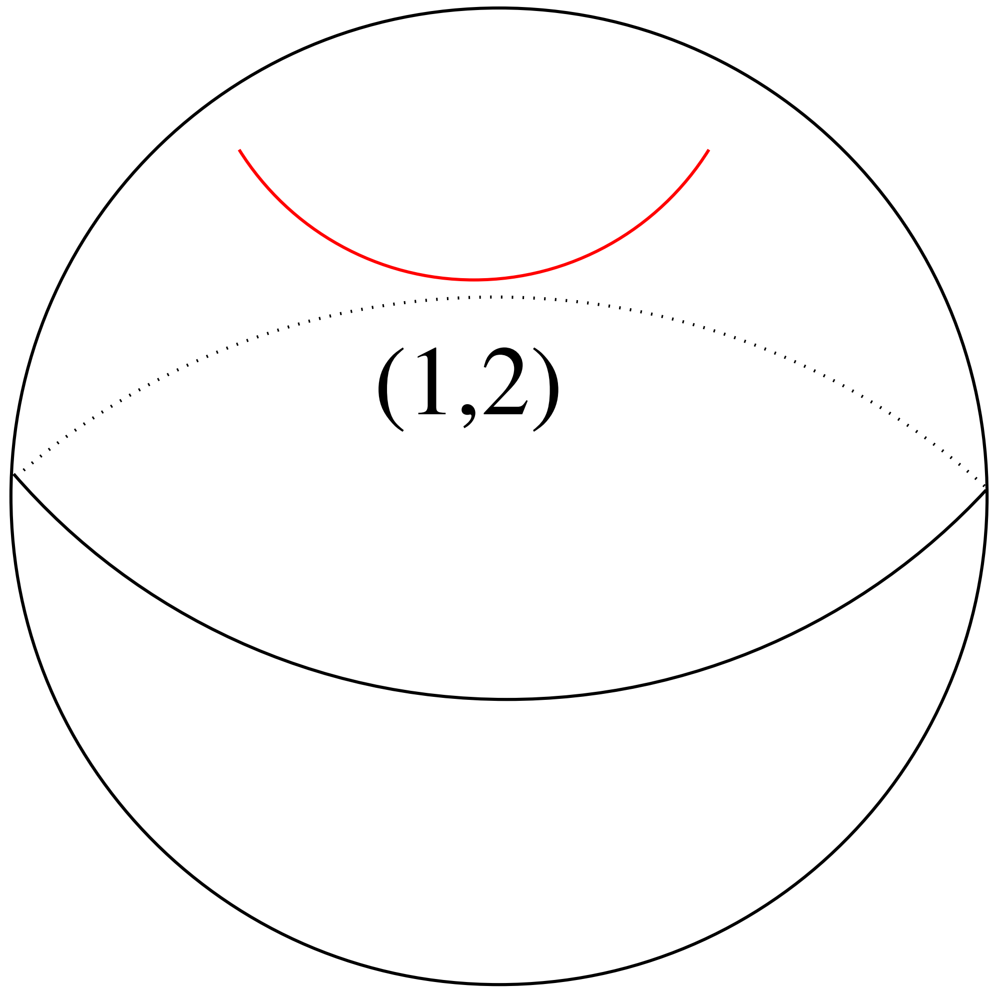
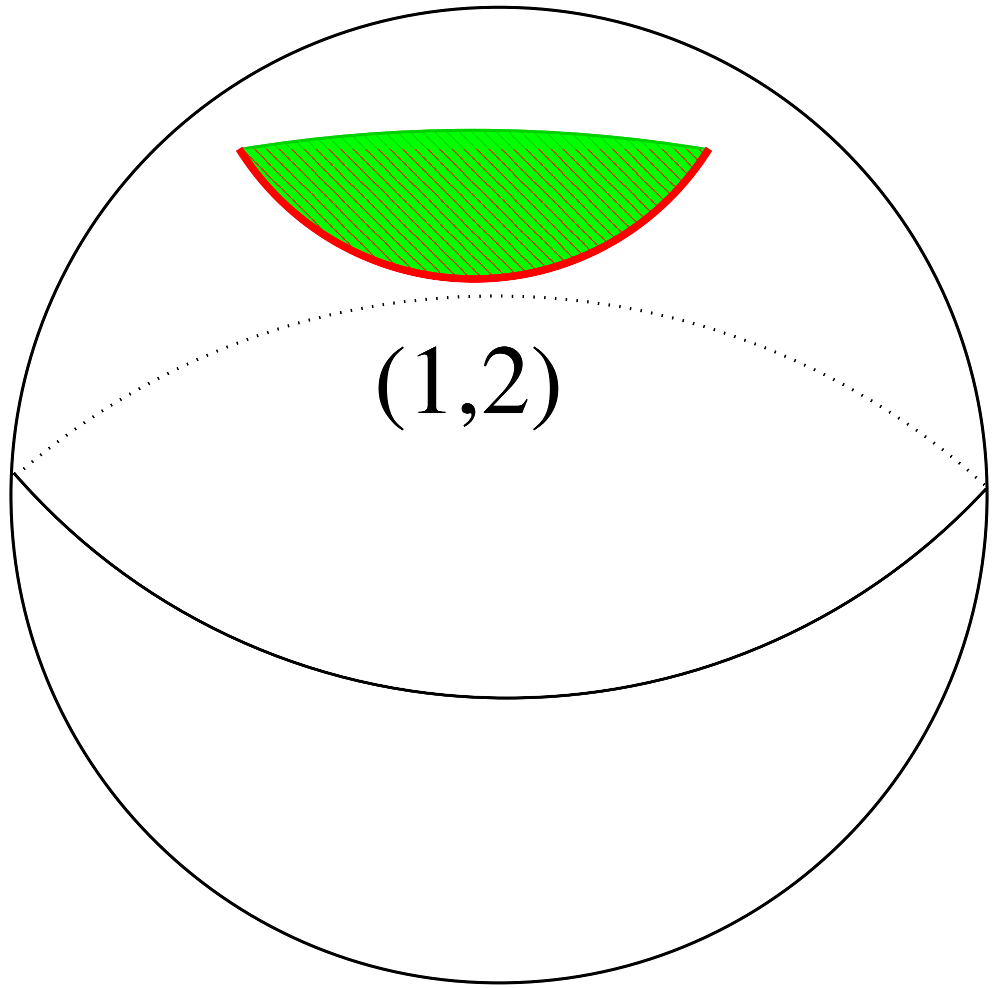
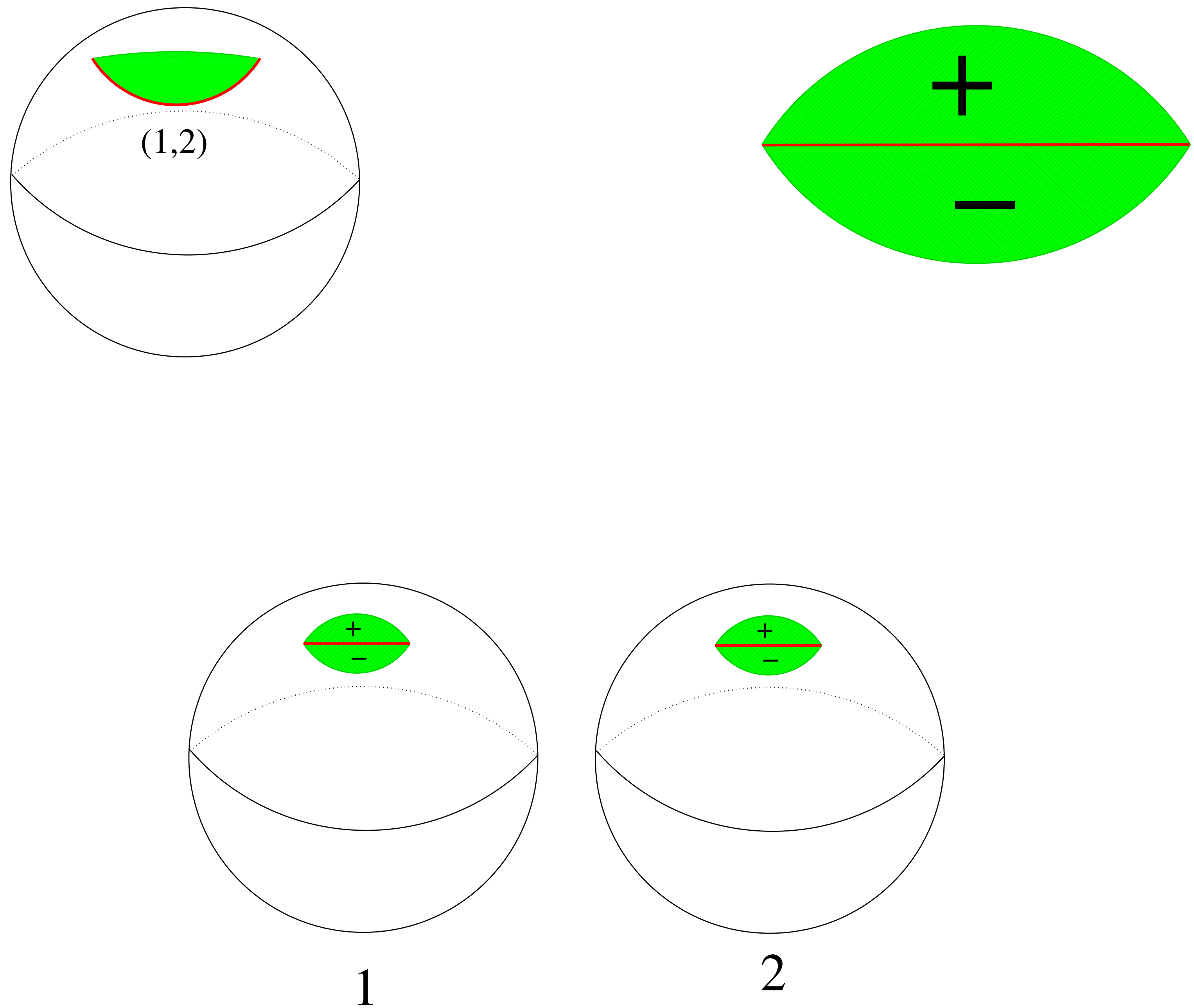
Nudos Universales

Víctor Núñez
Cimat



Una 3-bola B con
un arco $\alpha \subset B$ propiamente encajado y
una permutación $(1,2) \in S_n$.


(1,2)


(1,2)
+
−
+
−
+
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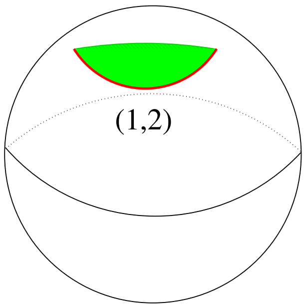
1
2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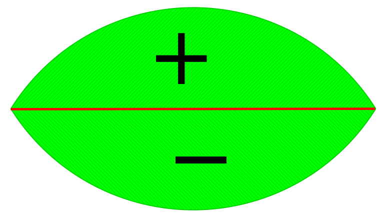
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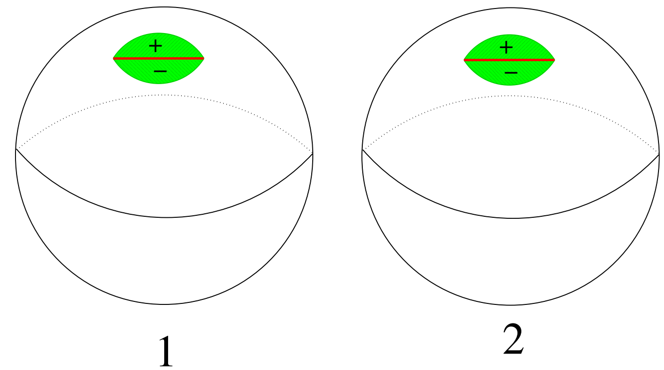
+
−
+
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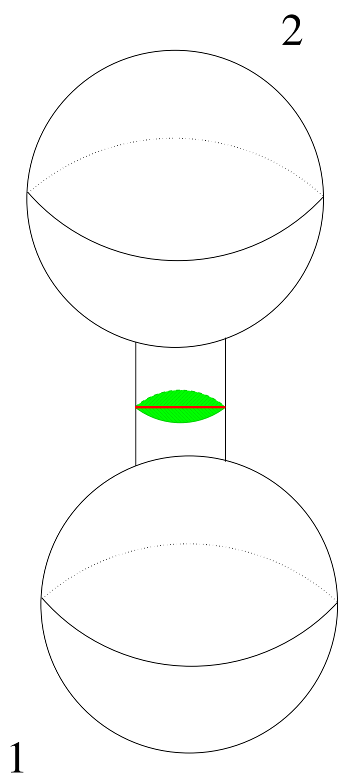
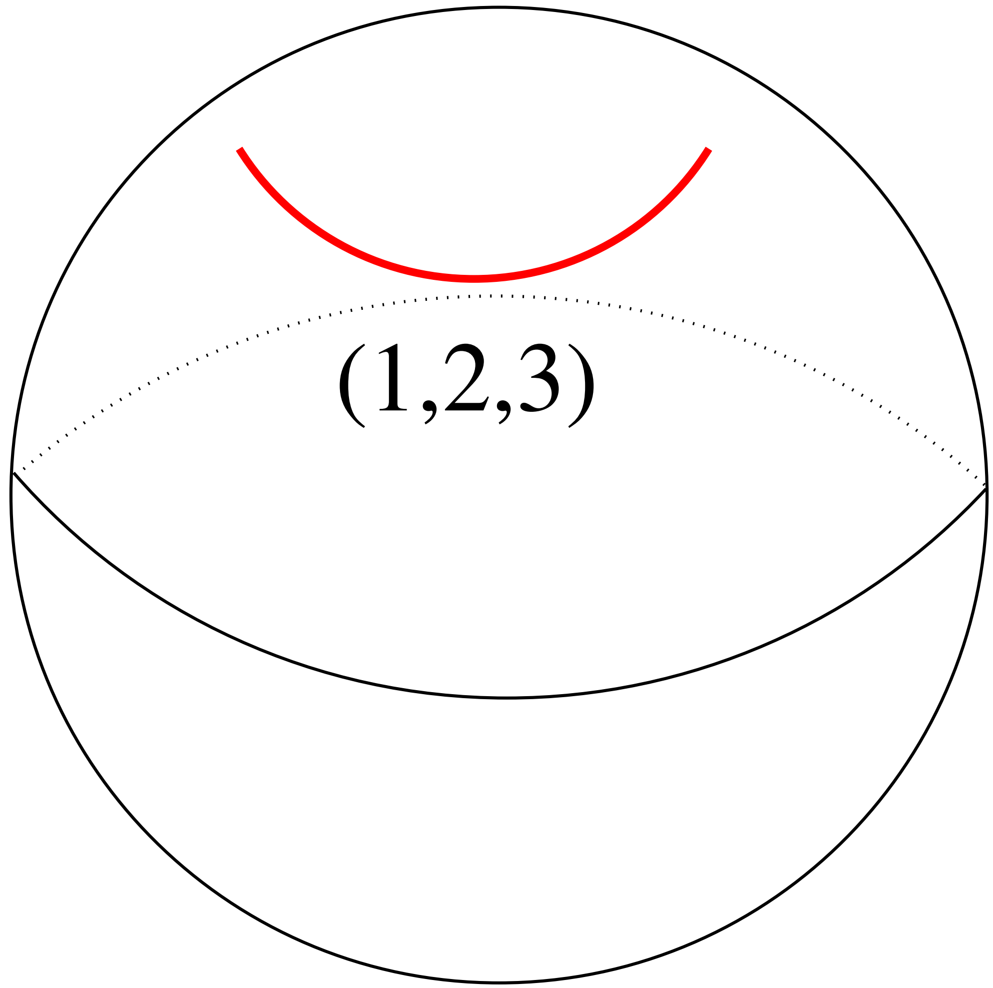
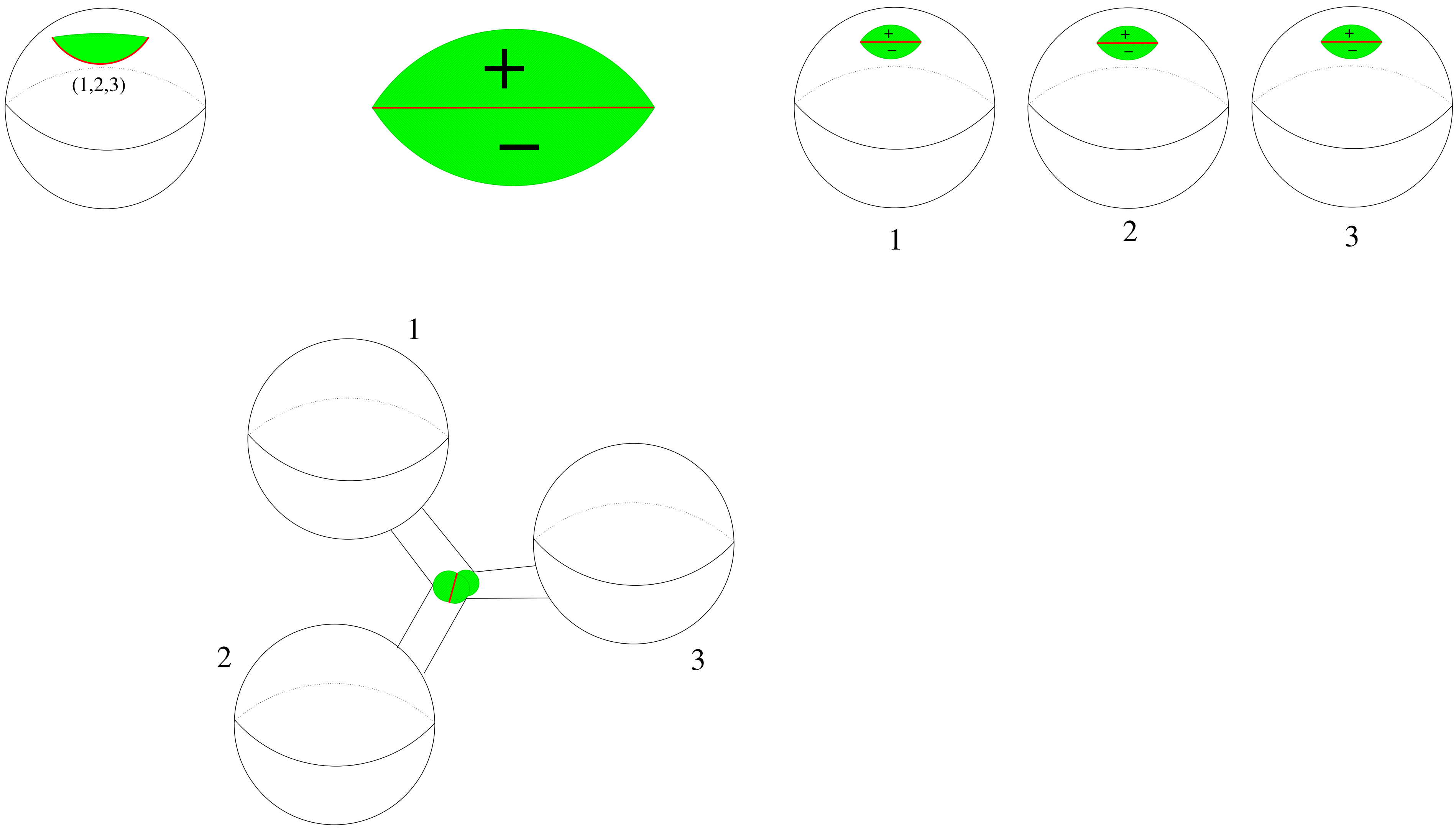
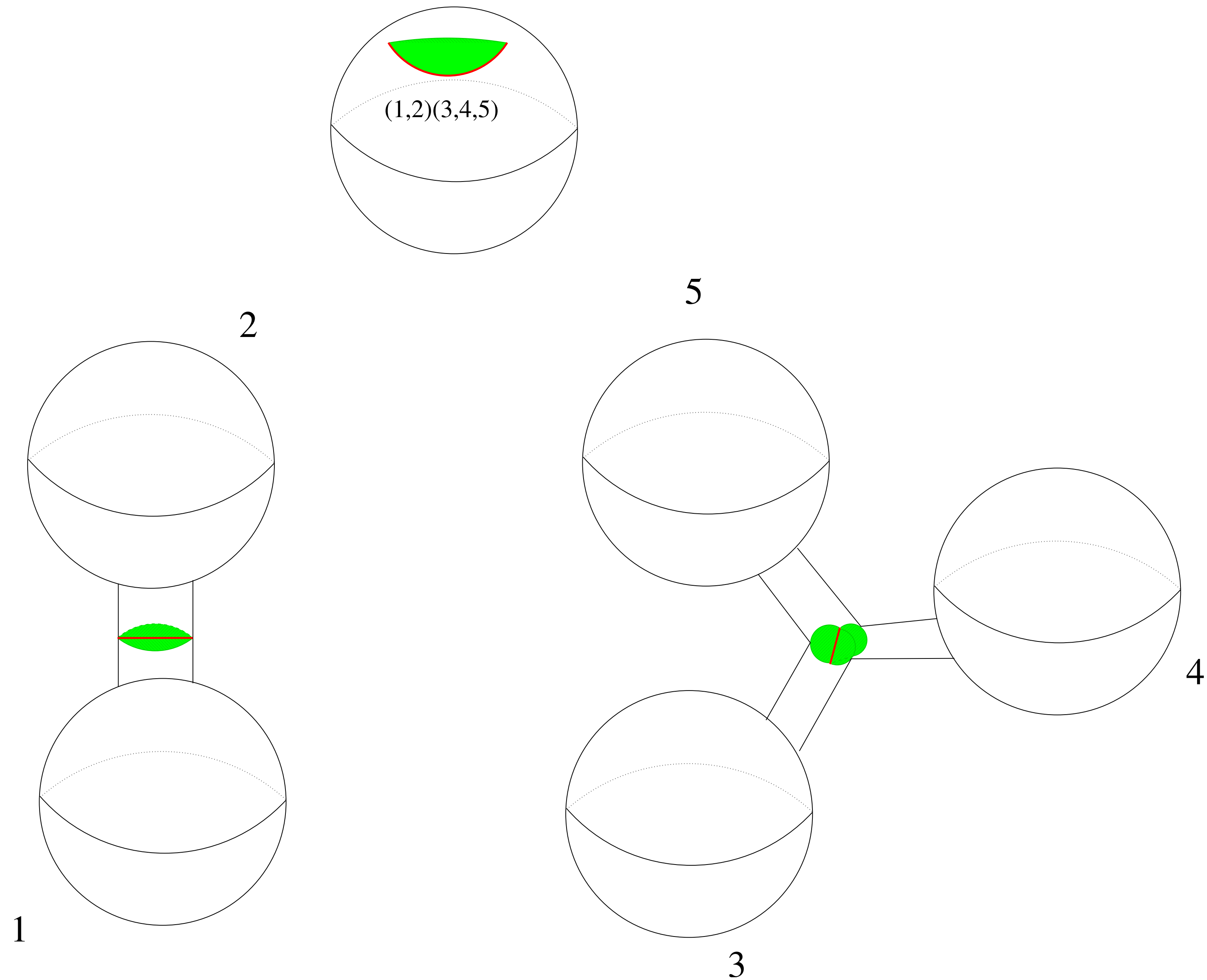
1
2

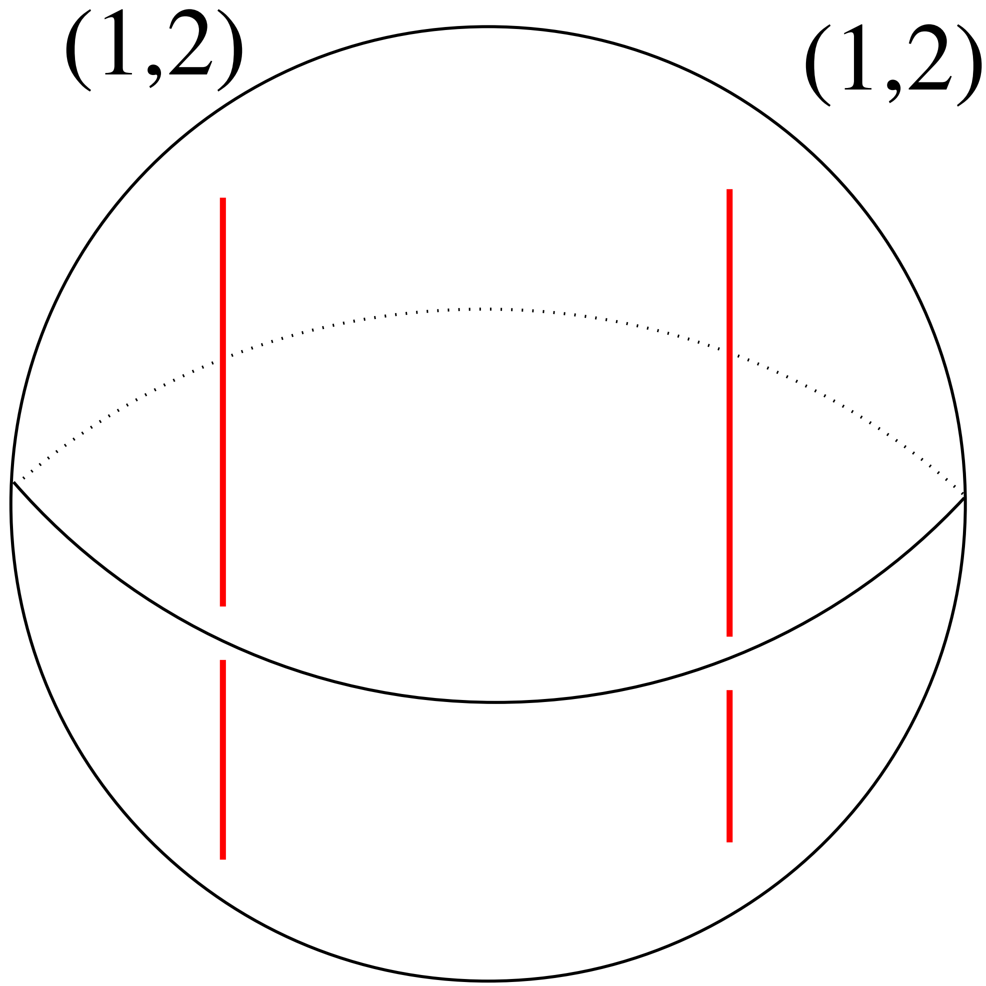

2
1

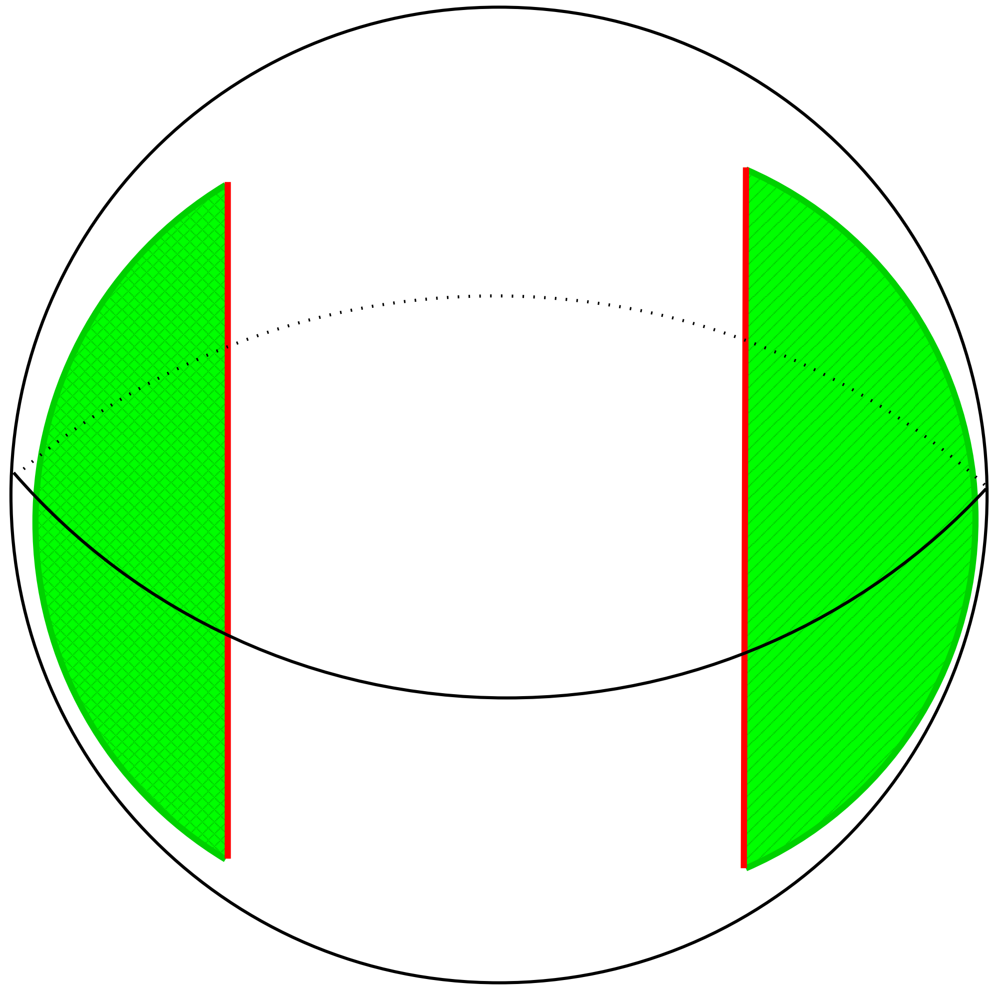

(1,2,3)

