - 3)^2}{16}}$

47 Dimensions of an arch An arch of a bridge is semielliptical, with major axis horizontal. The base of the arch is 30 feet across, and the highest part of the arch is 10 feet above the horizontal roadway, as shown in the figure. Find the height of the arch 6 feet from the center of the base.

Exercise 47




48 Designing a bridge A bridge is to be constructed across a river that is 200 feet wide. The arch of the bridge is to be semielliptical and must be constructed so that a ship less than 50 feet wide and 30 feet high can pass safely through the arch, as shown in the figure.

(a) Find an equation for the arch.

(b) Approximate the height of the arch in the middle of the bridge.

Exercise 48