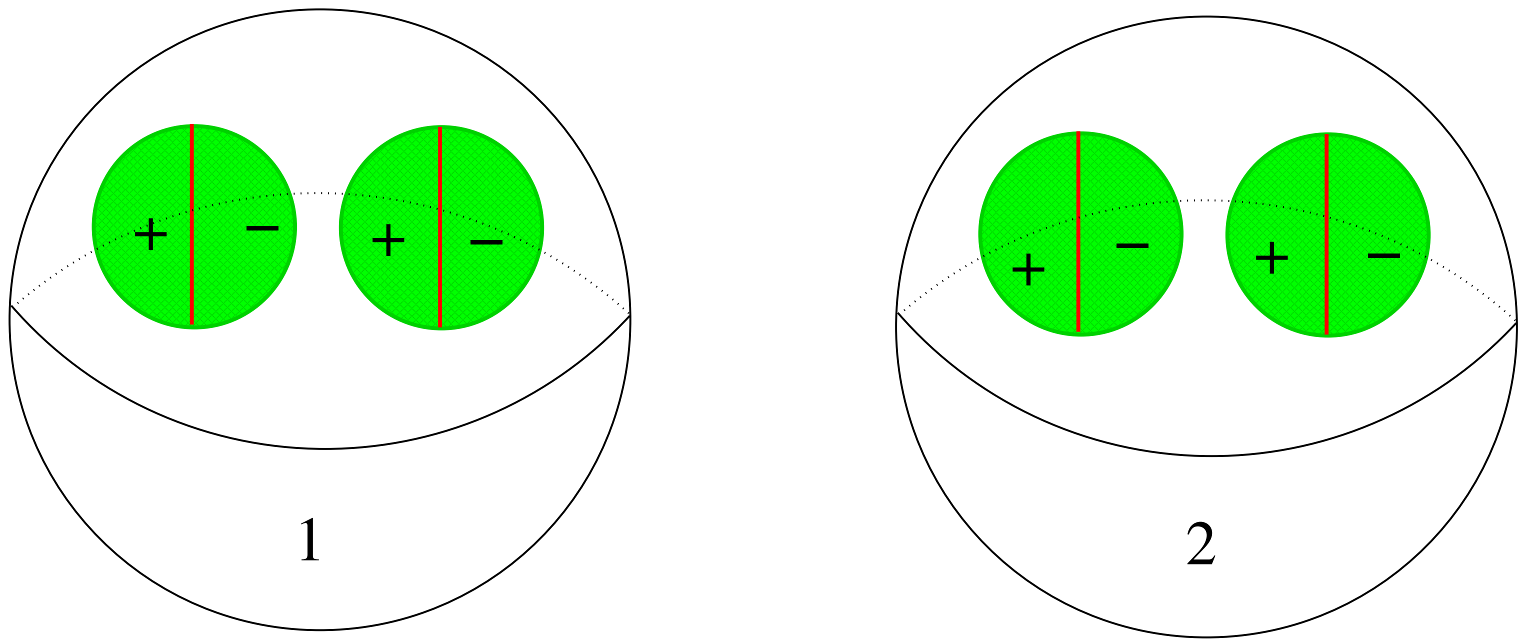

(1,2,3)
+
−
1
2
3
1
2
3

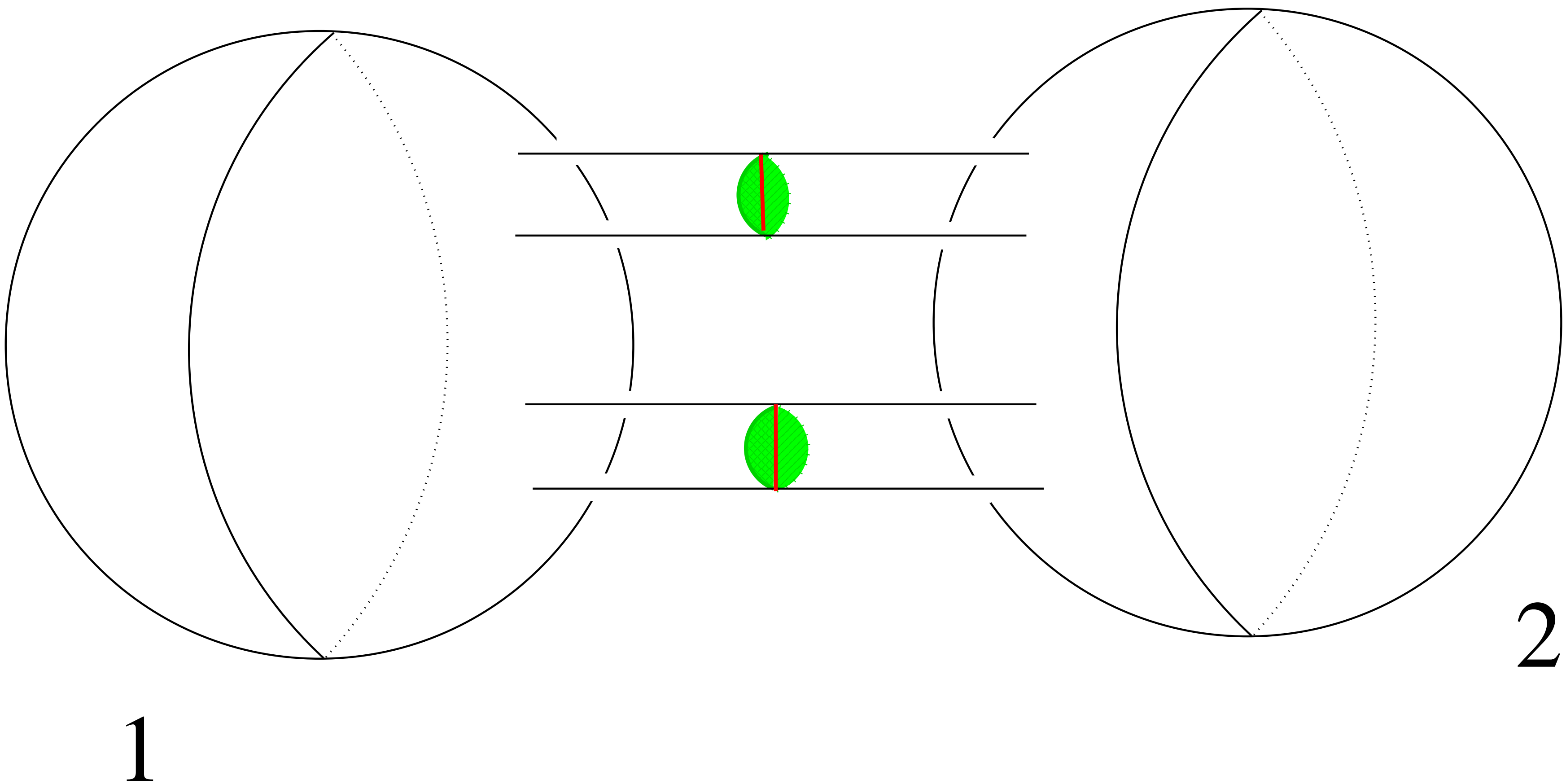


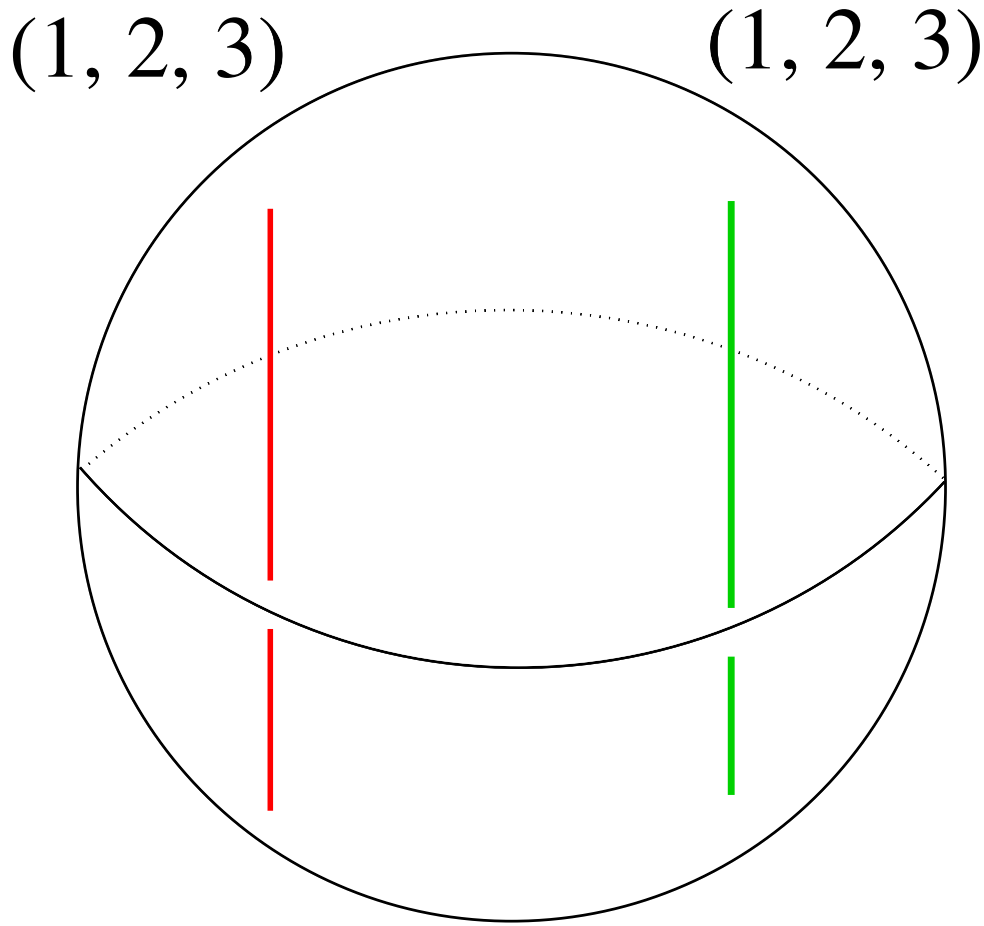
(1,2)(3,4,5)
2
1
5
4
3

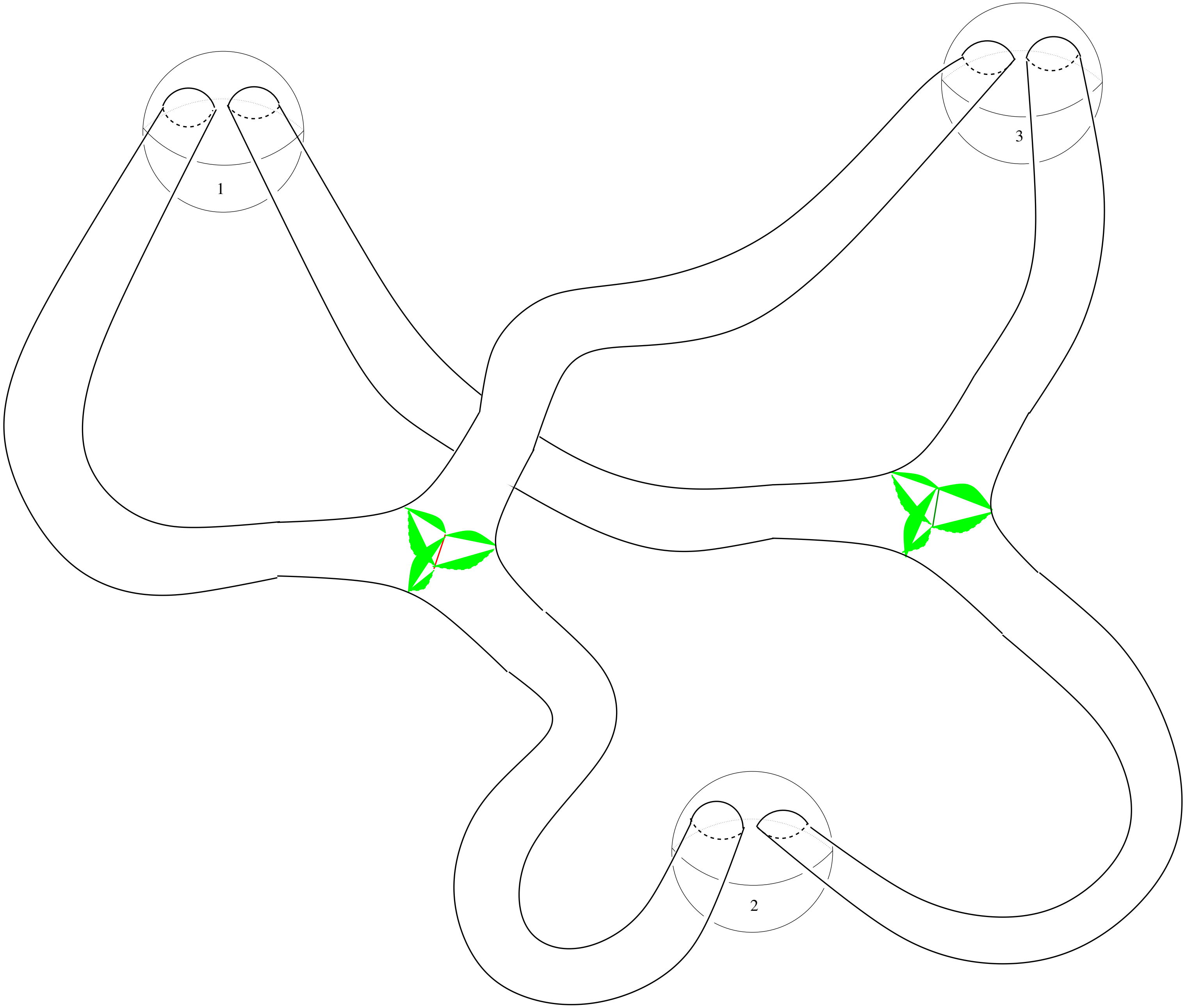

(1,2)
(1,2)

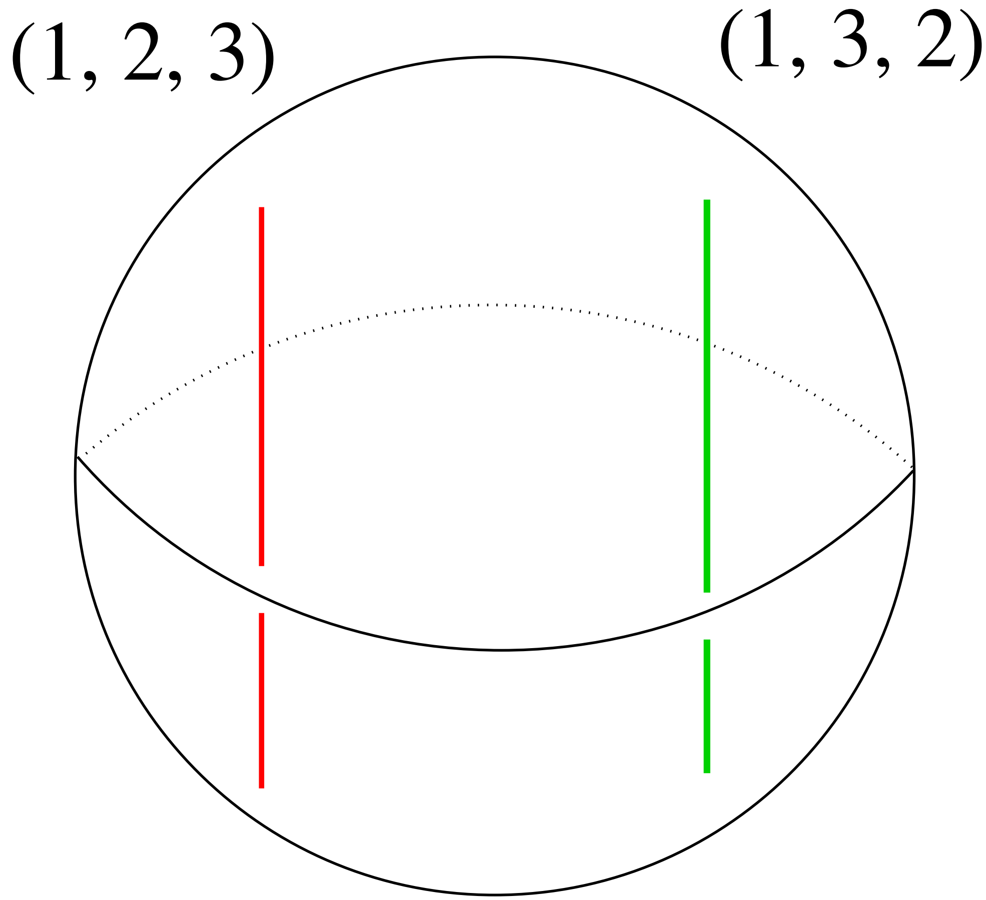


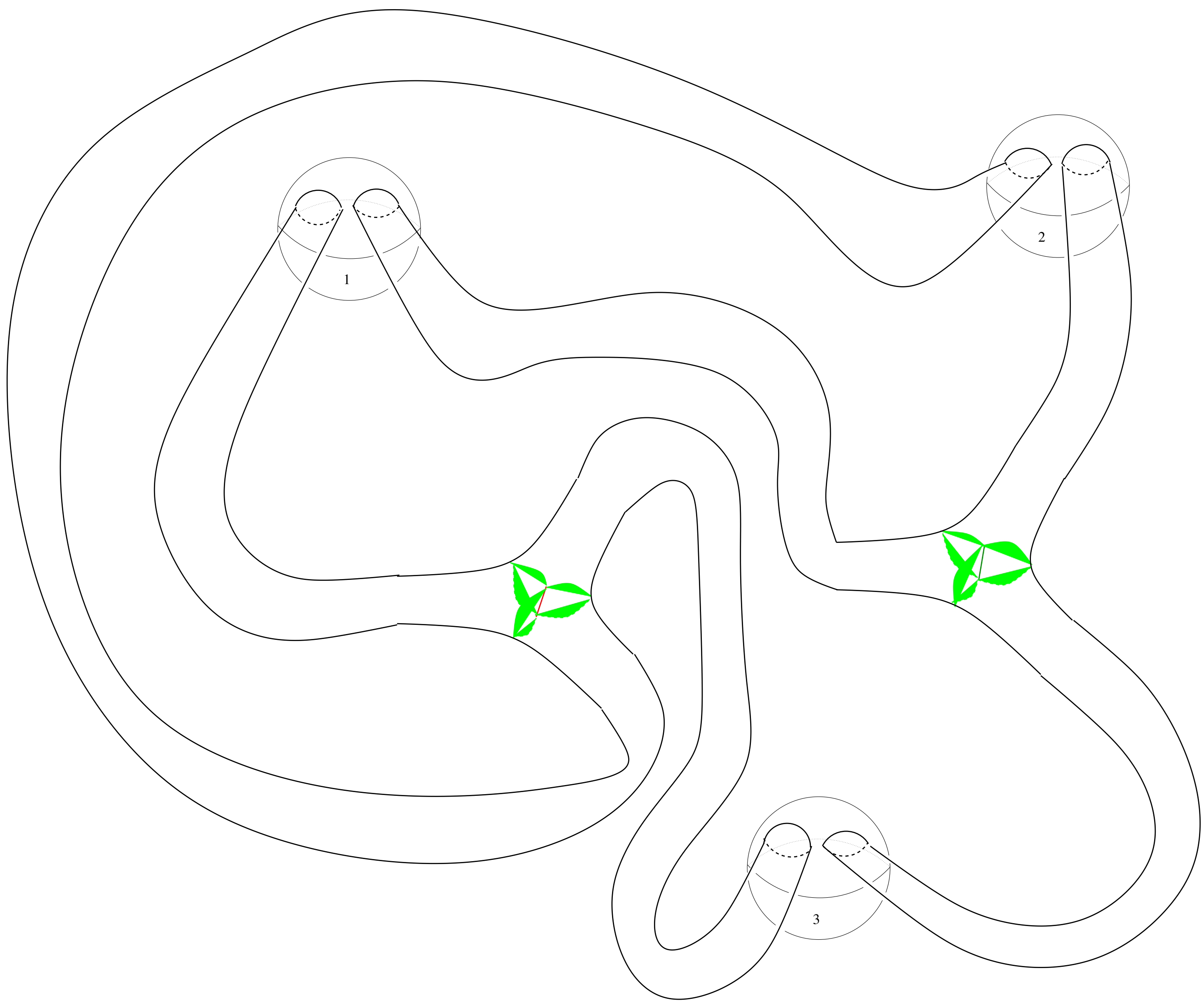

+
−
+
−
1
+
−
+
−
2

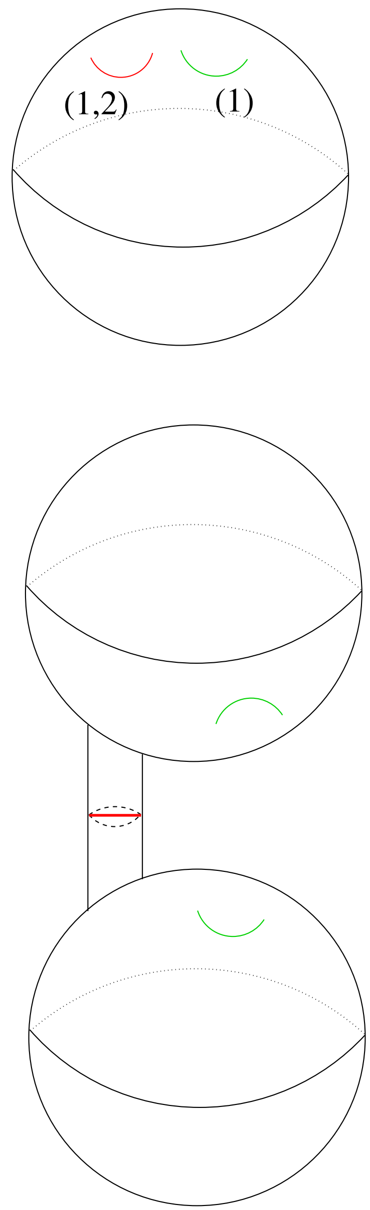

1
2


(1, 2, 3)
(1, 2, 3)

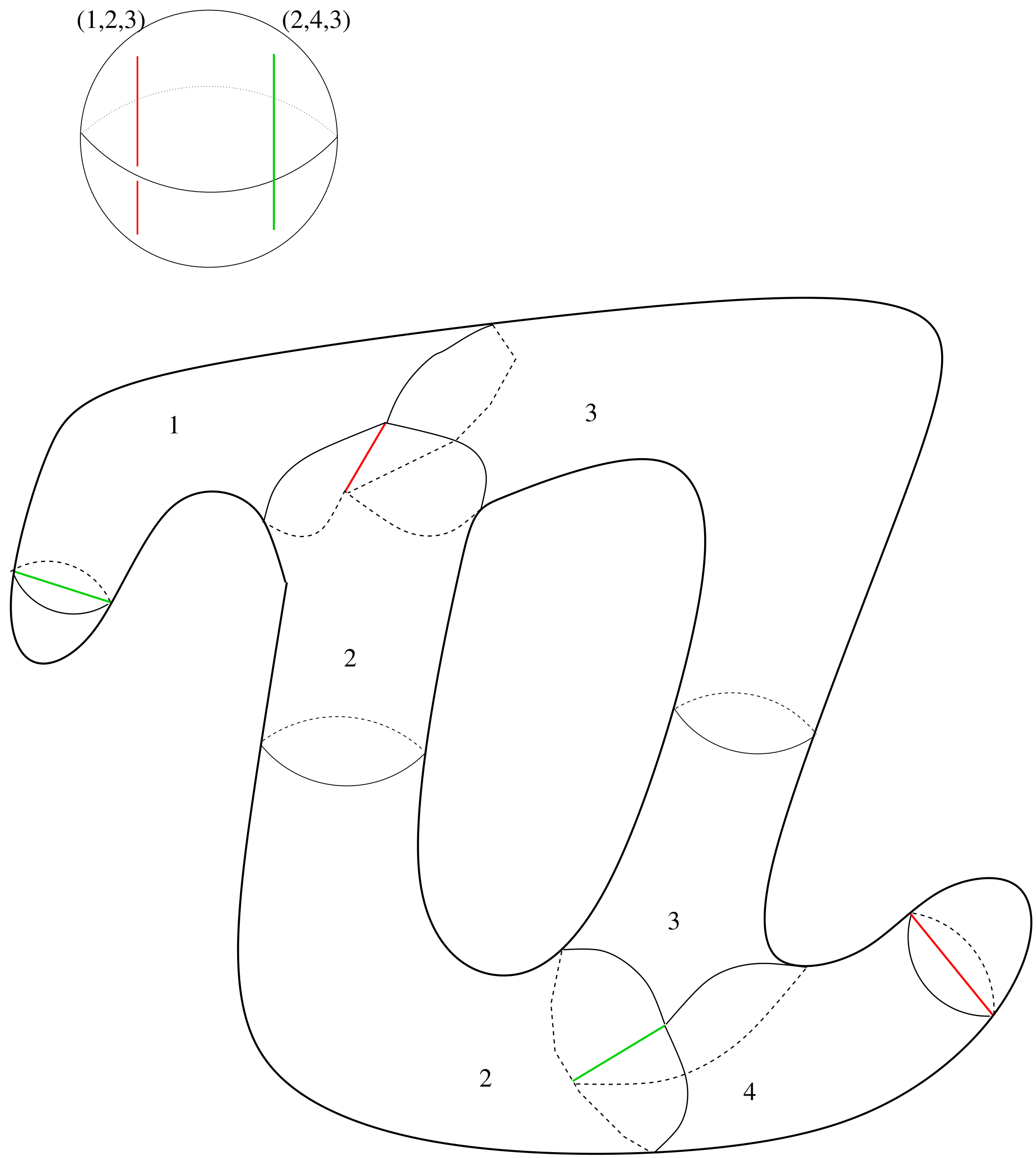

1
3
2

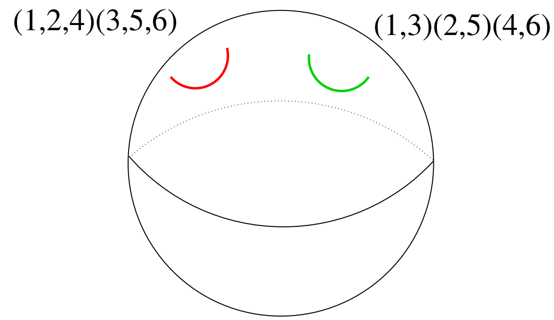

(1, 2, 3)
(1, 3, 2)

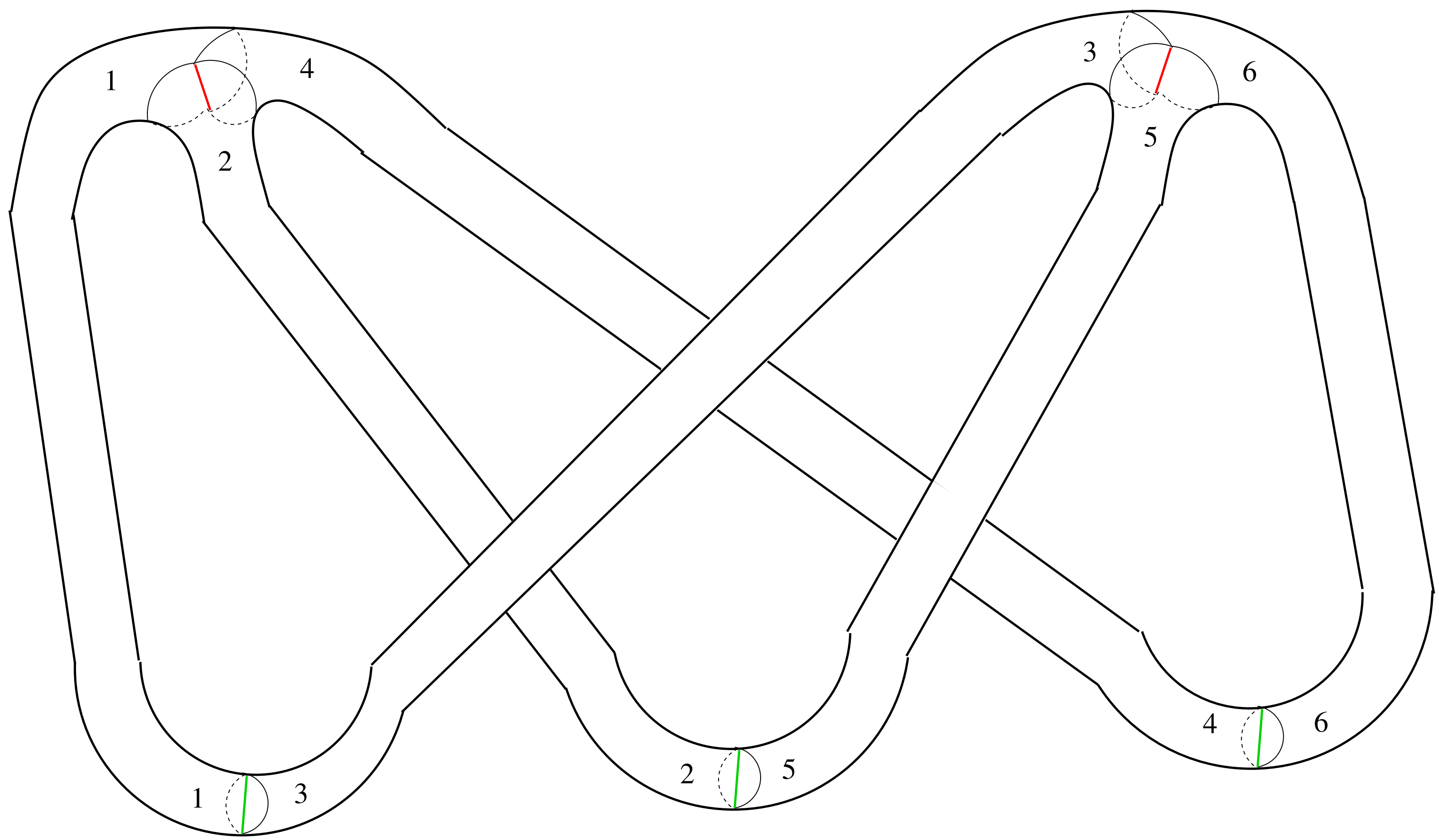

1
2
3



(1) =identidad


(1,2,3)
(2,4,3)
1
3
2
3
2
4


(1,2,4)(3,5,6)
(1,3)(2,5)(4,6)


1
4
2
3
6
5
1
3
2
5
4
6

Se obtuvo una función $\varphi : M \to N$ que es

- continua,
- abierta y
- propia.

Para cada $x \in N$ el número $\#\varphi^{-1}(x) = n$ está fijo, excepto para los puntos de un subconjunto $K \subset N$ de codimensión 2.

Definición. Una función $\varphi : M^m \to N^m$ se llama una cubierta ramificada de n hojas si φ es continua, abierta y propia y si existe una subvariedad $k \subset N$ de codimensión 2 tal que

$$\varphi : M - \varphi^{-1}(k) \to N - k$$

es un espacio cubriente de n hojas.

(k está propiamente encajada en N).

Se dice que φ está ramificada a lo largo de k.

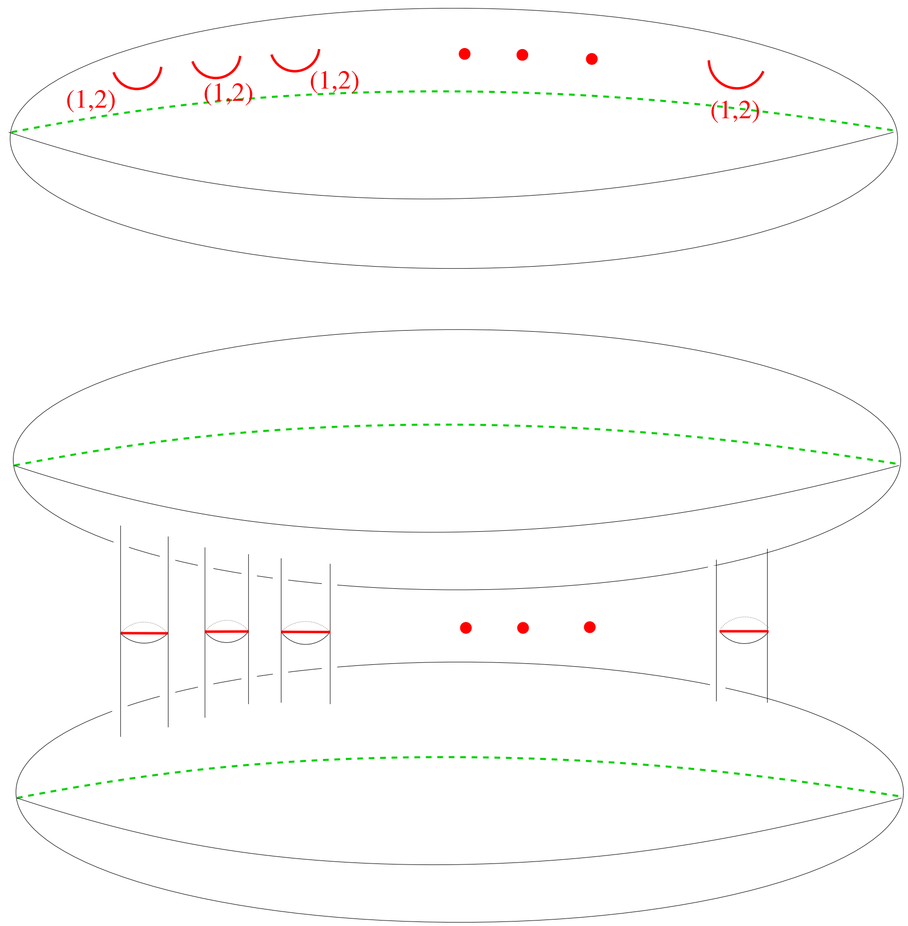
Para una cubierta ramificada $\varphi : M \longrightarrow (N, k)$ de n hojas se tiene una representación (un homomorfismo) asociada:

$$\omega_\varphi : \pi_1(N - k) \longrightarrow S_n.$$

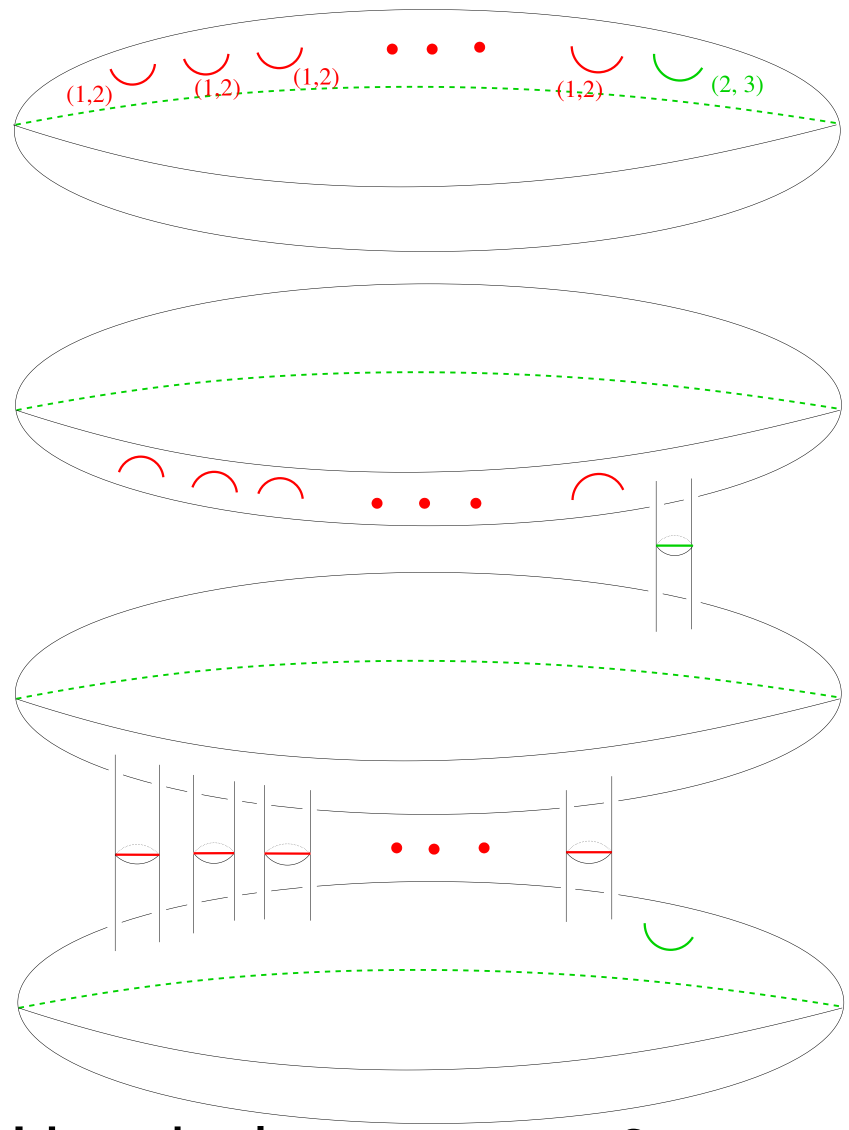
Para una representación $\omega : \pi_1(N - k) \longrightarrow S_n$ dada se tiene una cubierta ramificada

$$\varphi_\omega : M \longrightarrow (N, k).$$

Una cubierta ramificada $\varphi : M \longrightarrow (N, k)$ se llama simple si su representacion asociada manda cada meridiano de k en un 2-ciclo.



Una bola con $g + 1$ arcos da un cubo con g asas.



Una bola con $g + 2$ arcos da un cubo con g asas.

Teorema. (Heegaard)

M es una 3-variedad cerrada, conexa y orientable

$\Leftrightarrow$

M es la unión de dos cubos con asas (orientables) pegados a lo largo de sus fronteras.

$$\begin{array}{ccccc} V & & V_1 & & V_2 \\ {\scriptstyle\varphi}\big\downarrow & = & \big\downarrow{\scriptstyle\varphi_1} & = & \big\downarrow{\scriptstyle\varphi_2} \\ B^3 & & B_1 & & B_2 \end{array}$$

$$f : \partial V_1 \to \partial V_2$$
$$g : \partial B_1 \to \partial B_2$$

$$\begin{array}{ccc} V_1 \sqcup V_2 & \longrightarrow & V_1 \cup_f V_2 \\ {\scriptstyle\varphi_1 \sqcup \varphi_2}\big\downarrow & & \big\downarrow{\scriptstyle\varphi_1 \cup \varphi_2} \\ B_1 \sqcup B_2 & \longrightarrow & B_1 \cup_g B_2 (\cong S^3) \end{array}$$

$$\varphi_1 \cup \varphi_2 \text{ es función } \Leftrightarrow \begin{array}{ccc} \partial V_1 & \xrightarrow{f} & \partial V_2 \\ {\scriptstyle\varphi_1}\big\downarrow & & \big\downarrow{\scriptstyle\varphi_2} \\ \partial B_1 & \xrightarrow[g]{} & \partial B_2 \end{array} \text{ conmuta}$$

Teorema. (Berstein y Edmonds)

$\varphi : \partial V \to \partial B^3$ cubierta simple de al menos tres hojas

$f' : \partial V \to \partial V$ homeomorfismo

$\Rightarrow$

Existen $f : \partial V \to \partial V$ y $g : \partial B^3 \to B^3$ homeomorfismos tales que f es isotópica a f' y

$$\begin{array}{ccc} \partial V_1 & \xrightarrow{f} & \partial V_2 \\ \varphi_1 \downarrow & & \downarrow \varphi_2 \\ \partial B_1 & \xrightarrow[g]{} & \partial B_2 \end{array} \text{ conmuta}$$

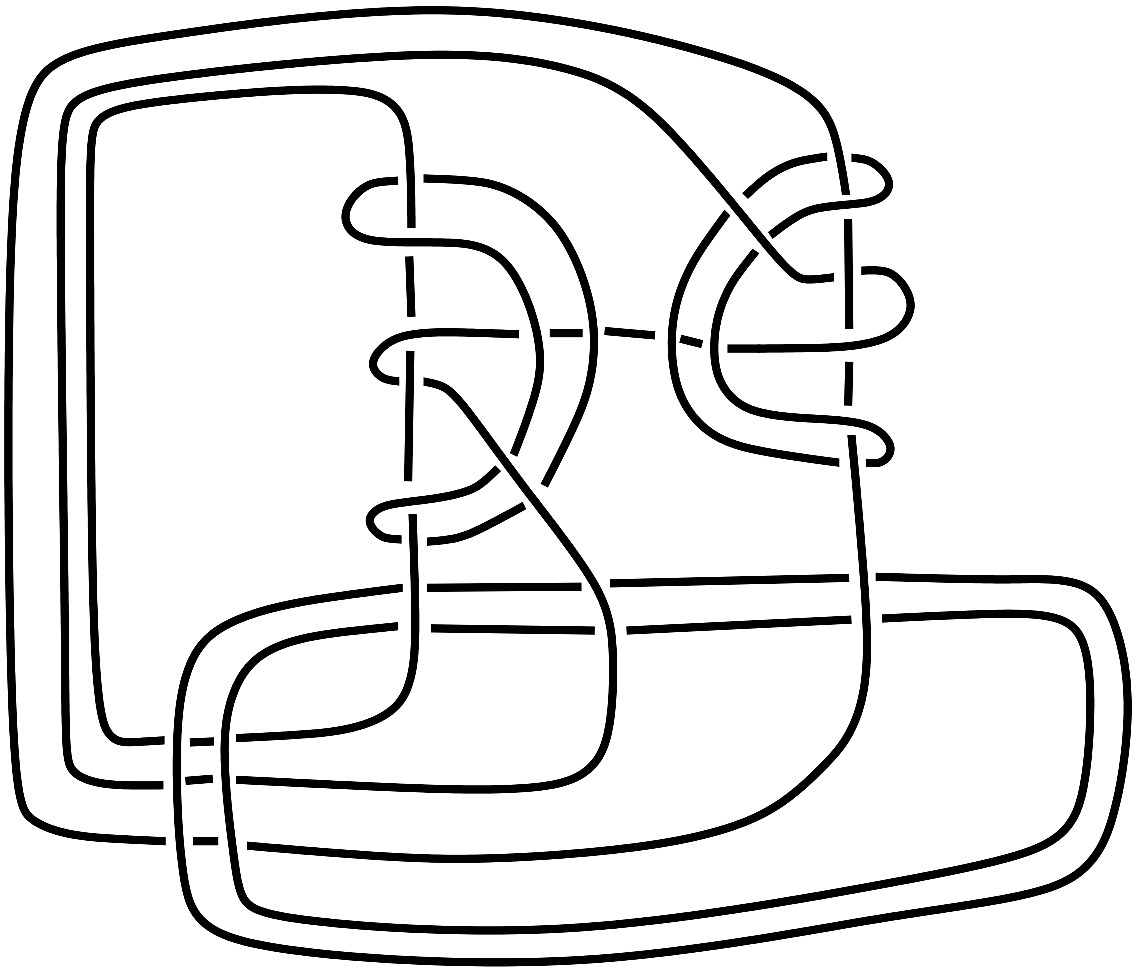
Teorema. (Hilden y Montesinos)

Toda 3-variedad cerrada, conexa y orientable es cubierta ramificada de la 3-esfera S^3 con una proyección cubriente simple de tres hojas y la ramificación es a lo largo de un enlace en S^3.

Pregunta: (A. Ramírez)

¿Existe algún enlace $L \subset S^3$ tal que toda 3-variedad cerrada, conexa y orientable es una cubierta ramificada de (S^3, L)?

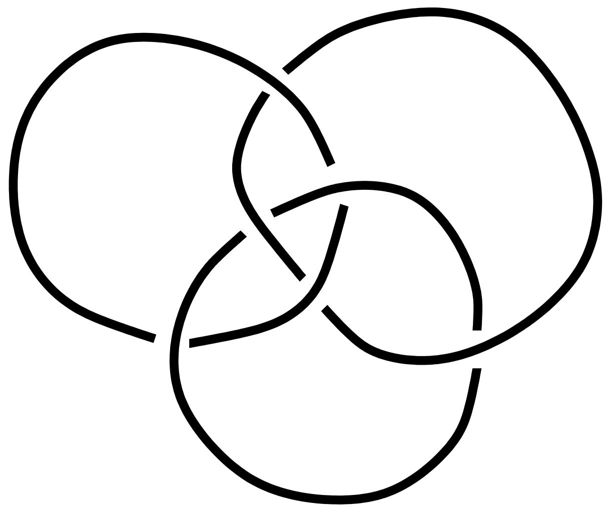
Teorema. (Thurston) El enlace

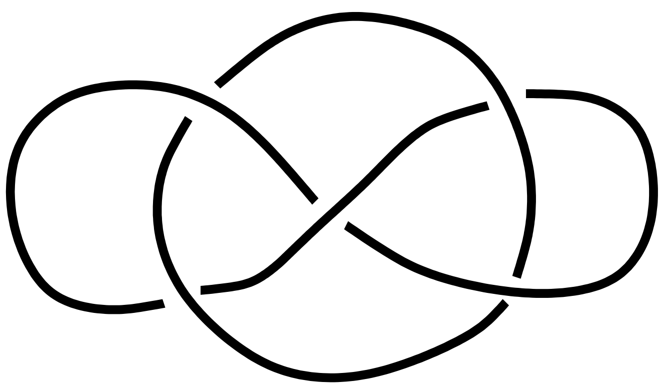


es universal.

Teorema. (Hilden–Lozano–Montesinos)

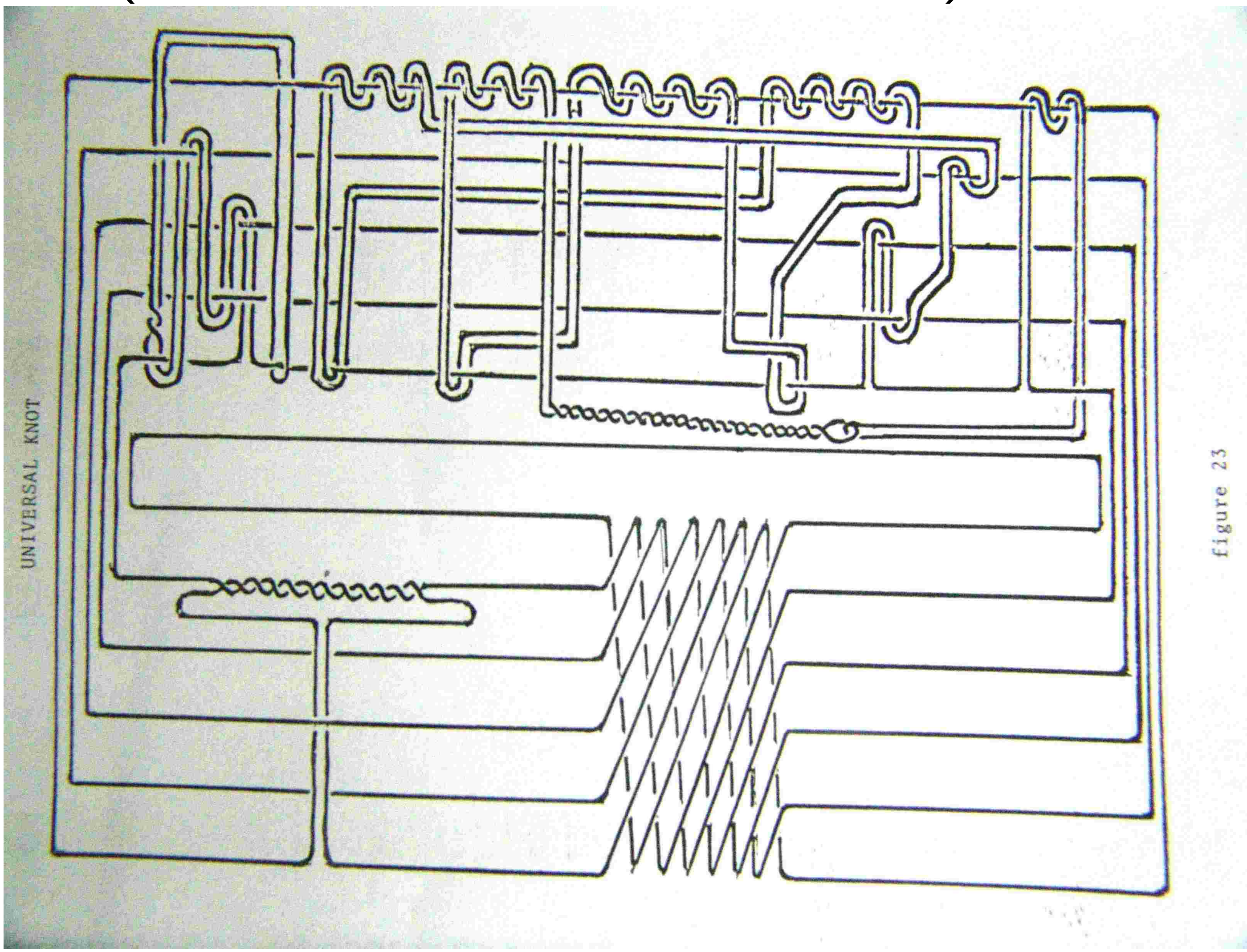
Los enlaces





son universales.

Teorema. (Hilden–Lozano–Montesinos) El nudo



es universal.

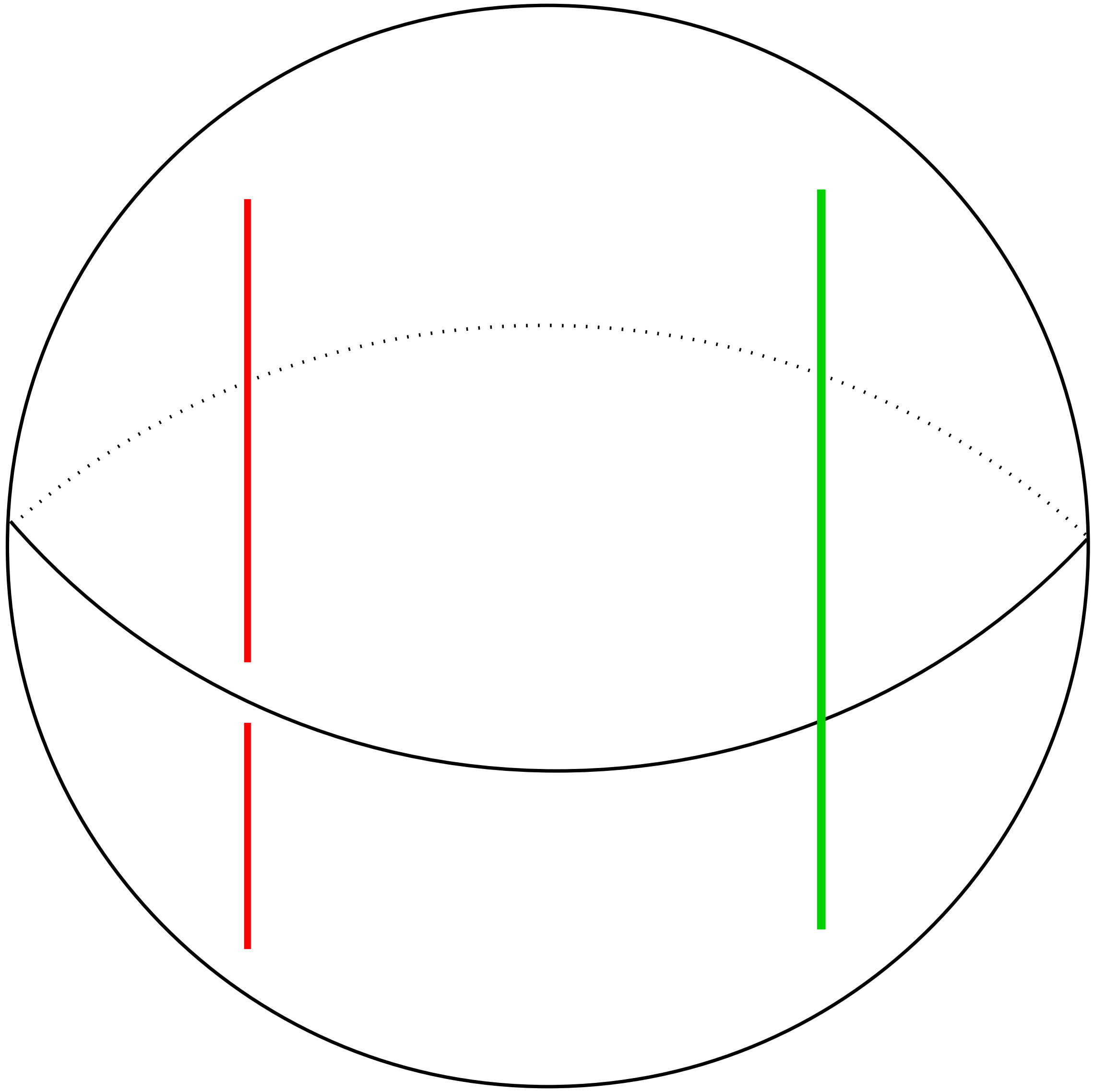
Nudos de n puentes.

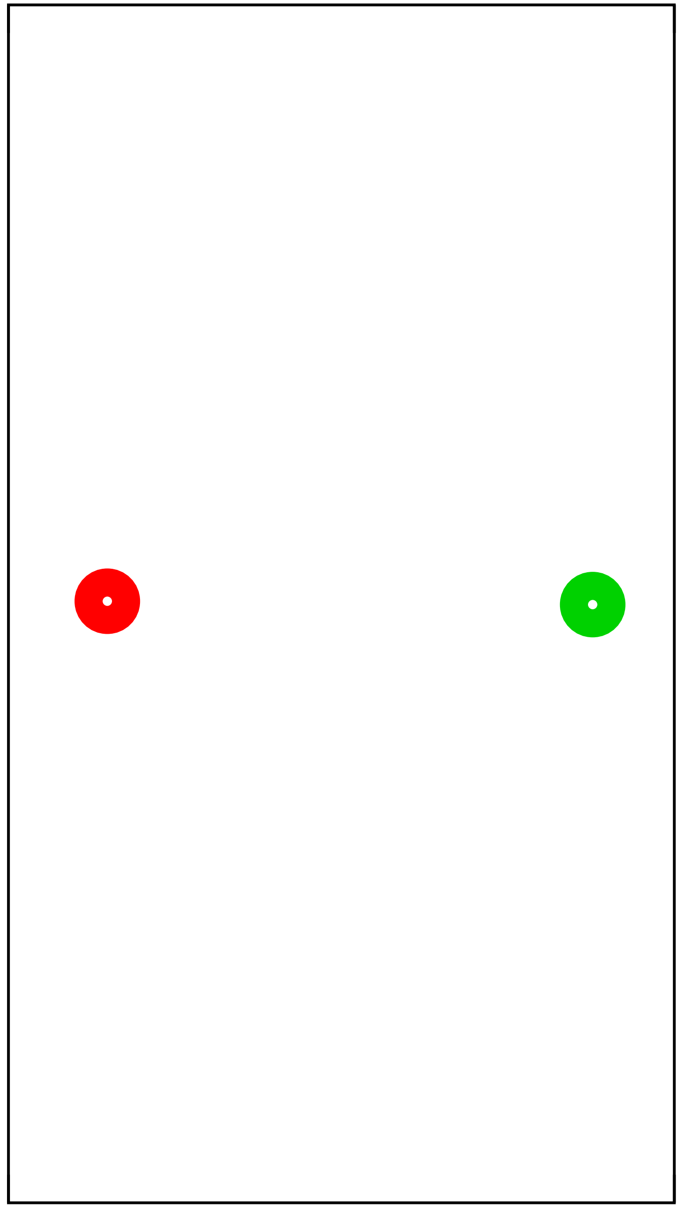
Un <u>n-ovillo trivial</u> es una pareja $(B, \{\alpha_i\}_{i=1}^n)$ donde

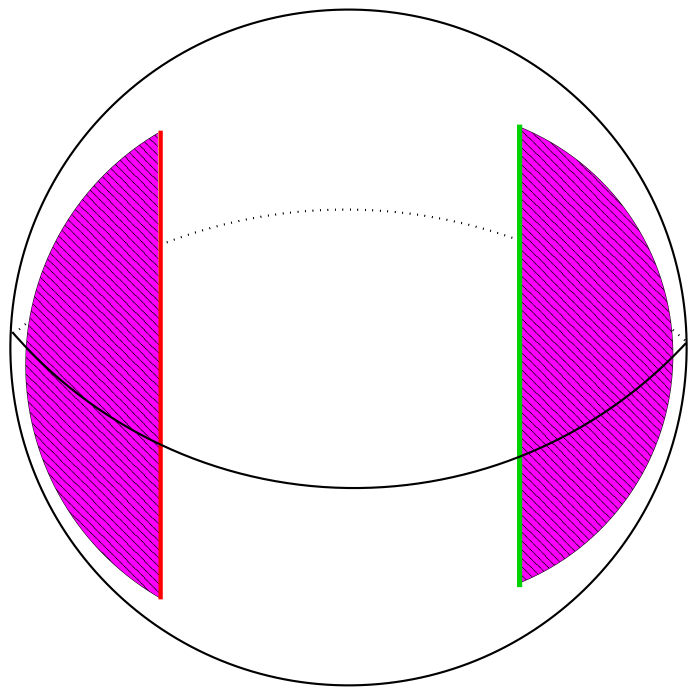
B es una 3-bola

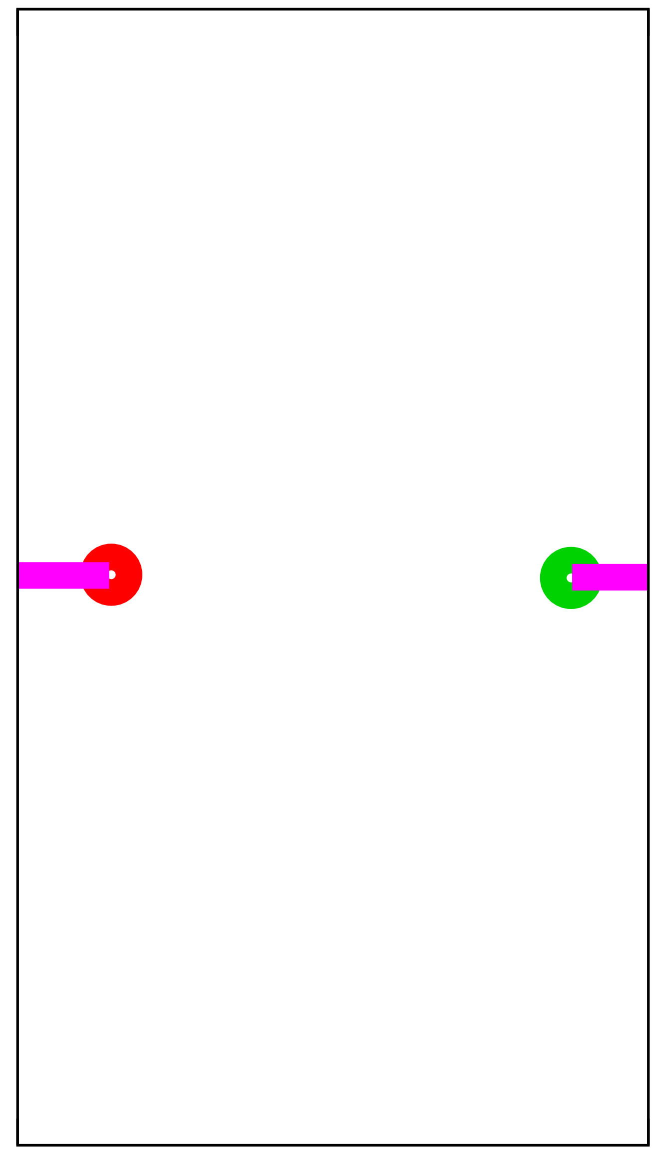
$\alpha_1, \ldots, \alpha_n \subset B$ son n arcos propiamente encajados <u>triviales</u>

(O sea, hay n 2-discos ajenos $D_1, \ldots, D_n \subset B$ tales que $\partial D_i = \alpha_i \cup \beta_i$ donde $\beta_i \subset \partial B$ y $\partial\alpha_i = \partial\beta_i$.)







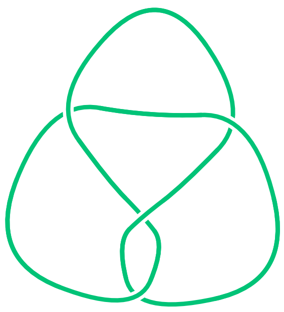


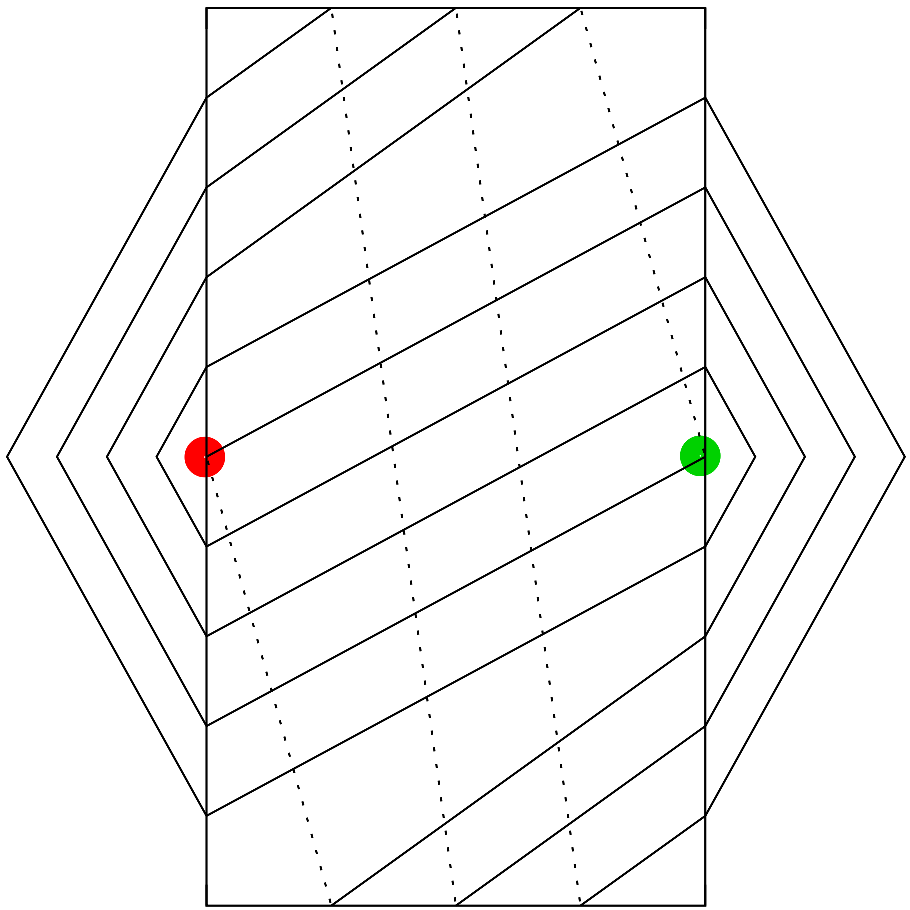
Un enlace $k \subset S^3$ está en <u>posición de n puentes</u> si existen dos n-ovillos triviales $(B, \{\alpha_i\})$ y $(B', \{\alpha'_i\})$ tales que

$$S^3 = B \cup_\partial B'$$

y

$$k = (\sqcup \alpha_1) \cup (\sqcup \alpha'_i)$$

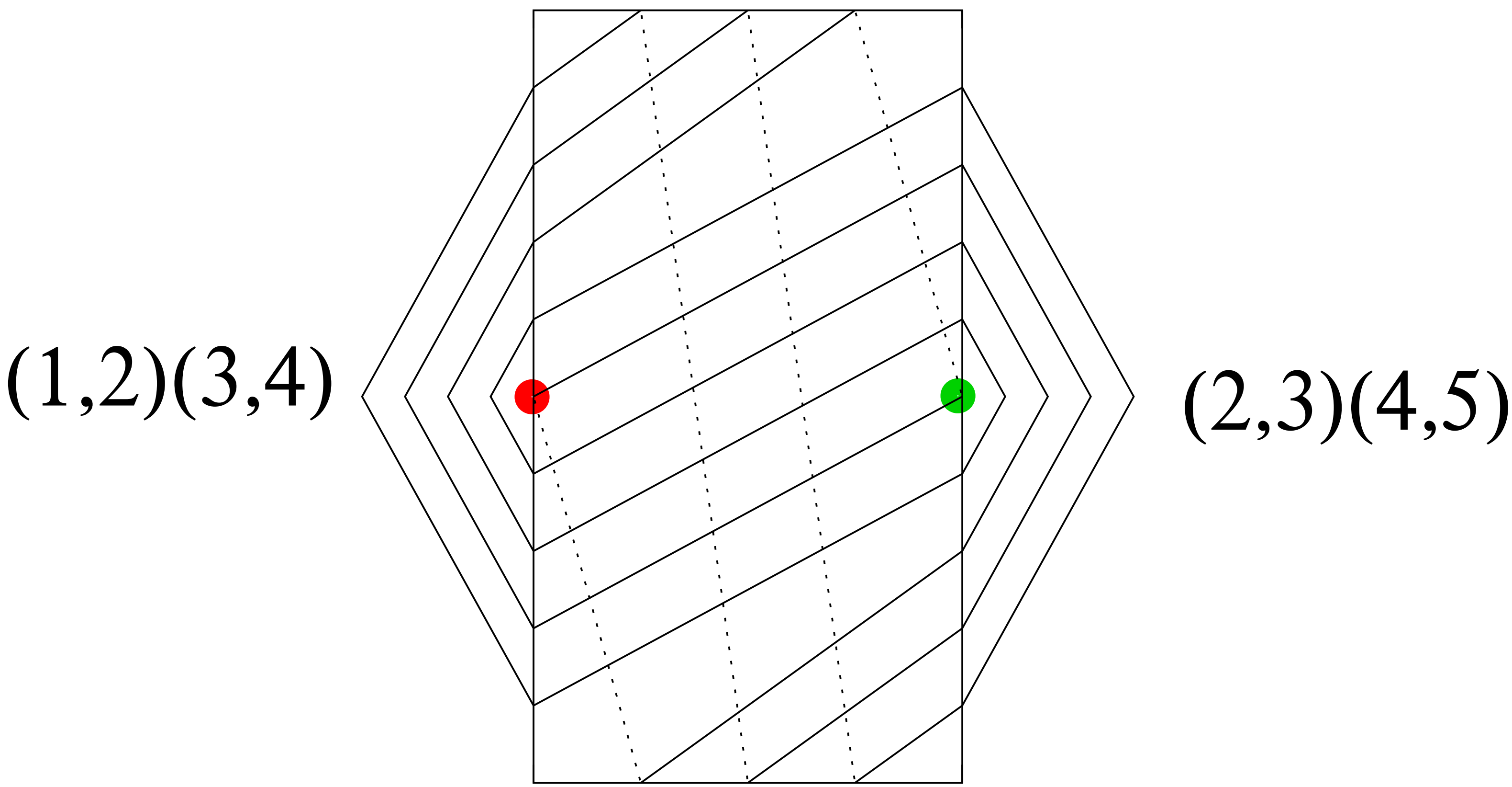




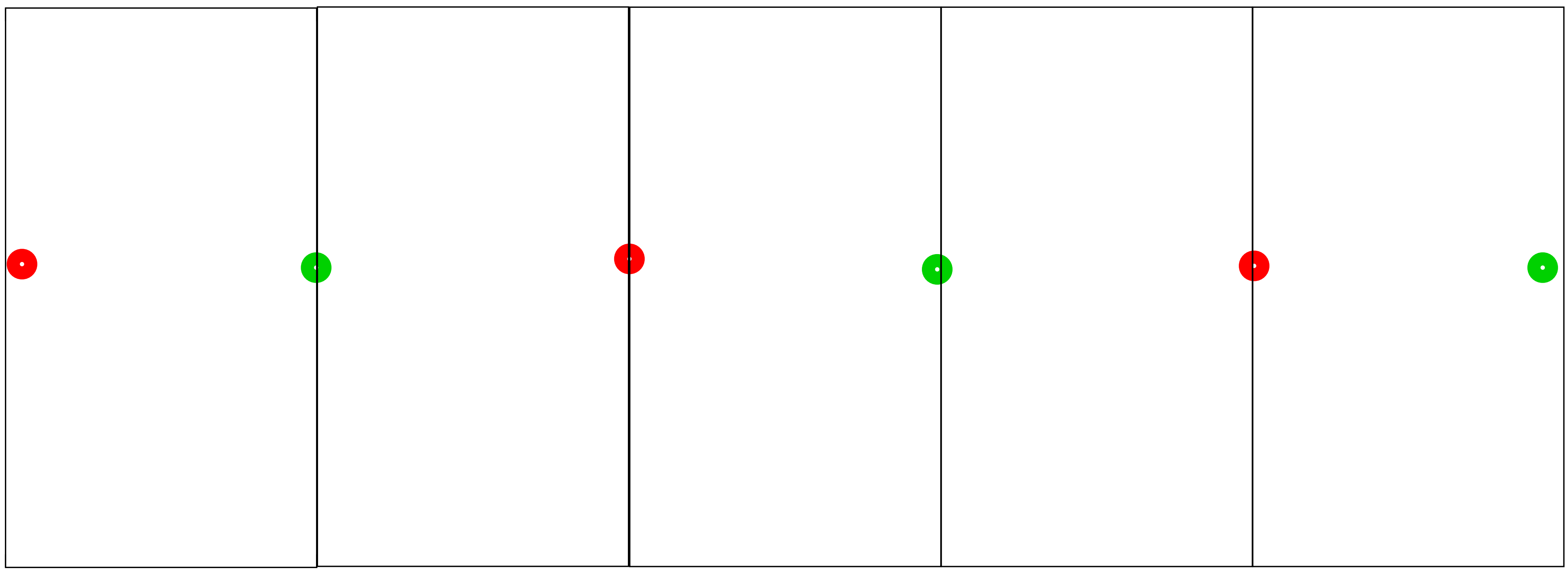
$\ell(5/3)$.

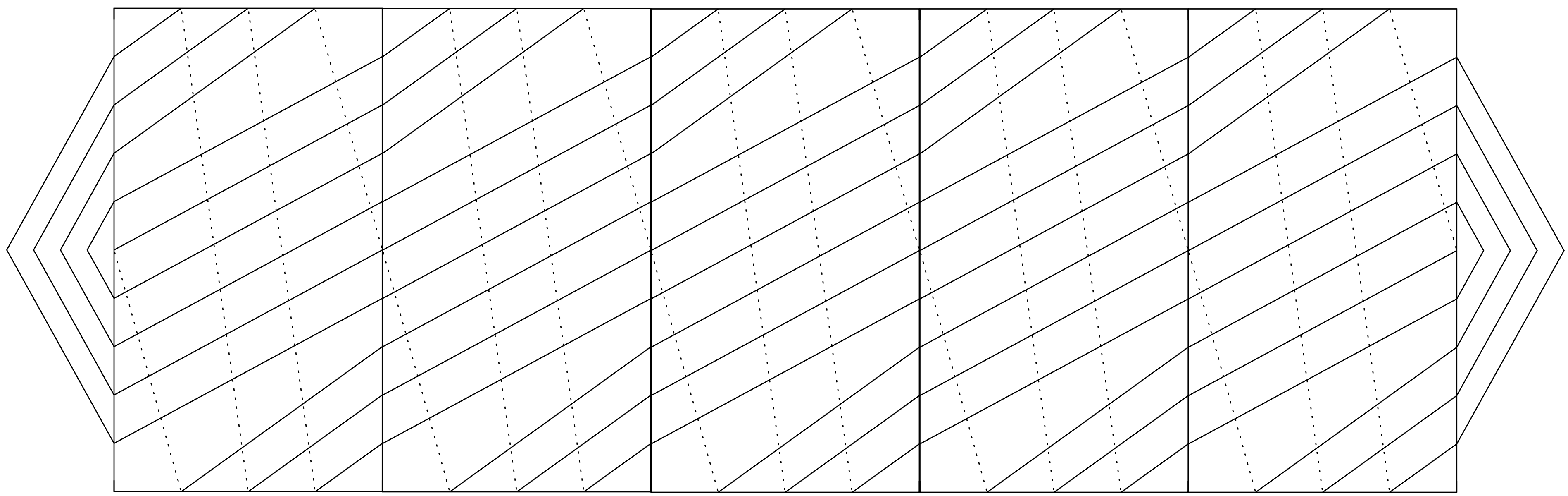
Cubiertas de nudos de dos puentes.

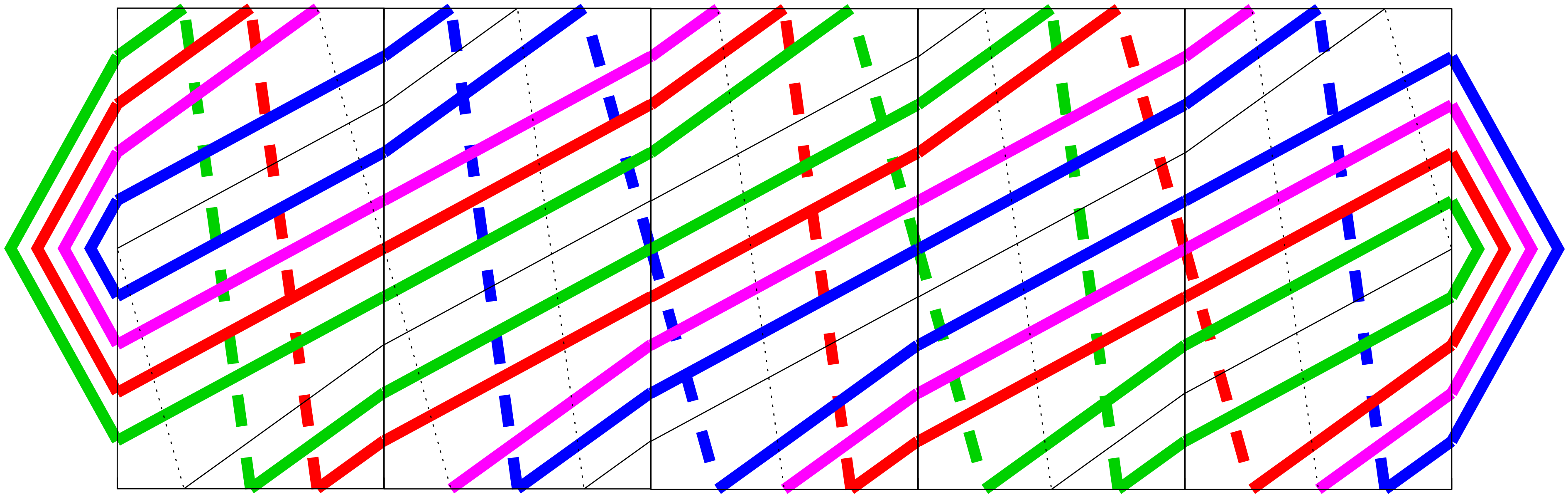
$k = \ell(5/3)$



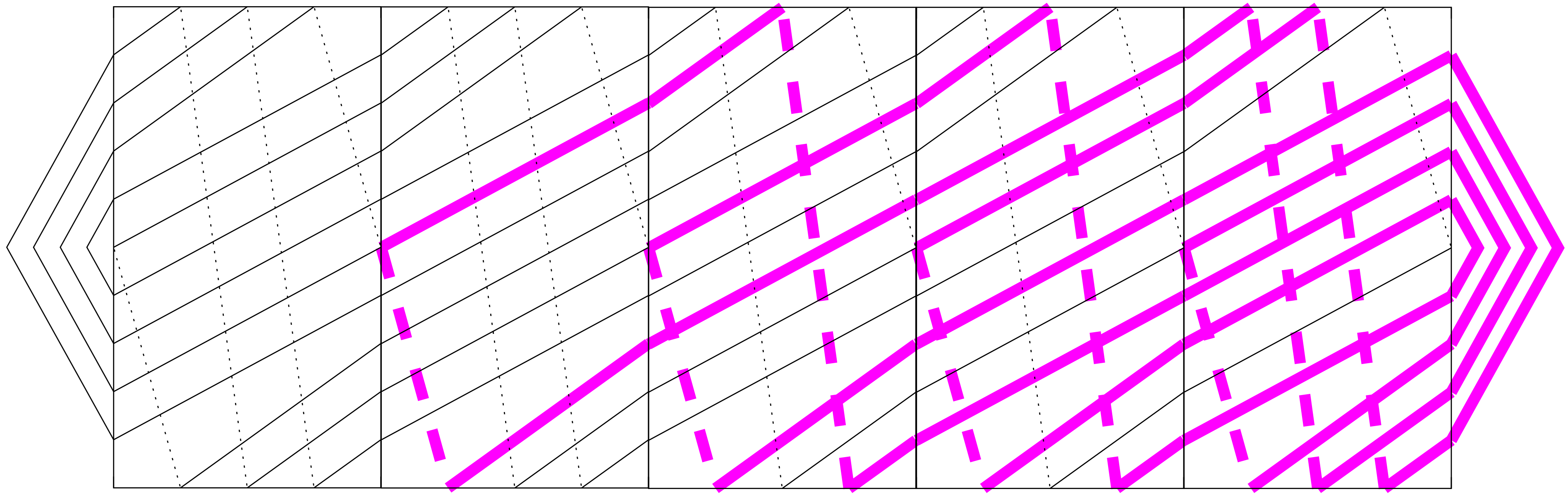
Conocemos la cubierta de la bola:

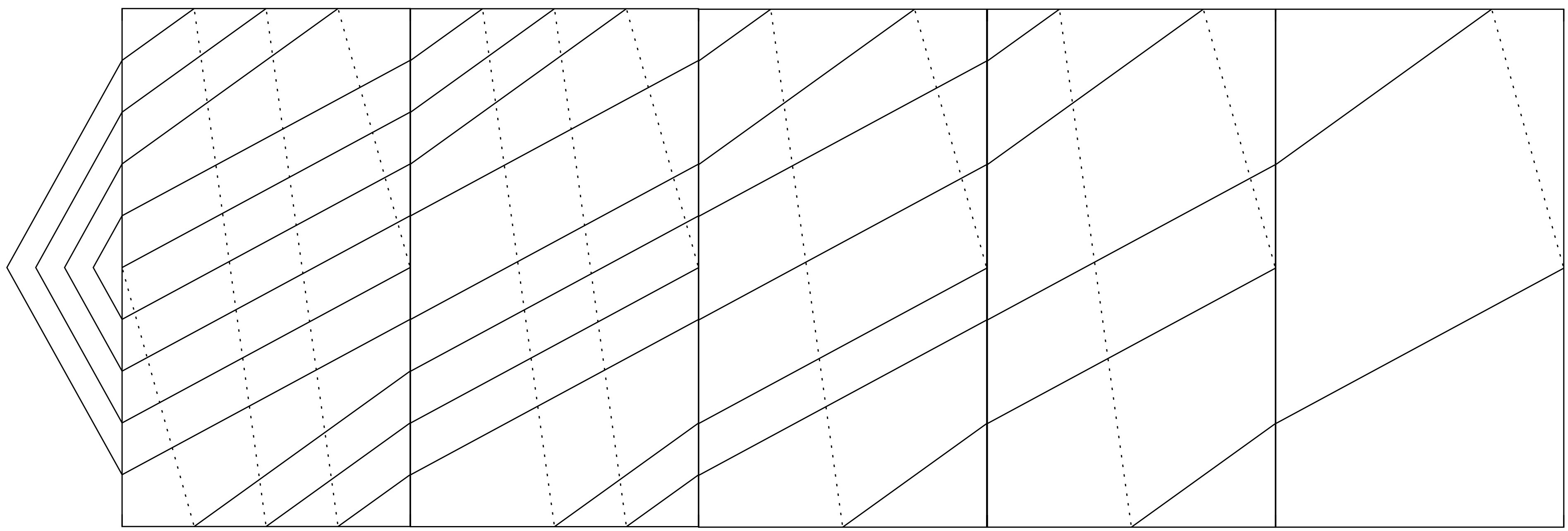


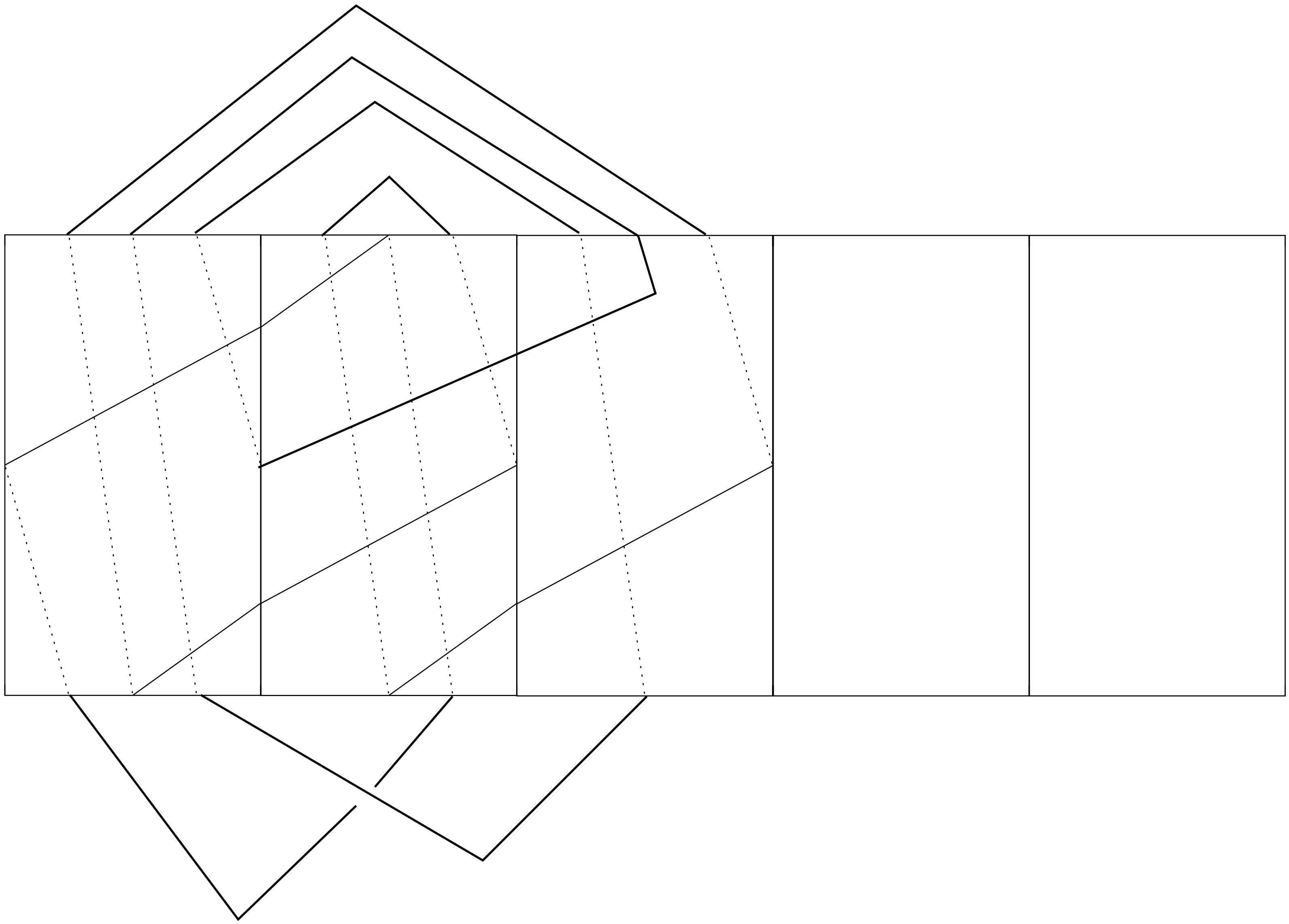


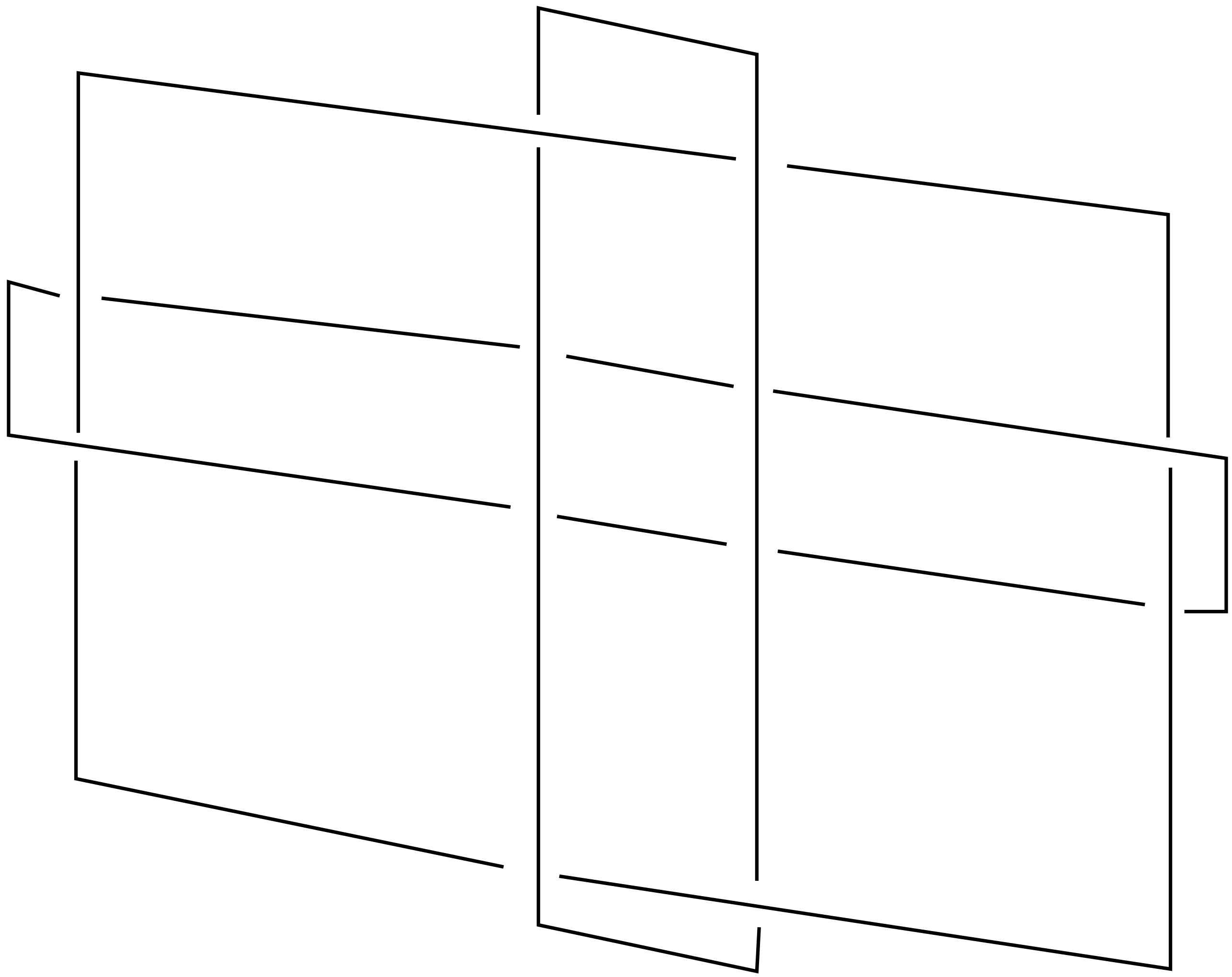


“Sobran” estos arcos (por ejemplo):









El enlace romano.

3.3 El Enlace Romano.

Teorema 3.4. *Existe una cubierta ramificada $p : S^3 \to S^3$ con ramificación sobre el enlace Romano tal que el conjunto singular contiene los anillos Borromeanos. Esto es, el enlace Romano es universal.*

Demostración. Paso 1: Dibujemos el enlace romano (Figura 36) como en la Figura 39. Sea $p_1 : S^3 \to S^3$ la cubierta diédrica de cuatro hojas con ramificación en las componentes A y B del enlace Romano (ver Figura 39). Observemos la esfera punteada; ésta separa a S^3 en dos ovillos triviales de 2-hilos. Aplicando la teoría de la sección 3.1 a estos dos ovillos, podemos dibujar la preimagen de la componente C como en la Figura 40 (a), esta preimagen es precisamente el enlace de dos puentes $L(12/5)$ (Figura 40 (b)).

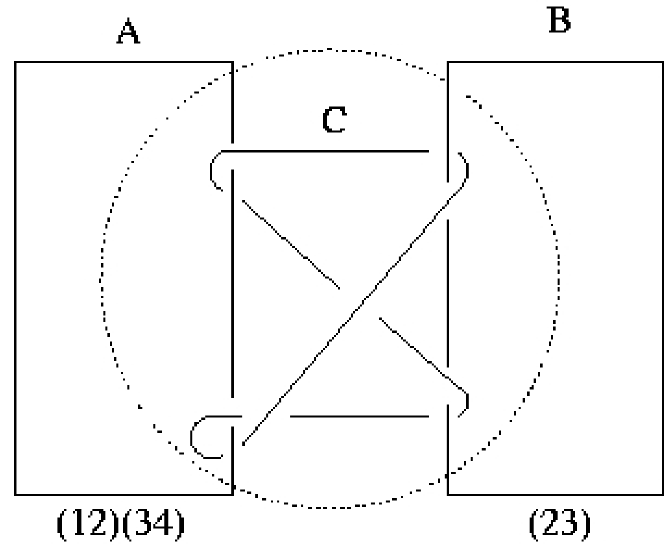


Figura 39.

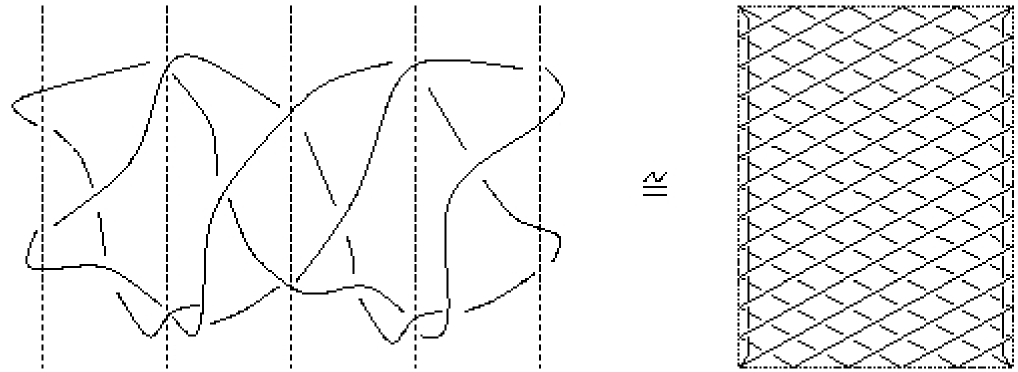


Figura 40. a) b)

Paso 2: Tomemos la cubierta diédrica de $p_2 : S^3 \to S^3$ de seis hojas (ver Figura 38) sobre $L(12/5)$ y de la preimagen de este enlace escogemos las componentes C_0, C_2, C_5 y C_6 marcadas en la Figura 38. Estas componentes están dibujadas en la Figura 41 (a).

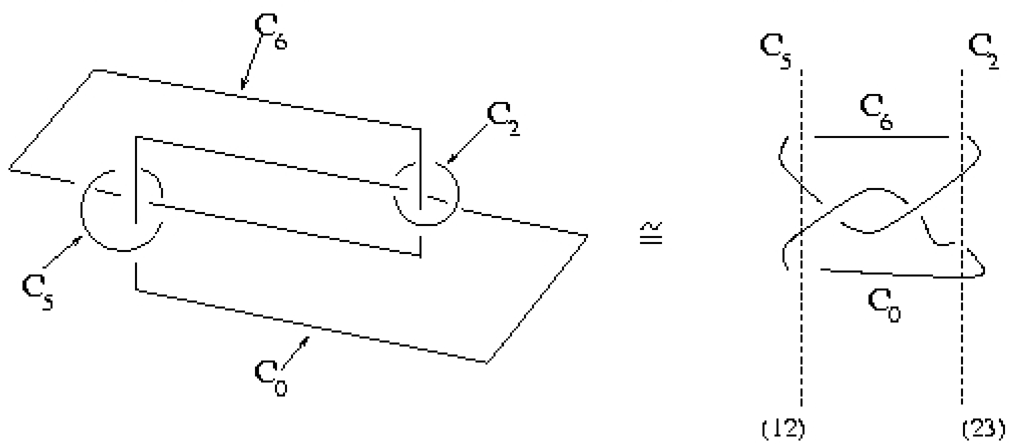


Figura 41. a) b)

Paso 3: Sobre el enlace anterior tomamos la cubierta diédrica $p_3 : S^3 \to S^3$ de tres hojas con ramificación sobre las componentes C_5 y C_2 (Figura 41 (b). La preimagen bajo p_3 de C_0 y C_6 es el enlace de la Figura 42.

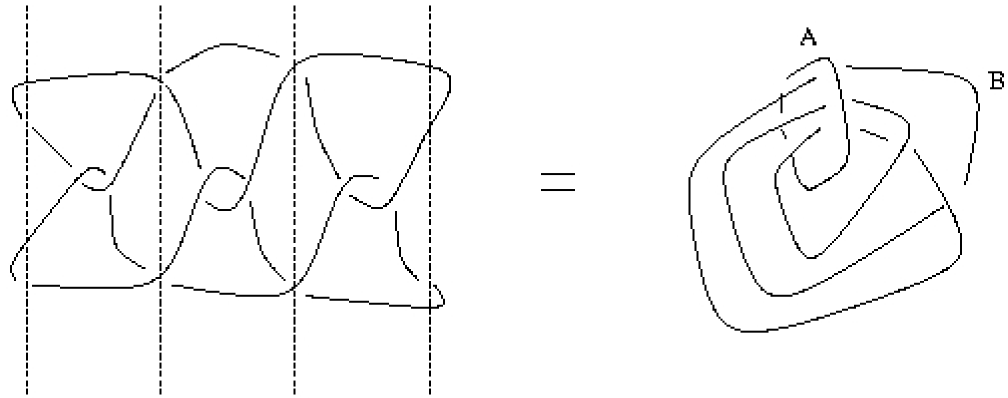


Figura 42.

Una cubierta cíclica de tres hojas sobre A:

B bajo esta cubierta son los Borromeanos. Por tanto la cubierta $p = p_4 \circ p_3 \circ p_2 \circ p_1$ es la cubierta que describe el teorema.

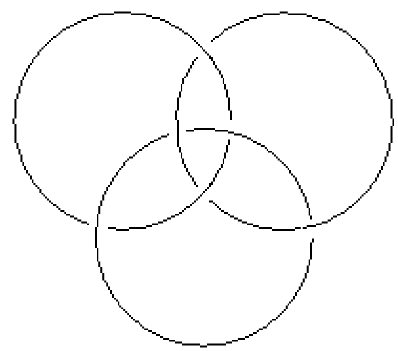


Figura 43.

■

$\varphi_1 : S^3 \longrightarrow (S^3, \ell(\frac{5}{3}))$ de **5** hojas, $\varphi_1^{-1}(\ell(\frac{5}{3}))$ =romano.

$\varphi_2 : S^3 \longrightarrow (S^3, \mathrm{romano})$ de **4** hojas, $\varphi_2^{-1}(\mathrm{romano}) \supset \ell(\frac{12}{5})$.

$\varphi_3 : S^3 \longrightarrow (S^3, \ell(\frac{12}{5}))$ de **6** hojas, $\varphi_3^{-1}(\ell(\frac{12}{5})) \supset L_2$.

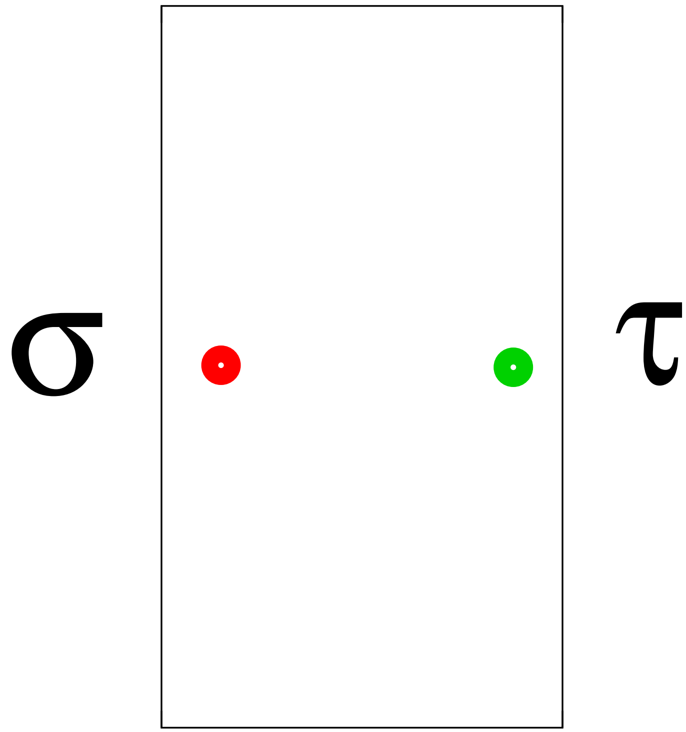
$\varphi_4 : S^3 \longrightarrow (S^3, L_2)$ de **3** hojas, $\varphi_4^{-1}(L_2) \supset L_3$.

$\varphi_5 : S^3 \longrightarrow (S^3, L_3)$ de **3** hojas, $\varphi_5^{-1}(L_3) \supset \mathrm{Borromeos}$.

Por lo tanto $\varphi = \varphi_5 \circ \varphi_4 \circ \varphi_3 \circ \varphi_2 \circ \varphi_1 : S^3 \longrightarrow (S^3, \ell(\frac{5}{3}))$ de **1080** hojas, $\varphi^{-1}(\ell(\frac{5}{3})) \supset \mathrm{Borromeos}$.

Luego $k = \ell(\frac{5}{3})$ es universal.

Para $k = \ell(a/b)$ con a impar



$$\sigma = (1,2)(3,4)\cdots(a-2,a-1)$$
$$\tau = (2,3)(4,5)\cdots(a-1,a)$$

Un nudo de dos puentes $\ell(b/a)$ es hiperbólico si y sólo si $b \not\equiv \pm 1 \pmod{a}$.

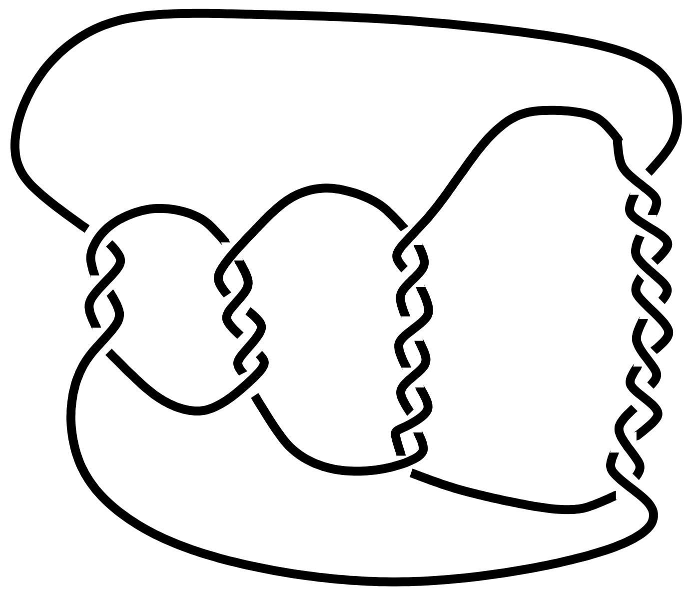
Teorema. (Hilden–Lozano–Montesinos) Un nudo de dos puentes k es universal si y sólo si k es hiperbólico.

Más nudos universales.

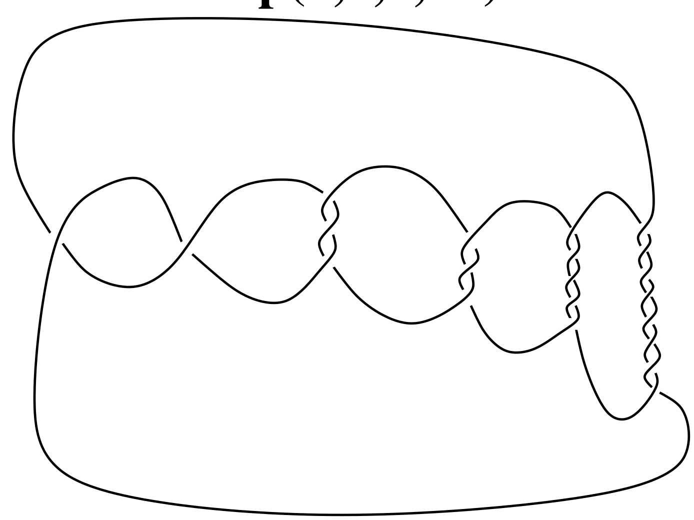
Definición. Un enlace de Uchida es un nudo pretzel $p(a_1, a_2, \ldots, a_t)$ con al menos dos a's pares.

Teorema. (Uchida) Todos los enlaces de Uchida son universales, excepto:

- $p(2s, 2t)$, $s, t \in Z - \{0\}$.
- $p(-2, 2, s)$, $s \in Z - \{0\}$
- $p(\pm 2, \pm 3, \mp 4)$, $p(\pm 2, \mp 3, \mp 6)$, $p(\pm 2, \mp 4, \mp 4)$ y
- $p(-2, -2, 2, 2)$.



p(2,4,6,−8)

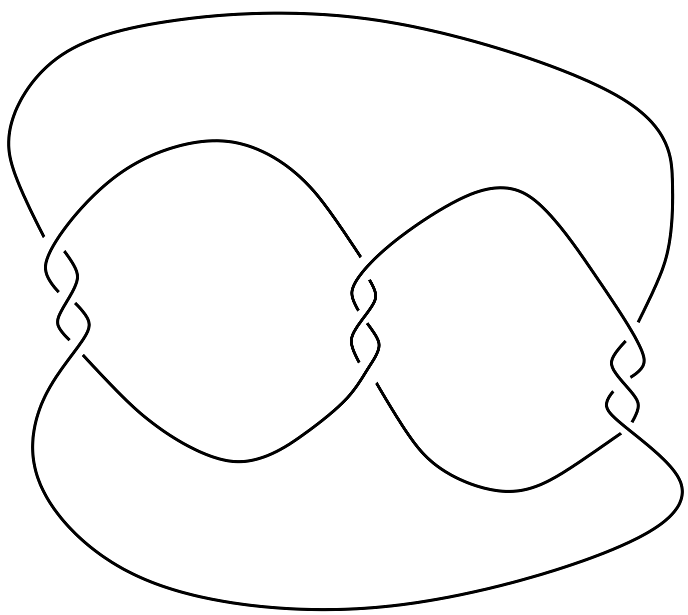


p(1, 1, 2, 3, 5, 8)

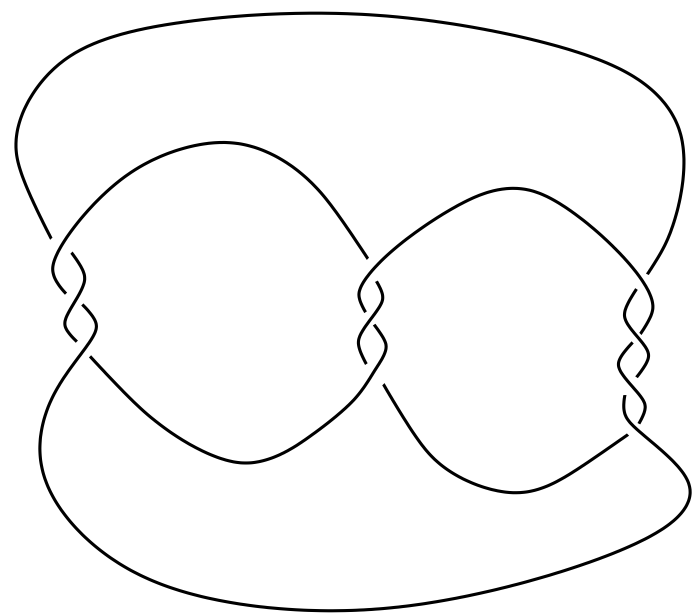
Teorema. (J. Rodríguez)

Si $|n| > 1$ y n es impar, entonces $p(n, n, -n)$ es universal.

Si $n \neq 2$ y n es par, entonces $p(3, 3, n)$ es universal.



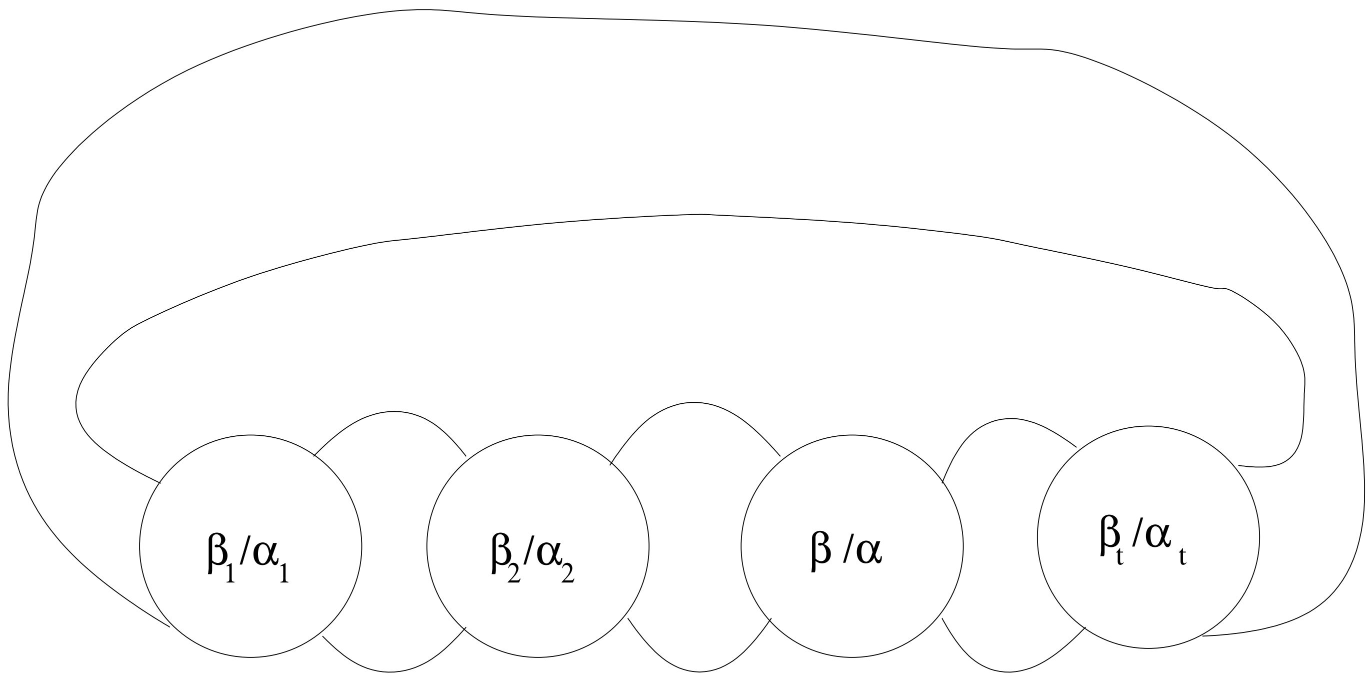
p(3, 3, –3)



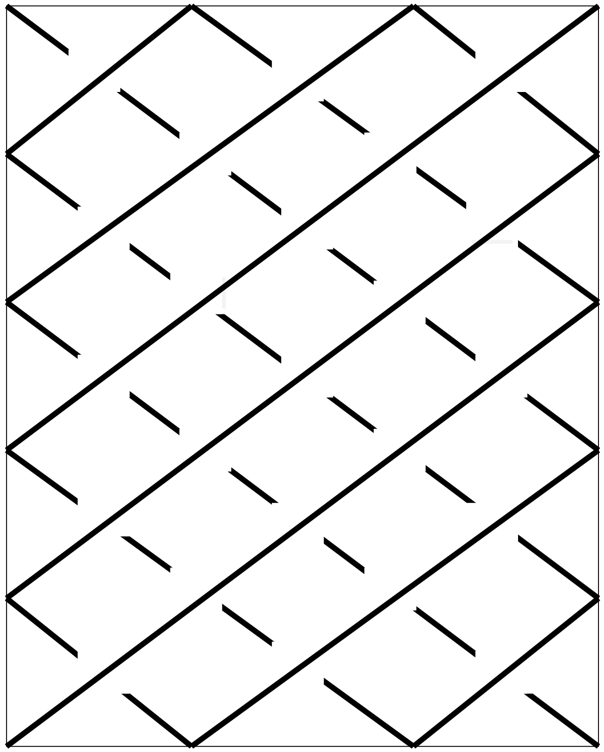
p(3, 3, –4)

Nudos de Montesinos.

Un <u>nudo de Montesinos</u> $m(\beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$ es un enlace de la forma:



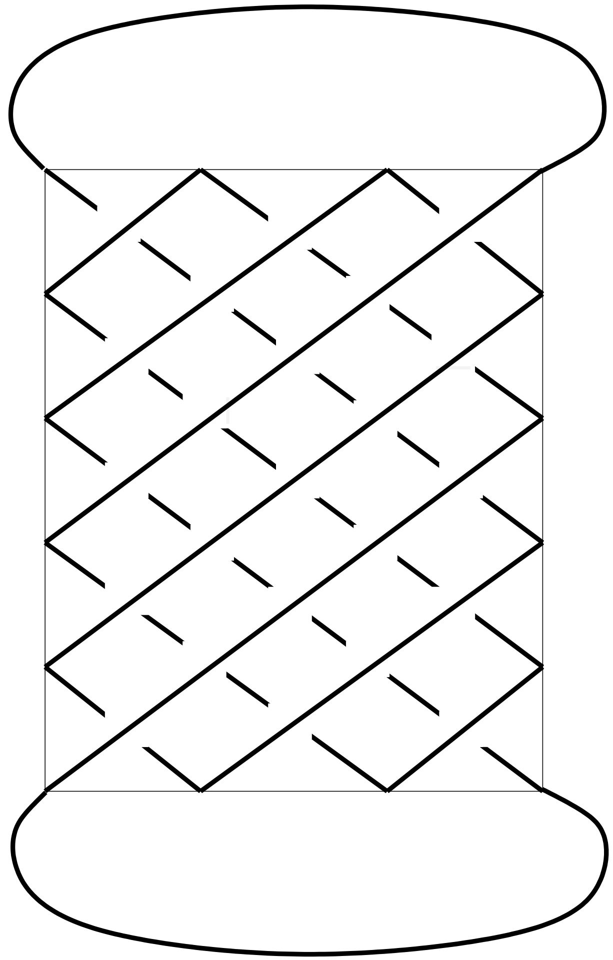
donde cada cajita (cada almohada cuadrada) contiene un ovillo racional



El ovillo racional 3/5

El lado vertical está dividido en **5** intervalos.

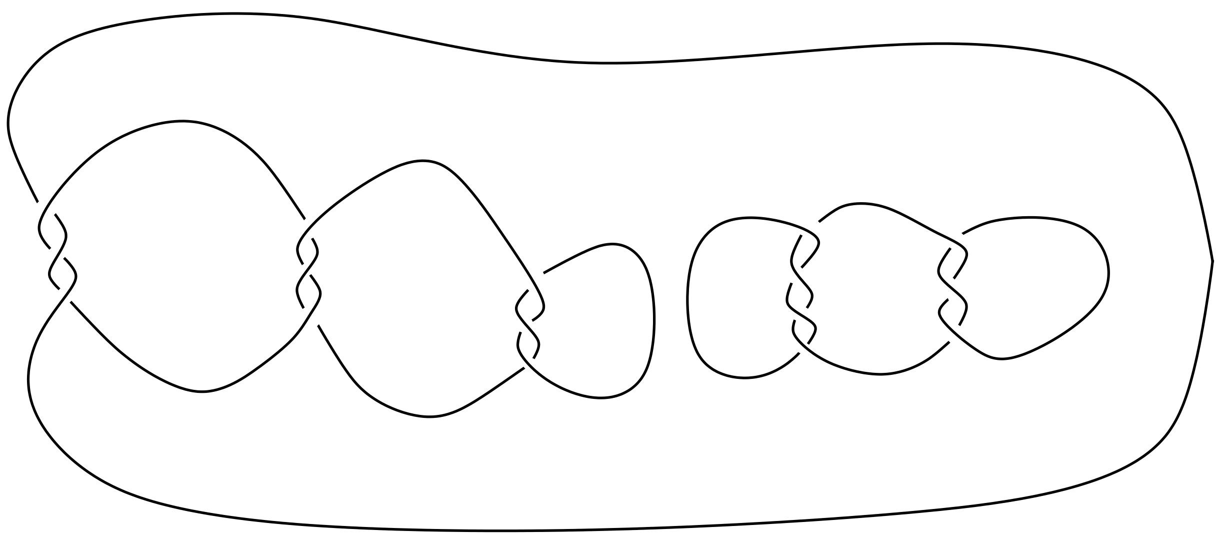
El lado horizontal está dividido en **3** intervalos.



El nudo racional $\ell(3/5) = m(3/5)$

Siempre supondremos que para cada i, $(\alpha_i, \beta_i) = 1$.

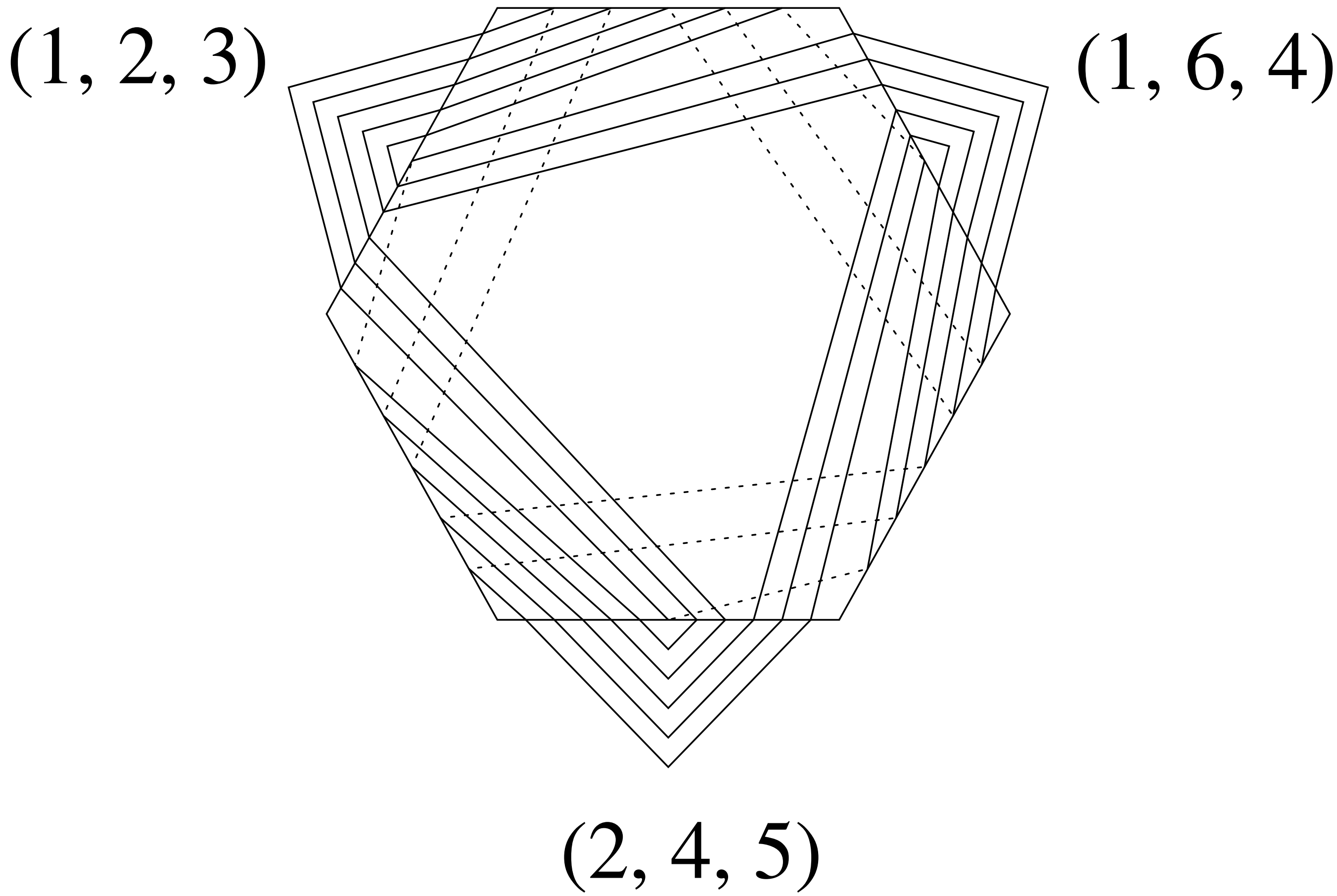
Es conveniente permitir que algunas α's sean cero (aunque en este caso el nudo de montesinos es una unión de sumas conexas de enlaces racionales



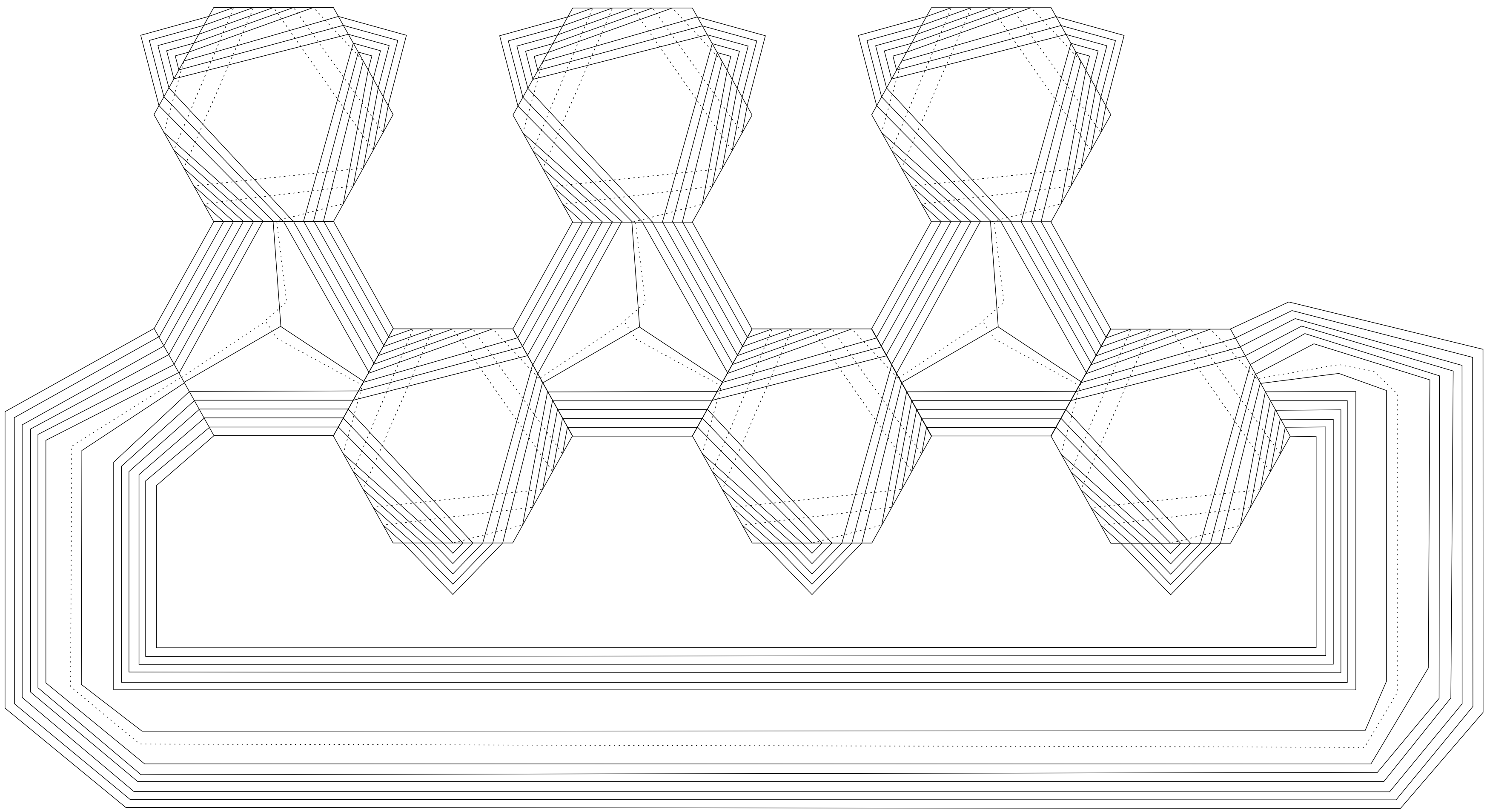
m(1/3, 1/3, –1/3, 1/0, –1/4, 1/3, 1/0)

).

Cubiertas de nudos de Montesinos.



$m(1/3, 1/3, 1/3)$



Necesitamos otro enfoque...

Cocientes diédricos.

Sea $k \subset S^3$ un enlace. Escribimos $B_2(k)$ para la cubierta cíclica ramificada de dos hojas de (S^3, k)

(es decir, es la cubierta que se obtiene de marcar cada meridiano de k con la permutación $(1, 2)$.

En este caso hay una involución

$$u : B_2(k) \to B_2(k)$$

cuyo cociente es la cubierta cíclica ramificada de dos hojas

$$p : B_2(k) \to (S^3, k)$$

y es tal que $p(\mathrm{fix}(u)) = k$).

Sea $\varphi : M \to (S^3, k)$ una cubierta ramificada de d-hojas. φ se llama un cociente diédrico si existe un diagrama conmutativo de cubiertas ramificadas

$$\begin{array}{ccc} & \tilde{M} & \\ {}^{q}\swarrow & & \searrow{}^{\psi} \\ M & & B_2(k) \\ {}_{\varphi}\searrow & & \swarrow{}_{p} \\ & (S^3, k) & \end{array}$$

tal que ψ es un espacio cubriente (sin ramificación) de d-hojas.

En este caso q es una cubierta cíclica ramificada de dos hojas con ramificación en el seudo-branch de φ (éste es un subenlace muy especial de $\varphi^{-1}(k)$).

Si k es el nudo de Montesinos $k = m(\beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$, entonces $B_2(k)$ es la variedad de Seifert

$$B_2(k) = (O, 0; \beta_1/\alpha_1, \ldots, \beta_t/\alpha_t).$$

Teorema. (E. Ramírez y V.) Es posible calcular, en términos de los invariantes de Seifert, las cubiertas de una variedad de Seifert con símbolo $(O, 0; \beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$.

El seudobranch.

Para el nudo de Montesinos $k = m(\frac{\beta_1}{\alpha_1}, \ldots, \frac{\beta_t}{\alpha_t})$ escribimos $\Delta(k) = \beta_1\alpha_2 \cdots \alpha_t + \alpha_1\beta_2 \cdots \alpha_t + \cdots + \alpha_1\alpha_2 \cdots \beta_t$.

Teorema. (J. Rodríguez y V.) Si n es un divisor positivo de $\Delta(k)$ y para cada $i = 1, \ldots, t$, $(n, \alpha_i) = 1$, entonces

$$k \sim m(\frac{n \cdot b_1}{\alpha_1}, \ldots, \frac{n \cdot b_t}{\alpha_t}),$$

y existe un cociente diédrico de n hojas $\varphi : S^3 \to (S^3, k)$ tal que

$$m(\frac{b_1}{\alpha_1}, \ldots, \frac{b_t}{\alpha_t}) \subset \varphi^{-1}(k).$$

($m(\frac{b_1}{\alpha_1}, \ldots, \frac{b_t}{\alpha_t})$ es el seudobranch de φ.)

Corolario.
Supongamos que $(n, \alpha_i) = 1$ para cada $i = 1, 2, \ldots, t$.

Si $m(\beta_1/\alpha_1, \ldots, \beta_t/\alpha_t)$ es universal,
entonces $m(n\beta_1/\alpha_1, \ldots, n\beta_t/\alpha_t)$ es universal.

Tabla de Conway: 2-enlaces de 9 cruces y alternantes

$m(1/4, 2/3, 1/2)$ $[\Delta = 17 \cdot 2] = m(17/4, 17/3, -17/2) \leftarrow m(1/4, 1/3, -1/2)$

$m(3/4, 1/3, 1/2)$ $[\Delta = 19 \cdot 2] = m(19/4, 19/3, -19/2) \leftarrow m(1/4, 1/3, -1/2)$

$m(3/4, 2/3, 1/2)$ $[\Delta = 23 \cdot 2] = m(23/4, 23/3, -23/2) \leftarrow m(1/4, 1/3, -1/2)$

$m(1/3, 1/3, 2/3)$ $[\Delta = 4 \cdot 3^2] = m(4/3, 4/3, -4/3) \leftarrow m(1/3, 1/3, -1/3)$

$m(2/3, 2/3, 2/3) \leftarrow m(1/3, 1/3, 1/3)$

$m(3/5, 1/2, 3/2)$ $[\Delta = 13 \cdot 2^2] = m(13/5, 13/2, -13/2) \leftarrow m(1/5, 1/2, -1/2)$

$m(1/3, 1/2, 5/2)$ $[\Delta = 5 \cdot 2^2] = m(-5/3, 5/2, 5/2) \leftarrow m(-1/3, 1/2, 1/2)$

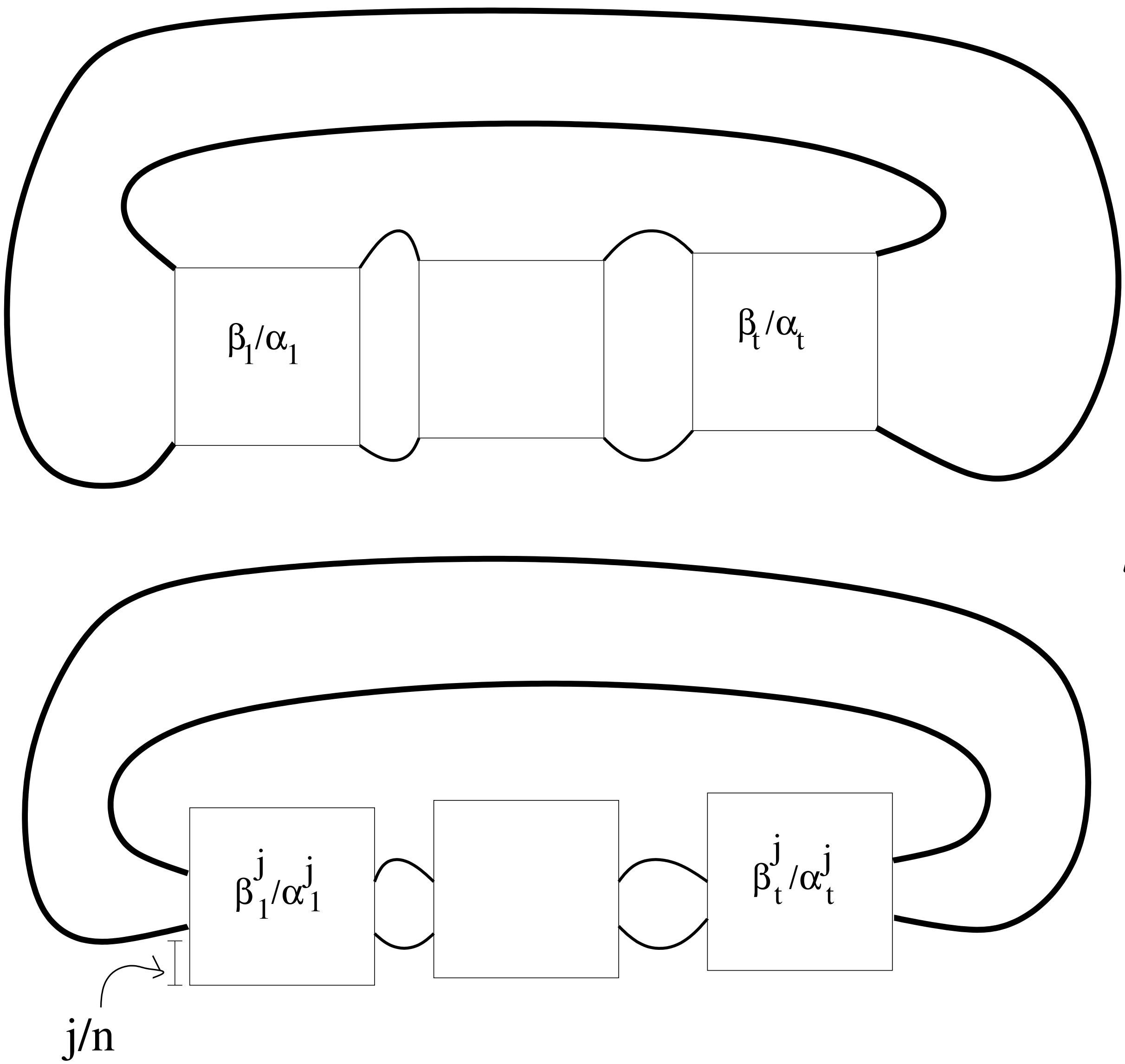
El branch.

Si $(n, \alpha_i) = 1$ para cada i y $k = m(n\beta_1/\alpha_1, \ldots, n\beta_t/\alpha_t)$ y $\varphi : S^3 \to (S^3, k)$ es un cociente diédrico de n-hojas, entonces

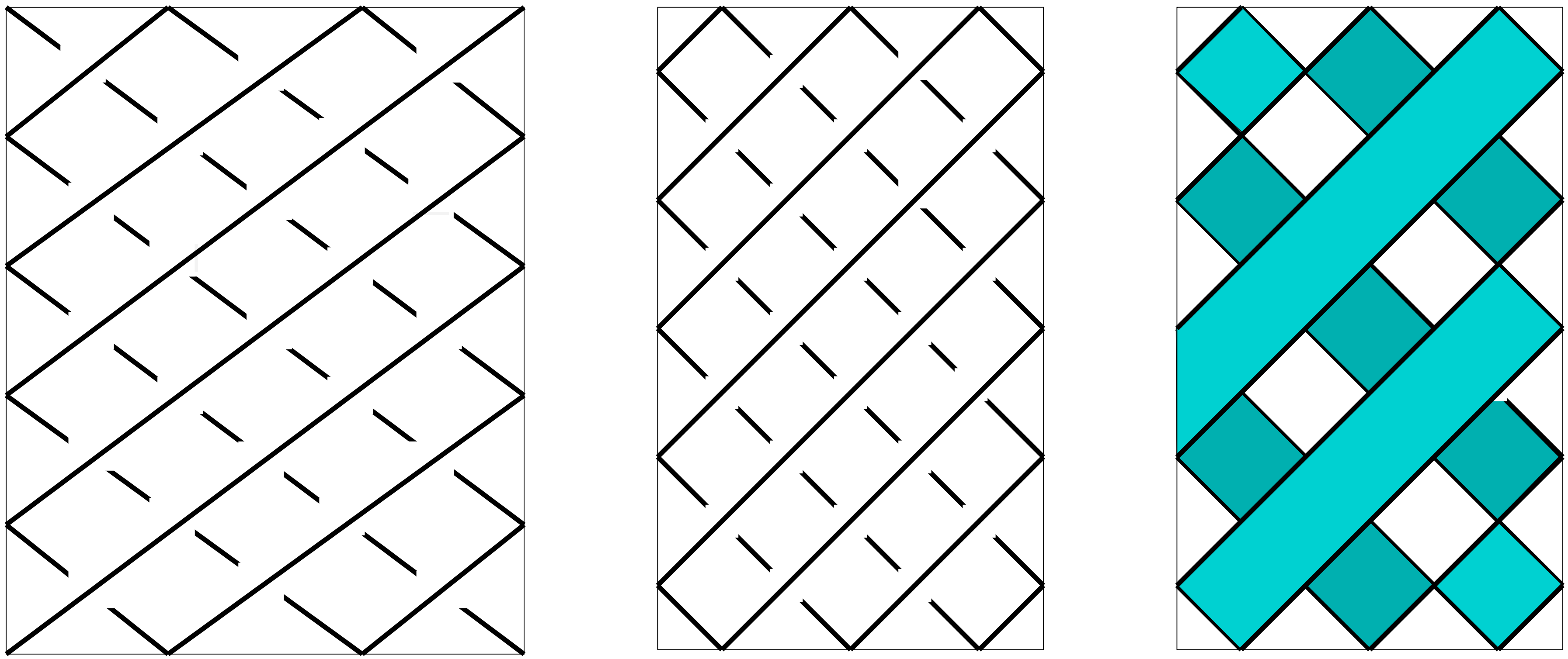
$\varphi^{-1}(k)$ tiene $(n-1)/2$ "componentes" $k_1, k_2, \ldots, k_{(n-1)/2}$, si n es impar

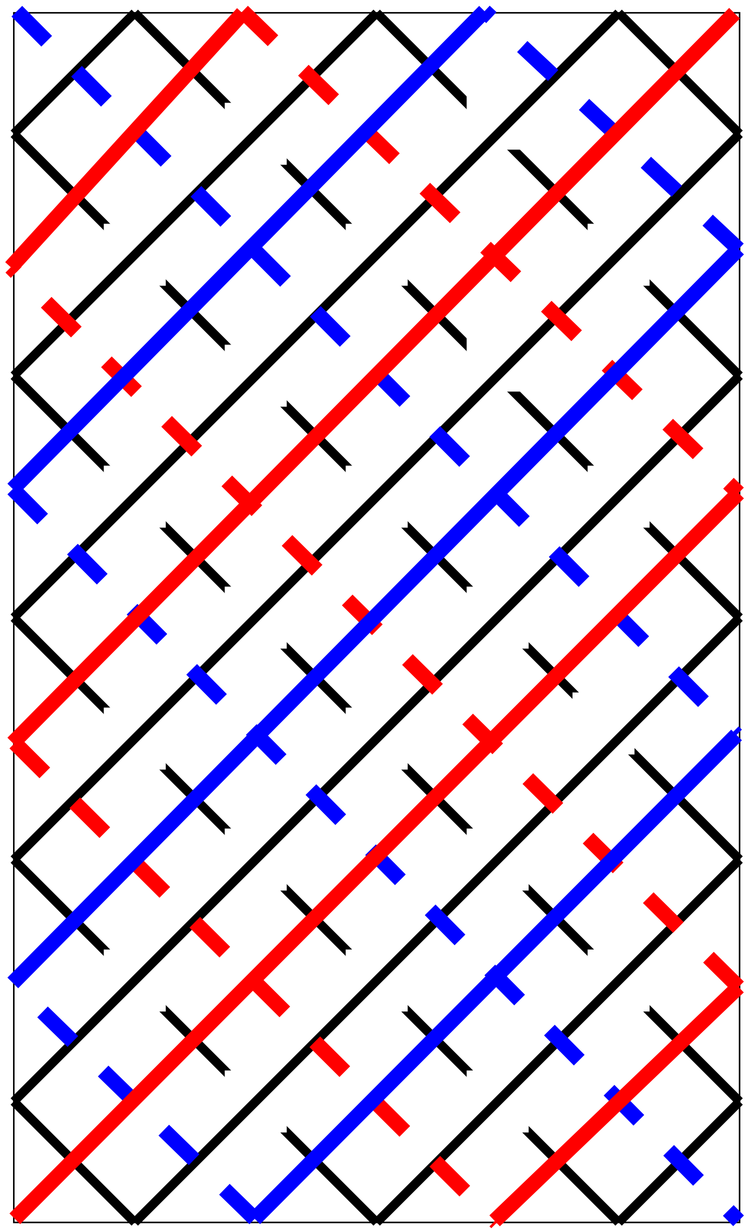
$\varphi^{-1}(k)$ tiene $n/2$ "componentes" $k_1, k_2, \ldots, k_{n/2}$, si n es par.

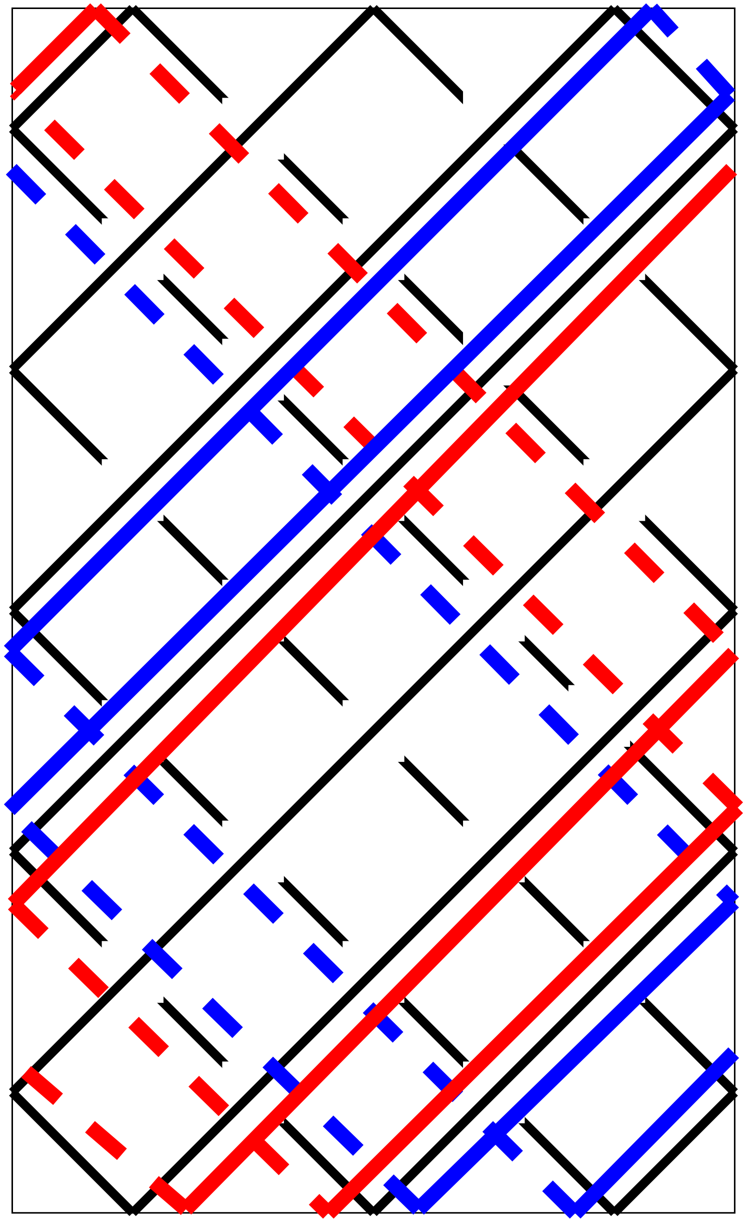
¿Cómo se ven esas "componentes"?



Las "componentes" k_1 y k_j de $\varphi^{-1}(k)$.







Algoritmo.

Sea ε el signo de la fracción β_i/α_i y dibuja el meridiano de $|\beta_i/\alpha_i|$.

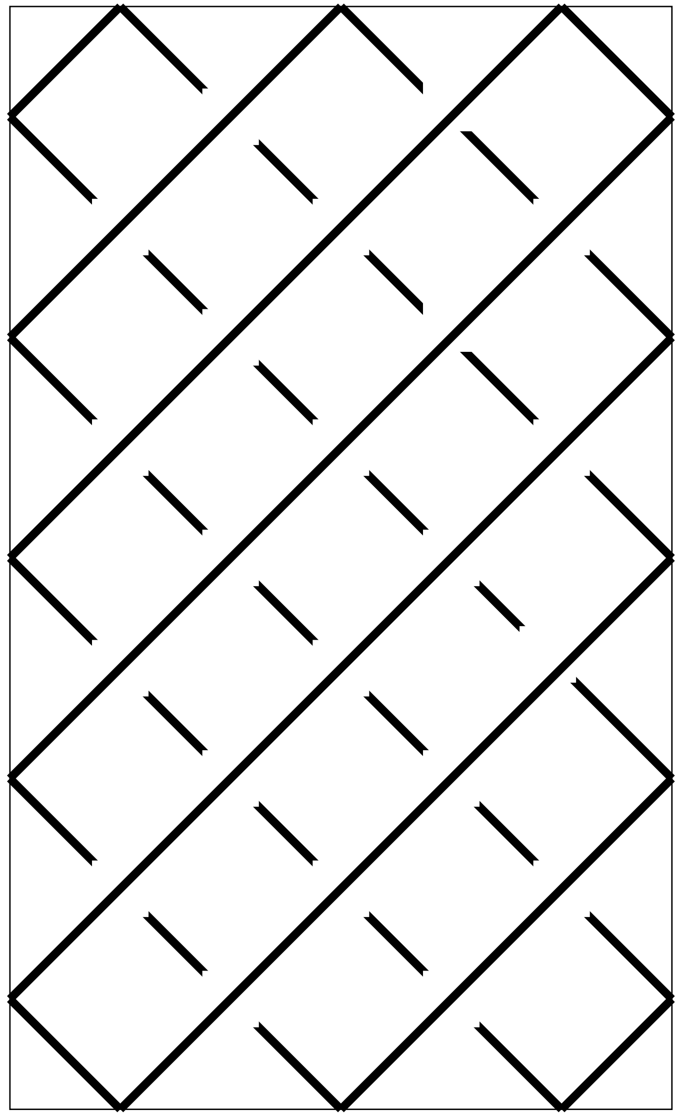
1. Marca el punto $p_0 = (0, 1/2\alpha_i)$ con $+1$.

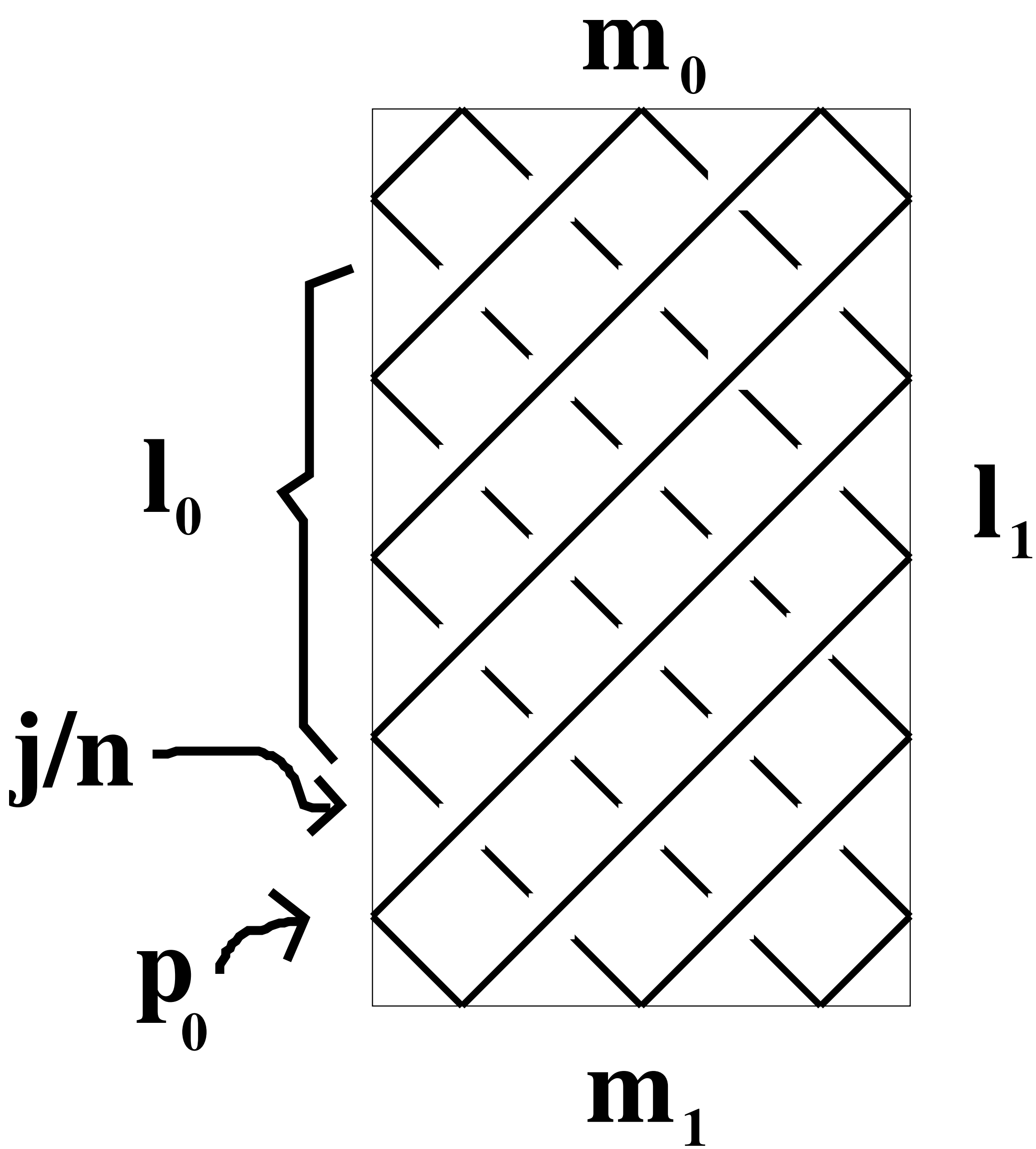
2. Si el punto $p_u \in d \cap \partial I^2$ está marcado con $\varepsilon_u \in \{-1, +1\}$, entonces el segmento de recta de d que comienza en p_u, si seguimos la orietación de'd, toca a ∂I^2 en el punto p_{u+1}.

   (a) Si p_{u+1} no tiene una marca, entonces
   - Si ($p_u \in m_0$ y $p_{u+1} \in m_0$) o ($p_u \in m_1$ y $p_{u+1} \in m_1$), entonces marca p_{u+1} con $\varepsilon_{u+1} = -\varepsilon_u$;
   - en otro caso marca p_{u+1} con $\varepsilon_{u+1} = \varepsilon_u$.

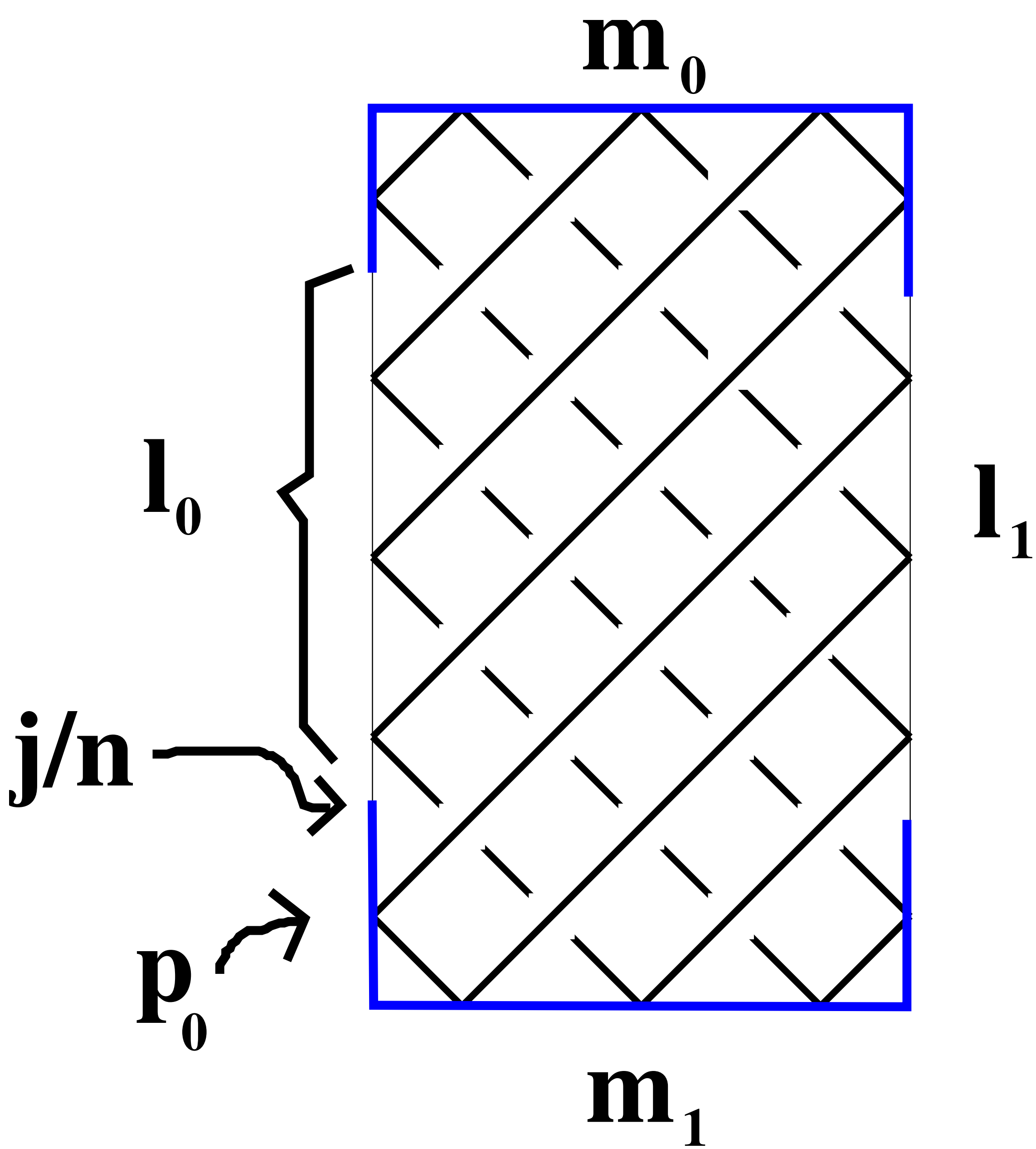
   GOTO 2 con "$u := u + 1$".

   (b) Si p_{u+1} ya está marcado, entonces escribe $b =$ la suma de las marcas de los puntos en m_0 y $\alpha =$ la suma de las marcas de los puntos en ℓ_0. Regresa $b_i^j/\alpha_i^j = \varepsilon b/\alpha$.

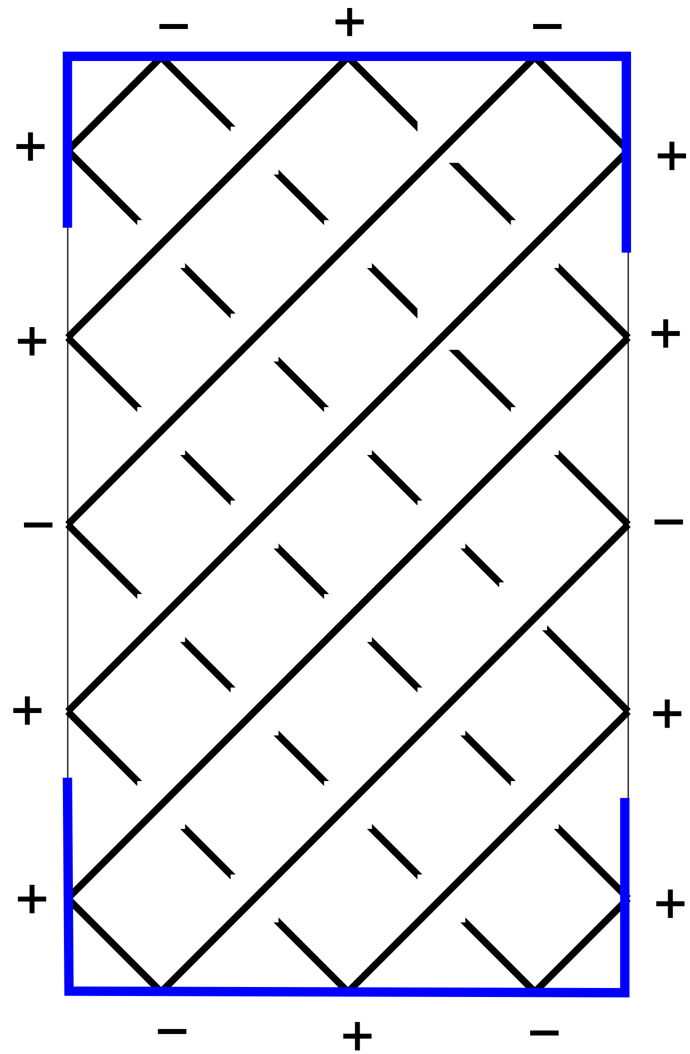




m_0
l_0
l_1
j/n
p_0
m_1

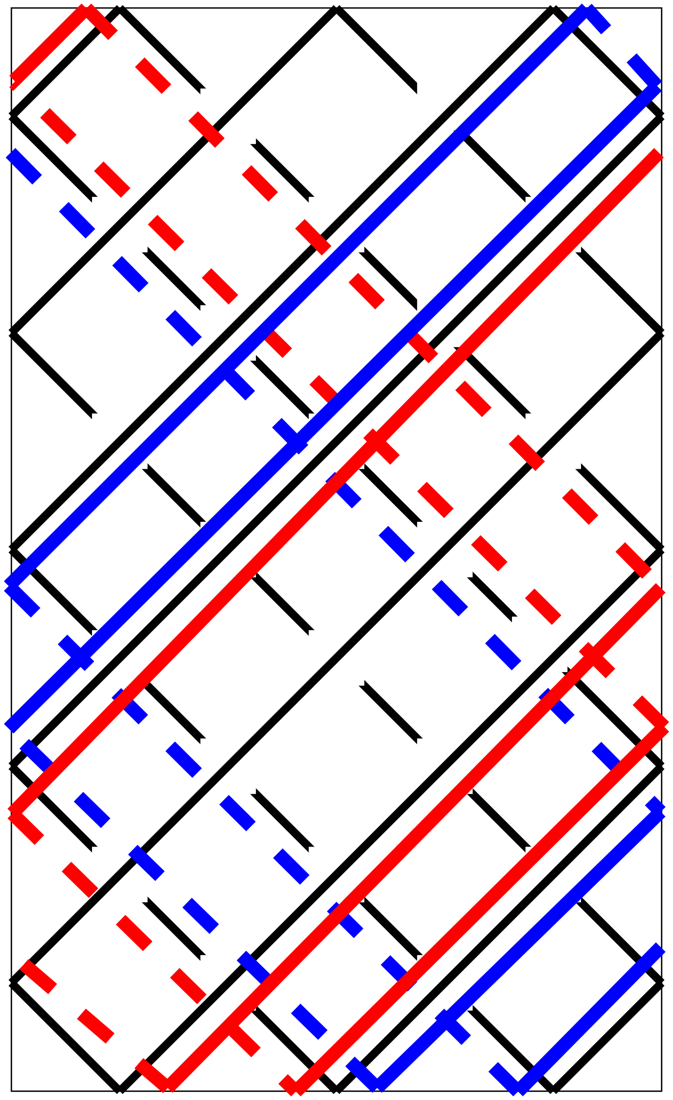


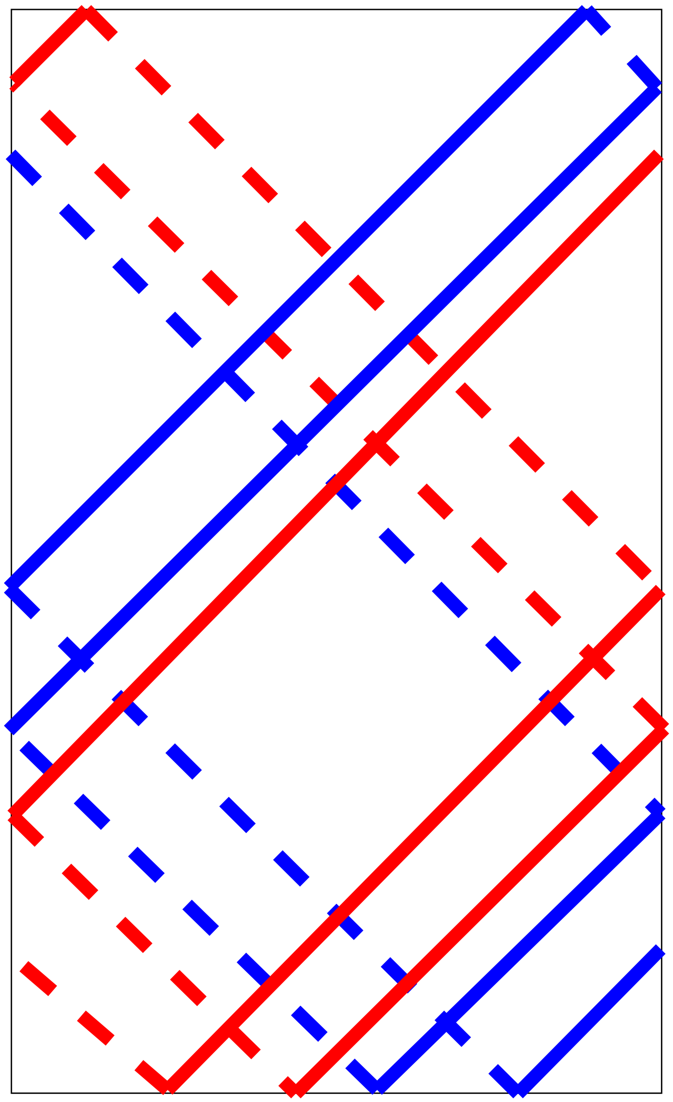
m0
l0
j/n
p0
l1
m1

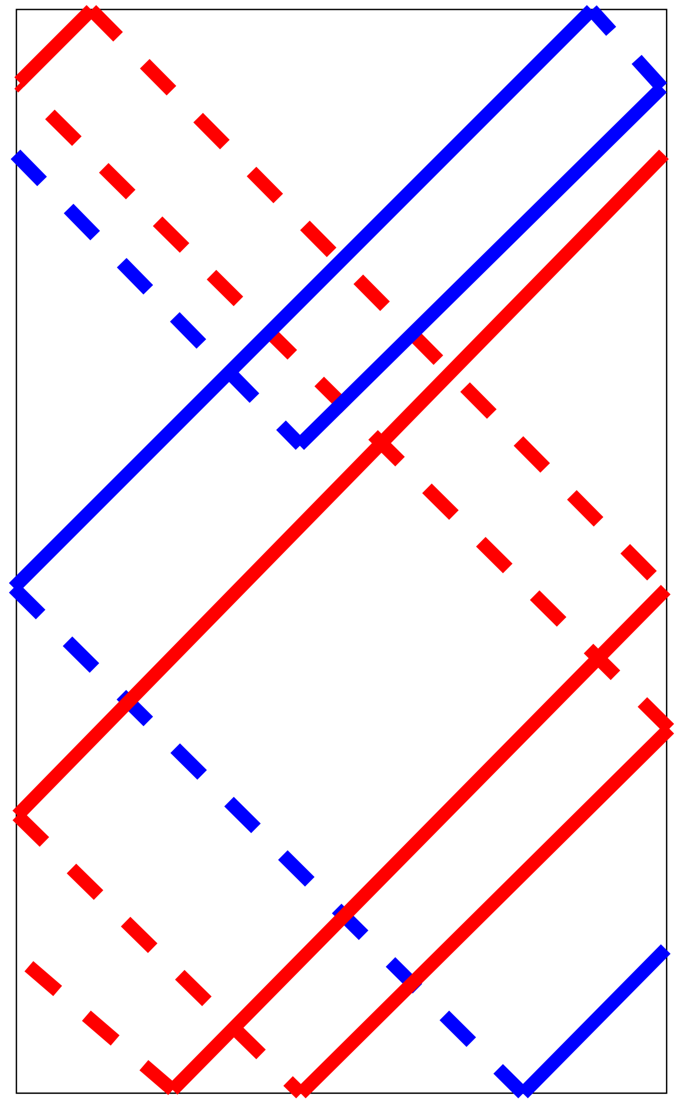


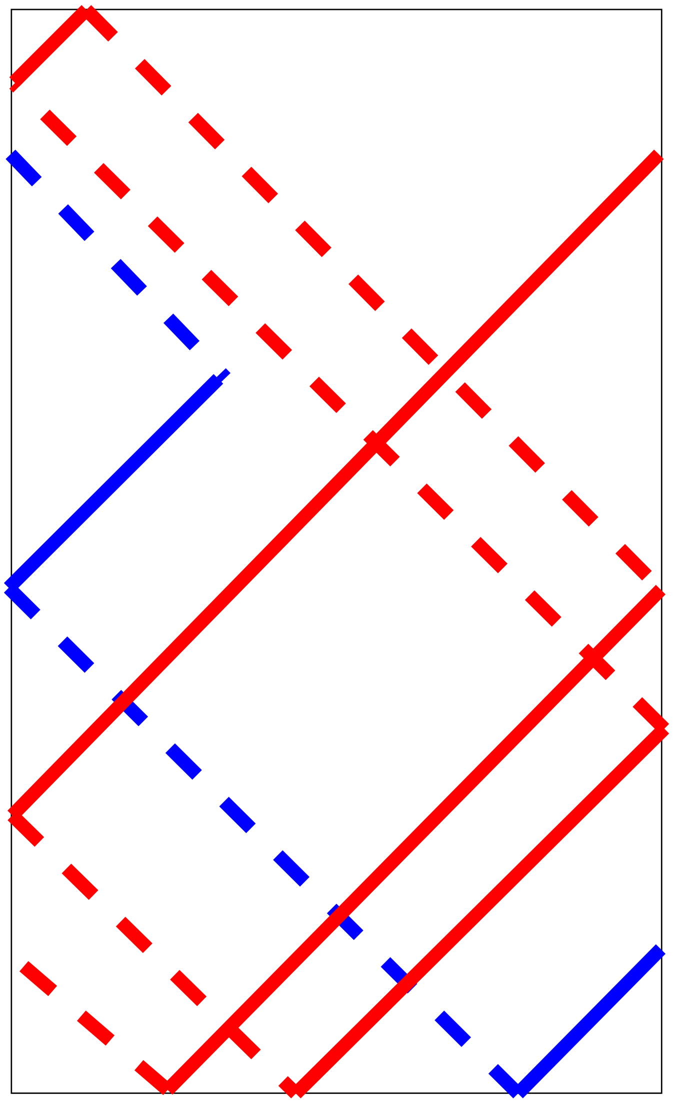
$$b_i^j/\alpha_i^j = 1/1.$$

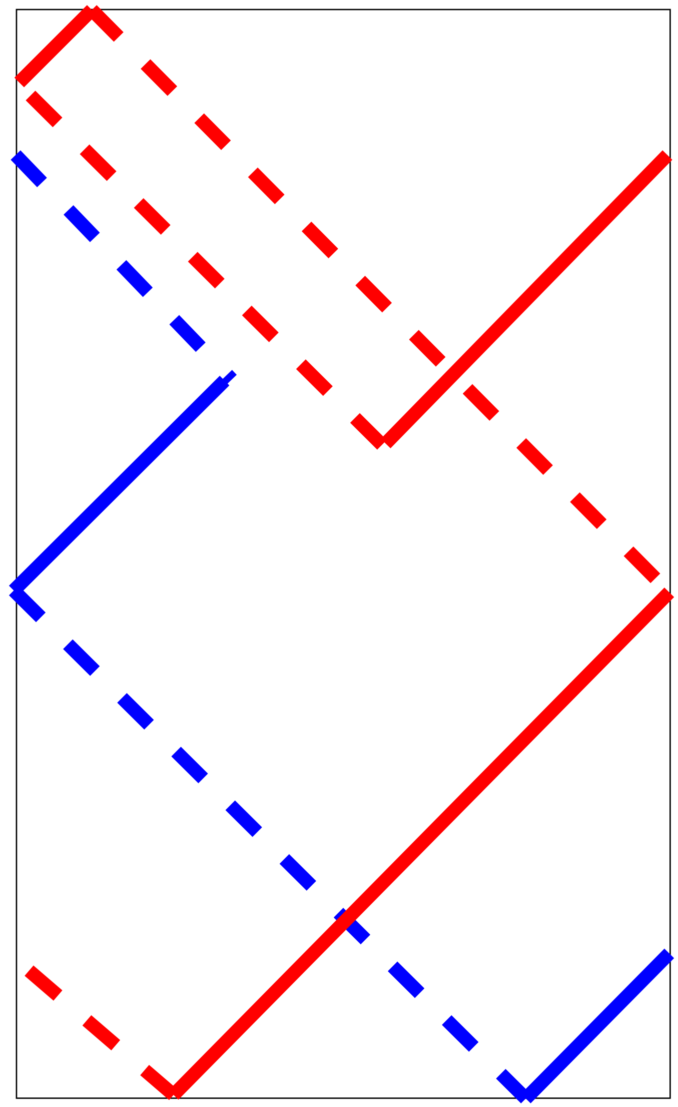
Con dibujos.

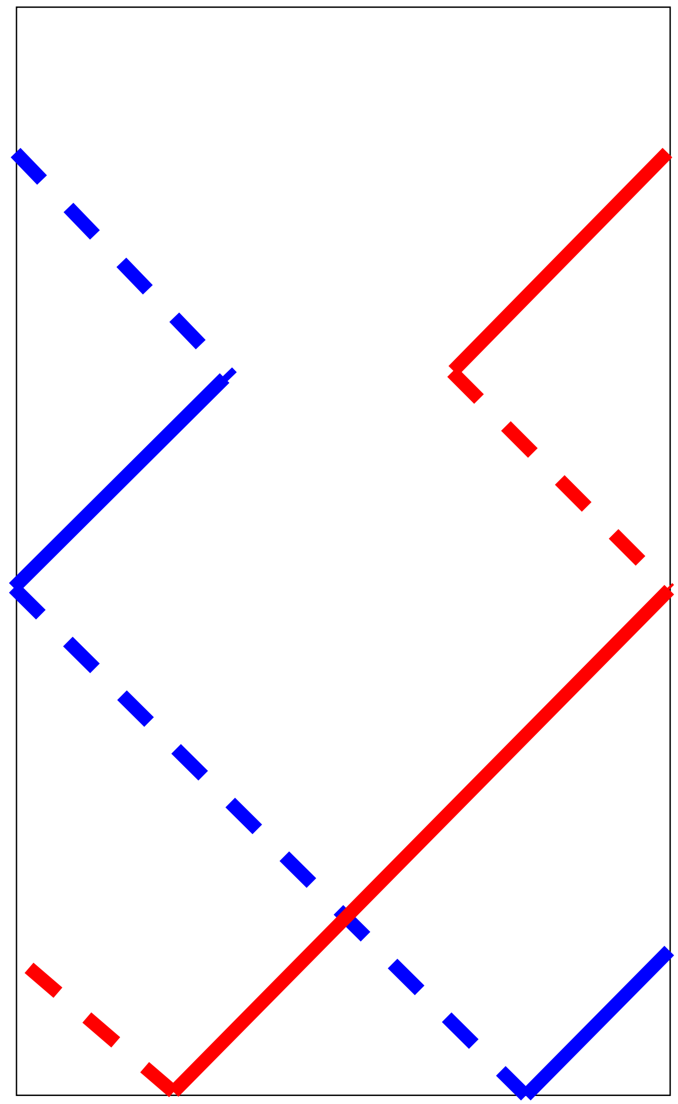












Una aplicación.

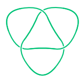
Proposición. (J. Rodríguez y V.) Sea q un entero impar, $q \notin \{-11, -7, -5, -3, -1, 1, 3, 5\}$ y sea k el nudo pretzel $k = p(2, q, q) = m(1/2, \pm 1/|q|, \pm 1/|q|)$.
Entonces existe una cubierta diédrica $\varphi : S^3 \to (S^3, k)$ de $|q + 4|$ hojas tal que

1. Si $|q| \equiv 1 \bmod 4$, entonces
   o bien el nudo de Montesinos $m(1/2, -1/5, -1/5) \subset \varphi^{-1}(k)$
   o bien $m(1/2, -2/9, -2/9) \subset \varphi^{-1}(k)$.

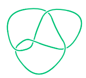
2. If $|q| \equiv -1 \bmod 4$, entonces
   o bien $m(-1/2, 3/5, 3/5) \subset \varphi^{-1}(k)$
   o bien $m(-1/2, 2/3, 2/3) \subset \varphi^{-1}(k)$.

¿Qué tantos nudos de Montesinos hay?

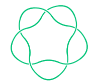
Nudos racionales.



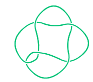
$3_1 = \mathrm{m}(\frac{1}{3})$



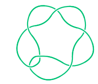
$4_1 = \mathrm{m}(\frac{2}{5})$



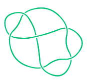
$5_1 = \mathrm{m}(\frac{1}{5})$



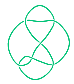
$5_2 = \mathrm{m}(\frac{3}{7})$



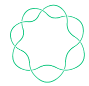
$6_1 = \mathrm{m}(\frac{4}{9})$



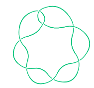
$6_2 = \mathrm{m}(\frac{4}{11})$



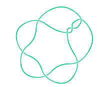
$6_3 = \mathrm{m}(\frac{5}{13})$



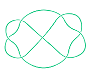
$7_1 = \mathrm{m}(\frac{1}{7})$



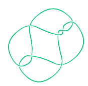
$7_2 = \mathrm{m}(\frac{5}{11})$



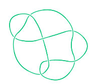
$7_3 = \mathrm{m}(\frac{4}{13})$



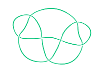
$7_4 = \mathrm{m}(\frac{4}{15})$



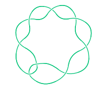
$7_5 = \mathrm{m}(\frac{7}{17})$



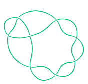
$7_6 = \mathrm{m}(\frac{7}{19})$



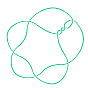
$7_7 = \mathrm{m}(\frac{8}{21})$



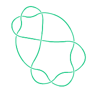
$8_1 = \mathrm{m}(\frac{6}{13})$



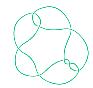
$8_2 = \mathrm{m}(\frac{6}{17})$



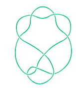
$8_3 = \mathrm{m}(\frac{4}{17})$



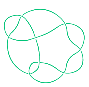
$8_4 = \mathrm{m}(\frac{5}{19})$



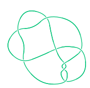
$8_6 = \mathrm{m}(\frac{10}{23})$



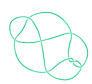
$8_7 = \mathrm{m}(\frac{9}{23})$



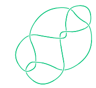
$8_8 = \mathrm{m}(\frac{9}{25})$



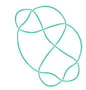
$8_9 = \mathrm{m}(\frac{7}{25})$



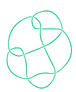
$8_{11} = \mathrm{m}(\frac{10}{27})$



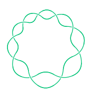
$8_{12} = \mathrm{m}(\frac{12}{29})$



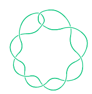
$8_{13} = \mathrm{m}(\frac{11}{29})$



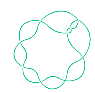
$8_{14} = \mathrm{m}(\frac{12}{31})$



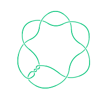
$9_1 = \mathrm{m}(\frac{1}{9})$



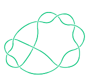
$9_2 = \mathrm{m}(\frac{7}{15})$



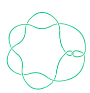
$9_3 = \mathrm{m}(\frac{6}{19})$



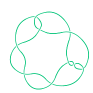
$9_4 = \mathrm{m}(\frac{5}{21})$



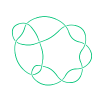
$9_5 = \mathrm{m}(\frac{6}{23})$



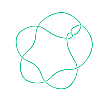
$9_6 = \mathrm{m}(\frac{11}{27})$



$9_7 = \mathrm{m}(\frac{13}{29})$

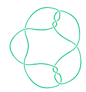


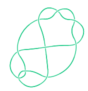
$9_8 = \mathrm{m}(\frac{11}{31})$

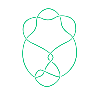


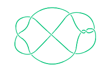
$9_9 = \mathrm{m}(\frac{9}{31})$

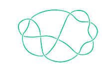
Nudos racionales.

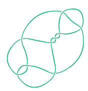

$9_{10} = \mathrm{m}(\frac{10}{33})$

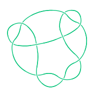

$9_{11} = \mathrm{m}(\frac{14}{33})$

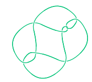

$9_{12} = \mathrm{m}(\frac{13}{35})$

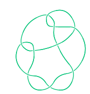

$9_{13} = \mathrm{m}(\frac{10}{37})$

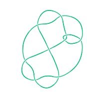

$9_{14} = \mathrm{m}(\frac{14}{37})$

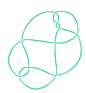

$9_{15} = \mathrm{m}(\frac{16}{39})$

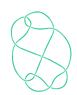

$9_{17} = \mathrm{m}(\frac{14}{39})$

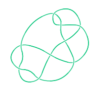

$9_{18} = \mathrm{m}(\frac{17}{41})$

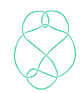

$9_{19} = \mathrm{m}(\frac{16}{41})$

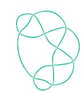

$9_{20} = \mathrm{m}(\frac{15}{41})$

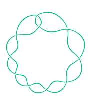

$9_{21} = \mathrm{m}(\frac{18}{43})$

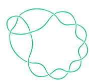

$9_{23} = \mathrm{m}(\frac{19}{45})$

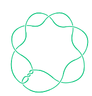

$9_{26} = \mathrm{m}(\frac{18}{47})$

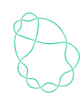

$9_{27} = \mathrm{m}(\frac{19}{49})$

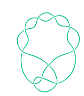

$9_{31} = \mathrm{m}(\frac{21}{55})$

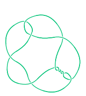

$10_{1} = \mathrm{m}(\frac{8}{17})$

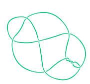

$10_{2} = \mathrm{m}(\frac{8}{23})$

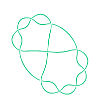

$10_{3} = \mathrm{m}(\frac{6}{25})$

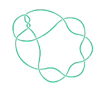

$10_{4} = \mathrm{m}(\frac{7}{27})$

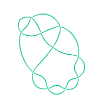

$10_{5} = \mathrm{m}(\frac{13}{33})$

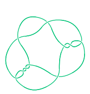

$10_{6} = \mathrm{m}(\frac{16}{37})$

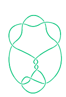

$10_{7} = \mathrm{m}(\frac{16}{43})$

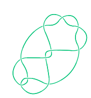

$10_{8} = \mathrm{m}(\frac{6}{29})$

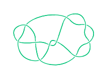

$10_{9} = \mathrm{m}(\frac{11}{39})$

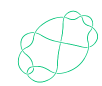

$10_{10} = \mathrm{m}(\frac{17}{45})$

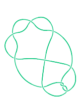

$10_{11} = \mathrm{m}(\frac{13}{43})$

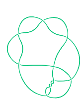

$10_{12} = \mathrm{m}(\frac{17}{47})$

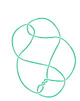

$10_{13} = \mathrm{m}(\frac{22}{53})$

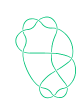

$10_{14} = \mathrm{m}(\frac{22}{57})$

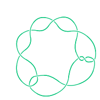

$10_{15} = \mathrm{m}(\frac{19}{43})$


$10_{16} = \mathrm{m}(\frac{14}{47})$

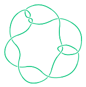

$10_{17} = \mathrm{m}(\frac{9}{41})$


$10_{18} = \mathrm{m}(\frac{23}{55})$

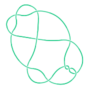

$10_{19} = \mathrm{m}(\frac{14}{51})$


$10_{20} = \mathrm{m}(\frac{16}{35})$

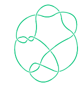
Nudos racionales.



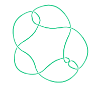
$10_{21} = \mathrm{m}(\frac{16}{45})$



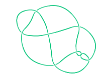
$10_{22} = \mathrm{m}(\frac{13}{49})$



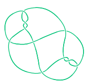
$10_{23} = \mathrm{m}(\frac{23}{59})$



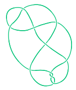
$10_{24} = \mathrm{m}(\frac{24}{55})$



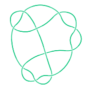
$10_{25} = \mathrm{m}(\frac{24}{65})$



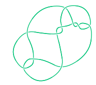
$10_{26} = \mathrm{m}(\frac{17}{61})$



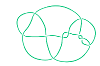
$10_{27} = \mathrm{m}(\frac{27}{71})$



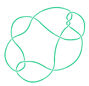
$10_{28} = \mathrm{m}(\frac{19}{53})$



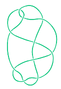
$10_{29} = \mathrm{m}(\frac{26}{63})$



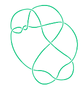
$10_{30} = \mathrm{m}(\frac{26}{67})$



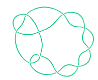
$10_{31} = \mathrm{m}(\frac{25}{57})$



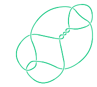
$10_{32} = \mathrm{m}(\frac{29}{69})$



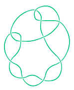
$10_{33} = \mathrm{m}(\frac{18}{65})$



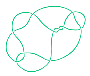
$10_{34} = \mathrm{m}(\frac{13}{37})$



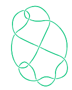
$10_{35} = \mathrm{m}(\frac{20}{49})$



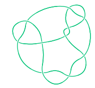
$10_{36} = \mathrm{m}(\frac{20}{51})$



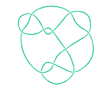
$10_{37} = \mathrm{m}(\frac{23}{53})$



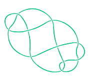
$10_{38} = \mathrm{m}(\frac{25}{59})$



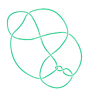
$10_{39} = \mathrm{m}(\frac{22}{61})$



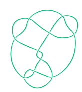
$10_{40} = \mathrm{m}(\frac{29}{75})$



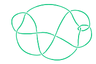
$10_{41} = \mathrm{m}(\frac{26}{71})$



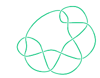
$10_{42} = \mathrm{m}(\frac{31}{81})$



$10_{43} = \mathrm{m}(\frac{27}{73})$

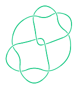


$10_{44} = \mathrm{m}(\frac{30}{79})$

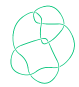


$10_{45} = \mathrm{m}(\frac{34}{89})$

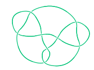
Nudos de Montesinos.



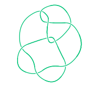
$8_5 = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{2})$



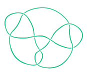
$8_{10} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{1}{2})$



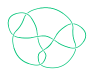
$8_{15} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{1}{2})$



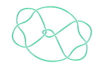
$8_{19} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{2})$



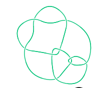
$8_{20} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{-1}{2})$



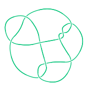
$8_{21} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{2})$



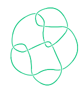
$9_{16} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{3}{2})$



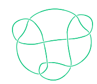
$9_{22} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{2})$



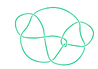
$9_{24} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{3}{2})$



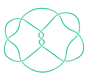
$9_{25} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{1}{2})$



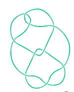
$9_{28} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{3}{2})$



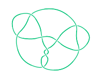
$9_{30} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{1}{2})$



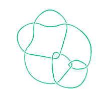
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



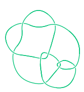
$9_{36} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{1}{2})$



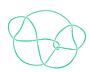
$9_{37} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$



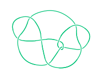
$9_{42} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-1}{2})$



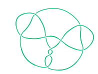
$9_{43} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{2})$



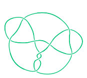
$9_{44} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{2})$



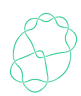
$9_{45} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{-1}{2})$



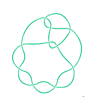
$9_{46} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{3})$



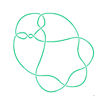
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



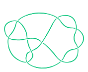
$10_{46} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{1}{2})$



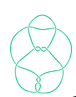
$10_{47} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{1}{2})$



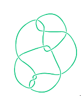
$10_{48} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{1}{2})$



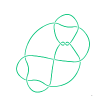
$10_{49} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{1}{2})$



$10_{50} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{1}{2})$

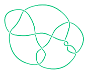


$10_{51} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{1}{2})$

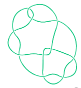


$10_{52} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{1}{2})$

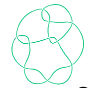
Nudos de Montesinos.



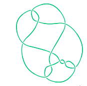
$10_{53} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{1}{2})$



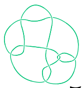
$10_{54} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{1}{2})$



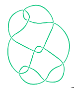
$10_{55} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{1}{2})$



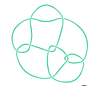
$10_{56} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{1}{2})$



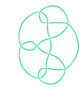
$10_{57} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{1}{2})$



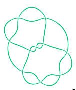
$10_{58} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{1}{2})$



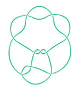
$10_{59} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{1}{2})$



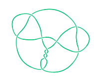
$10_{60} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{1}{2})$



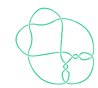
$10_{61} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{1}{3})$



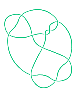
$10_{62} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{2}{3})$



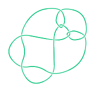
$10_{63} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{2}{3})$



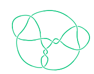
$10_{64} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{1}{3})$



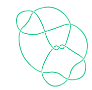
$10_{65} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{2}{3})$



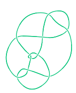
$10_{66} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{2}{3})$



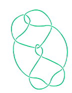
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



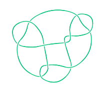
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



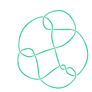
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



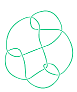
$10_{70} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{3}{2})$



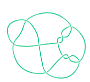
$10_{71} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{3}{2})$



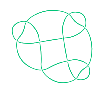
$10_{72} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{3}{2})$



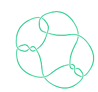
$10_{73} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{3}{2})$



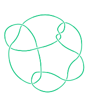
$10_{74} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{3})$



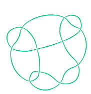
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



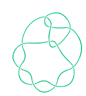
$10_{76} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{2})$



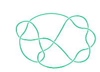
$10_{77} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{5}{2})$



$10_{78} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{2})$

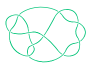


$10_{124} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{-1}{2})$

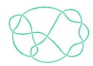


$10_{125} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{-1}{2})$

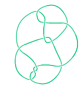
Nudos de Montesinos.



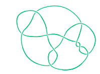
$10_{126} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{-1}{2})$



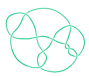
$10_{127} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{-1}{2})$



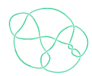
$10_{128} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{-1}{2})$



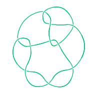
$10_{129} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{-1}{2})$



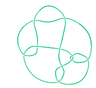
$10_{130} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{-1}{2})$



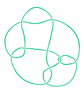
$10_{131} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{-1}{2})$



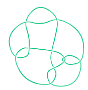
$10_{132} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{-1}{2})$



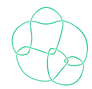
$10_{133} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{-1}{2})$



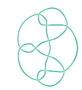
$10_{134} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{-1}{2})$



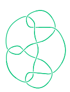
$10_{135} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{-1}{2})$



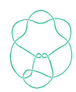
$10_{136} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{-1}{2})$



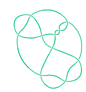
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



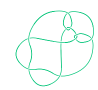
$10_{138} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{-1}{2})$



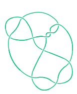
$10_{139} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-2}{3})$



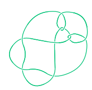
$10_{140} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-1}{3})$



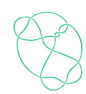
$10_{141} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{-1}{3})$



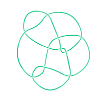
$10_{142} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-2}{3})$



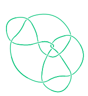
$10_{143} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-1}{3})$



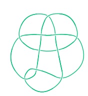
$10_{144} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{-1}{3})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$

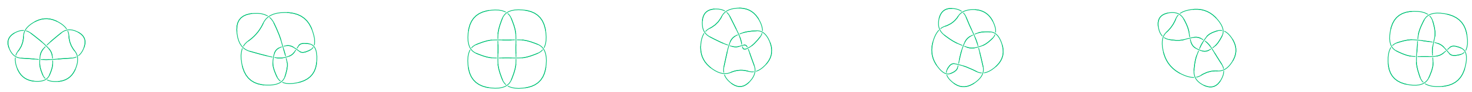


$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$

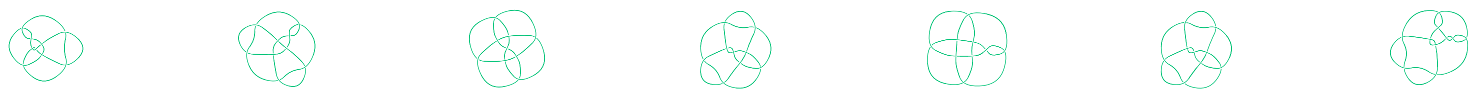


$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

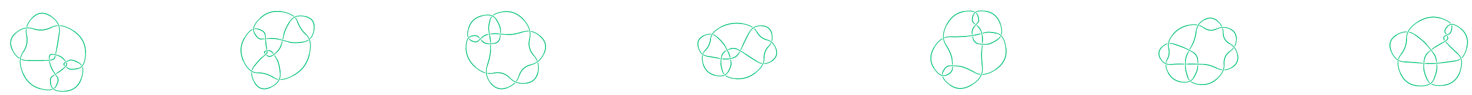
Los otros.



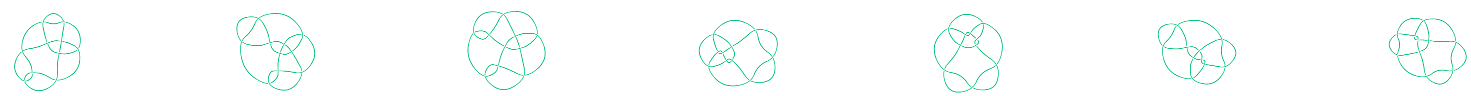
8_{16} 8_{17} 8_{18} 9_{29} 9_{32} 9_{33} 9_{34}



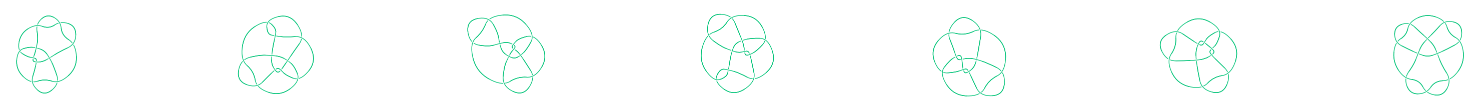
9_{38} 9_{39} 9_{40} 9_{41} 9_{47} 9_{49} 10_{79}



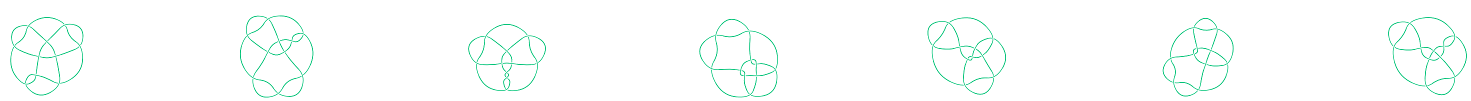
10_{80} 10_{81} 10_{82} 10_{83} 10_{84} 10_{85} 10_{86}



10_{87} 10_{88} 10_{89} 10_{90} 10_{91} 10_{92} 10_{93}

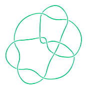


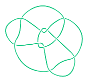
10_{94} 10_{95} 10_{96} 10_{97} 10_{98} 10_{99} 10_{100}

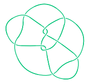


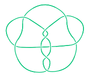
10_{101} 10_{102} 10_{103} 10_{104} 10_{105} 10_{106} 10_{107}

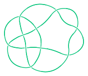
Los otros.

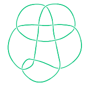

10_{108}

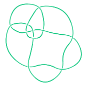

10_{109}

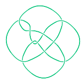

10_{110}

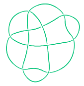

10_{111}

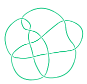

10_{112}

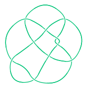

10_{113}

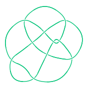

10_{114}

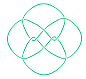

10_{115}

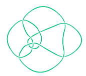

10_{116}

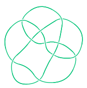

10_{117}

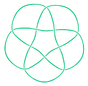

10_{118}

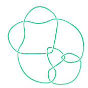

10_{119}

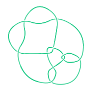

10_{120}


10_{121}


10_{122}


10_{123}


10_{148}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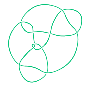
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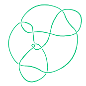
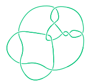
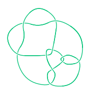
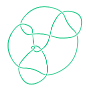
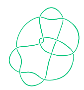
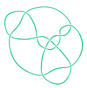

10_{151}

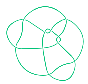

10_{152}

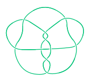

10_{153}

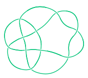

10_{154}

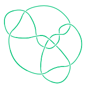

10_{155}


10_{156}


10_{157}


10_{158}


10_{159}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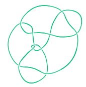
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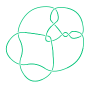
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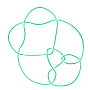
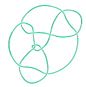
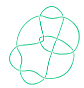
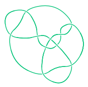

10_{153}


10_{154}


10_{155}


10_{156}

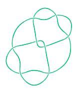
Necesitamos nudos universales específicos.

Cubiertas tipo diédricas

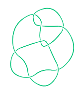
$$\begin{tikzcd}[column sep=small] & \tilde{M} \arrow[dl, "q"'] \arrow[dr, "\psi"] & \\ M \arrow[dr, "\varphi"'] & & B_2(k) \arrow[dl, "p"] \\ & (S^3, k) & \end{tikzcd}$$

ψ es un espacio cubriente arbitrario.

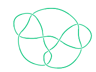
1. $k = m(\beta_1/2, \beta_2/3, \beta_3/3)$ es universal $\Leftrightarrow \Delta(k) \neq \pm 3$.



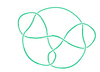
$8_5 = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{2})$



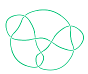
$8_{10} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{1}{2})$



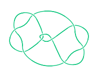
$8_{15} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{1}{2})$



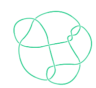
$8_{20} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{-1}{2})$



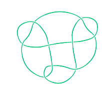
$8_{21} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{2})$



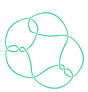
$9_{16} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{3}{2})$



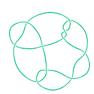
$9_{24} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{3}{2})$



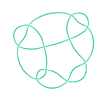
$9_{28} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{3}{2})$



$10_{76} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{2})$

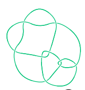


$10_{77} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{5}{2})$

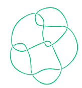


$10_{78} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{2})$

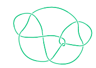
2. $k = m(\beta_1/2, \beta_2/3, \beta_3/5)$ es universal $\Leftrightarrow \Delta(k) \neq \pm 1$.



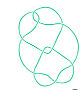
$9_{22} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{2})$



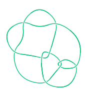
$9_{25} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{1}{2})$



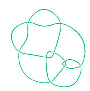
$9_{30} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{1}{2})$



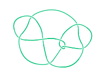
$9_{36} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{1}{2})$



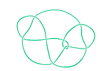
$9_{42} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-1}{2})$



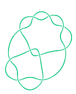
$9_{43} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{2})$



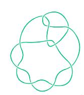
$9_{44} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{2})$



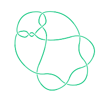
$9_{45} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{-1}{2})$



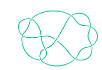
$10_{46} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{1}{2})$



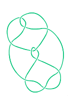
$10_{47} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{1}{2})$



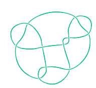
$10_{48} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{1}{2})$



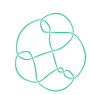
$10_{49} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{1}{2})$



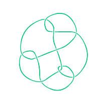
$10_{70} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{3}{2})$



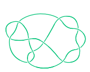
$10_{71} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{3}{2})$



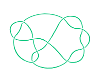
$10_{72} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{3}{2})$



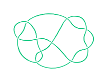
$10_{73} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{3}{2})$



$10_{125} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{-1}{2})$

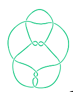


$10_{126} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{-1}{2})$

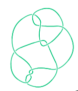


$10_{127} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{-1}{2})$

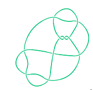
3. $k = m(\beta_1/2, \beta_2/3, \beta_3/7)$ es universal.



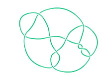
$10_{50} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{1}{2})$



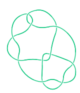
$10_{51} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{1}{2})$



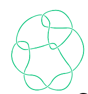
$10_{52} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{1}{2})$



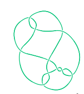
$10_{53} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{1}{2})$



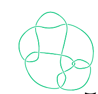
$10_{54} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{1}{2})$



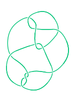
$10_{55} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{1}{2})$



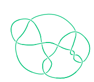
$10_{56} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{1}{2})$



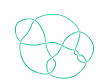
$10_{57} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{1}{2})$



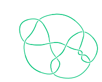
$10_{128} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{-1}{2})$



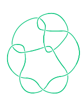
$10_{129} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{-1}{2})$



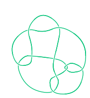
$10_{130} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{-1}{2})$



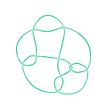
$10_{131} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{-1}{2})$



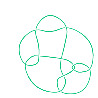
$10_{132} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{-1}{2})$



$10_{133} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{-1}{2})$



$10_{134} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{-1}{2})$

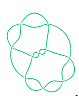


$10_{135} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{-1}{2})$

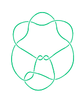
4. (a) $|x| > 1 \Rightarrow k = p(e; 2x, 3y, 3z)$ es universal.

(a.1) $|y| > 1$ o $|z| > 1 \Rightarrow k = p(2, 3y, 3z)$ es universal.

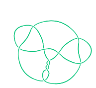
(a.2) $|y| > 1$ o $|z| > 1$ y $\beta_2 \equiv \pm 1 \pmod{y}$ y $\beta_3 \equiv \pm 1 \pmod{z}$
$\Rightarrow k = m(1/2, \beta_2/3y, \beta_3/3z)$ es universal.



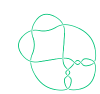
$10_{61} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{1}{3})$



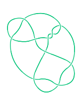
$10_{62} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{2}{3})$



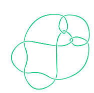
$10_{63} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{2}{3})$



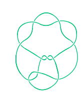
$10_{64} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{1}{3})$



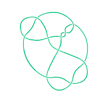
$10_{65} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{2}{3})$



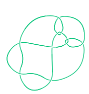
$10_{66} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{2}{3})$



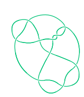
$10_{139} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-2}{3})$



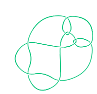
$10_{140} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-1}{3})$



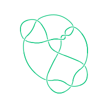
$10_{141} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{-1}{3})$



$10_{142} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-2}{3})$



$10_{143} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-1}{3})$

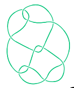


$10_{144} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{-1}{3})$

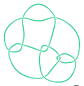
(b) $|x| > 1 \Rightarrow k = p(\pm 2, \pm 3y, \pm 5z)$ es universal.

(c) $z > 0 \Rightarrow k = p(\pm 2, \pm 3, \pm 7)$ es universal.

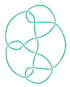
5. $y, z \neq 0 \Rightarrow p(\pm 2, 5y, 5z)$ es universal.



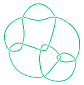
$10_{58} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{1}{2})$



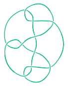
$10_{59} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{1}{2})$



$10_{60} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{1}{2})$



$10_{136} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{-1}{2})$



$10_{138} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{-1}{2})$

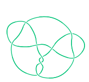
Teorema.

Si $p(b;\alpha_1,\ldots,\alpha_t)$ es un enlace de Uchida universal y $(n,\alpha_i)=1 \quad \forall i$ $\Rightarrow m(nb/1, n/\alpha_1,\ldots,n/\alpha_t)$ es universal.

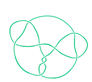
Teorema.

Si $|p|>1$ y $(n,p)=1$ y p impar $\Rightarrow m(n/p,n/p,-n/p)$ es universal.

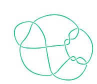
Si $p\neq 2$ y $(n,p)=1$ y p es par $\Rightarrow m(n/3,n/3,n/p)$ es universal.



$9_{37}=\mathrm{m}(\frac{1}{3},\frac{2}{3},\frac{2}{3})$



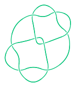
$9_{46}=\mathrm{m}(\frac{1}{3},\frac{1}{3},\frac{-1}{3})$



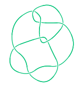
$10_{74}=\mathrm{m}(\frac{1}{3},\frac{1}{3},\frac{5}{3})$

¡Sesenta y seis nudos de Montesinos universales!

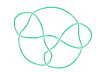
Nudos de Montesinos universales.



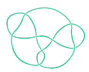
$8_5 = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{2})$



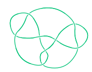
$8_{10} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{1}{2})$



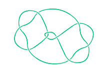
$8_{15} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{1}{2})$



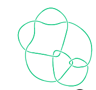
$8_{20} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{-1}{2})$



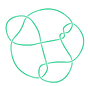
$8_{21} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{2})$



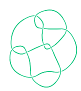
$9_{16} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{3}{2})$



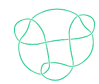
$9_{22} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{2})$



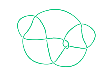
$9_{24} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{3}{2})$



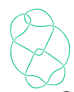
$9_{25} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{1}{2})$



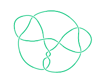
$9_{28} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{3}{2})$



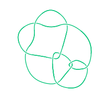
$9_{30} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{1}{2})$



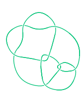
$9_{36} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{1}{2})$



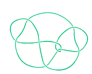
$9_{37} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$



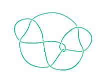
$9_{42} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-1}{2})$



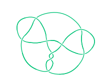
$9_{43} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{2})$



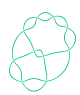
$9_{44} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{2})$



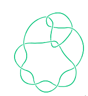
$9_{45} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{-1}{2})$



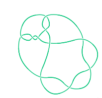
$9_{46} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{3})$



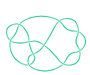
$10_{46} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{1}{2})$



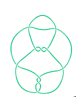
$10_{47} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{1}{2})$



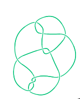
$10_{48} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{1}{2})$



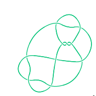
$10_{49} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{1}{2})$



$10_{50} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{1}{2})$

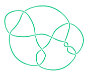


$10_{51} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{1}{2})$

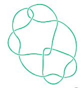


$10_{52} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{1}{2})$

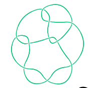
Nudos de Montesinos universales.



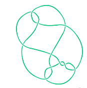
$10_{53} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{1}{2})$



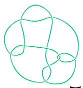
$10_{54} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{1}{2})$



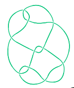
$10_{55} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{1}{2})$



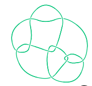
$10_{56} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{1}{2})$



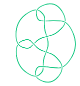
$10_{57} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{1}{2})$



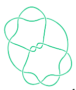
$10_{58} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{1}{2})$



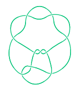
$10_{59} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{1}{2})$



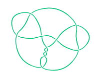
$10_{60} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{1}{2})$



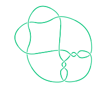
$10_{61} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{1}{3})$



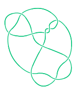
$10_{62} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{2}{3})$



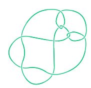
$10_{63} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{2}{3})$



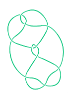
$10_{64} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{1}{3})$



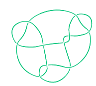
$10_{65} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{2}{3})$



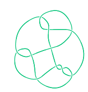
$10_{66} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{2}{3})$



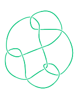
$10_{70} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{3}{2})$



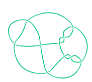
$10_{71} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{3}{2})$



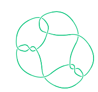
$10_{72} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{3}{2})$



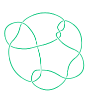
$10_{73} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{3}{2})$



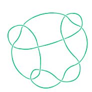
$10_{74} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{3})$



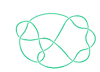
$10_{76} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{5}{2})$



$10_{77} = \mathrm{m}(\frac{1}{3}, \frac{2}{3}, \frac{5}{2})$

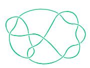


$10_{78} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{2})$

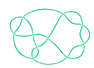


$10_{125} = \mathrm{m}(\frac{1}{5}, \frac{2}{3}, \frac{-1}{2})$

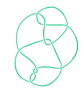
Nudos de Montesinos universales.



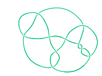
$10_{126} = \mathrm{m}(\frac{4}{5}, \frac{1}{3}, \frac{-1}{2})$



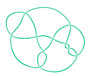
$10_{127} = \mathrm{m}(\frac{4}{5}, \frac{2}{3}, \frac{-1}{2})$



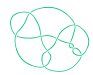
$10_{128} = \mathrm{m}(\frac{3}{7}, \frac{1}{3}, \frac{-1}{2})$



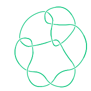
$10_{129} = \mathrm{m}(\frac{3}{7}, \frac{2}{3}, \frac{-1}{2})$



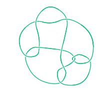
$10_{130} = \mathrm{m}(\frac{4}{7}, \frac{1}{3}, \frac{-1}{2})$



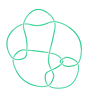
$10_{131} = \mathrm{m}(\frac{4}{7}, \frac{2}{3}, \frac{-1}{2})$



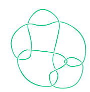
$10_{132} = \mathrm{m}(\frac{2}{7}, \frac{1}{3}, \frac{-1}{2})$



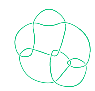
$10_{133} = \mathrm{m}(\frac{2}{7}, \frac{2}{3}, \frac{-1}{2})$



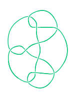
$10_{134} = \mathrm{m}(\frac{5}{7}, \frac{1}{3}, \frac{-1}{2})$



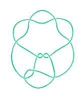
$10_{135} = \mathrm{m}(\frac{5}{7}, \frac{2}{3}, \frac{-1}{2})$



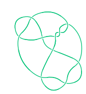
$10_{136} = \mathrm{m}(\frac{2}{5}, \frac{2}{5}, \frac{-1}{2})$



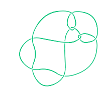
$10_{138} = \mathrm{m}(\frac{3}{5}, \frac{3}{5}, \frac{-1}{2})$



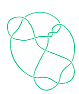
$10_{139} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-2}{3})$



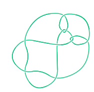
$10_{140} = \mathrm{m}(\frac{1}{4}, \frac{1}{3}, \frac{-1}{3})$



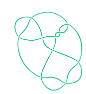
$10_{141} = \mathrm{m}(\frac{1}{4}, \frac{2}{3}, \frac{-1}{3})$



$10_{142} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-2}{3})$

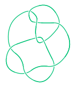


$10_{143} = \mathrm{m}(\frac{3}{4}, \frac{1}{3}, \frac{-1}{3})$

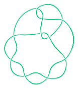


$10_{144} = \mathrm{m}(\frac{3}{4}, \frac{2}{3}, \frac{-1}{3})$

Nudos toroidales.

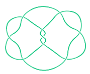


$8_{19} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{-1}{2})$

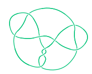


$10_{124} = \mathrm{m}(\frac{1}{5}, \frac{1}{3}, \frac{-1}{2})$

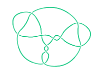
Indecisos.



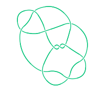
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



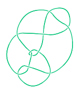
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



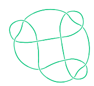
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



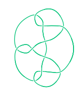
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



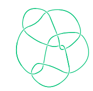
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



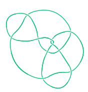
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



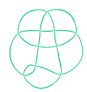
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$



$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$



$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

Unas aplicaciones.

Teorema. Sea q impar, $q \notin \{-1, -3, -7, -11\}$. Entonces $p(2, q, q)$ es universal.

Nótese que $p(2, -1, -1) =$ nudo trébol y $p(2, -3, -3) = \tau_{3,4}$ no son universales.

Pregunta: ¿Los nudos $p(2, -7, -7)$ y $p(2, -11, -11)$ son universales?

Conjetura. Sea q impar. El nudo $p(2, q, q)$ es impar si y sólo si $q \neq -1, -3$.

Ejemplo.

$k = m(1/3, 3/5, -3/4, -2/7, 3/11, -5/13)$, $\Delta(k) = 12869$.

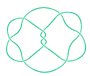
En la cubierta diédrica de 12869 hojas la "componente"

$$k_{2758} = m(1/1, -1/2, 1/1, 1/1) = m(7/2)$$

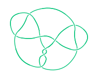
Luego k es universal.

Cubiertas de almohadas de nuevo.

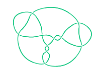
Indecisos.



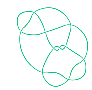
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



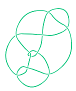
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



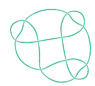
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



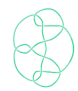
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



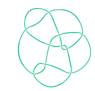
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



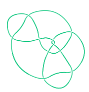
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



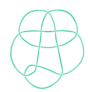
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$

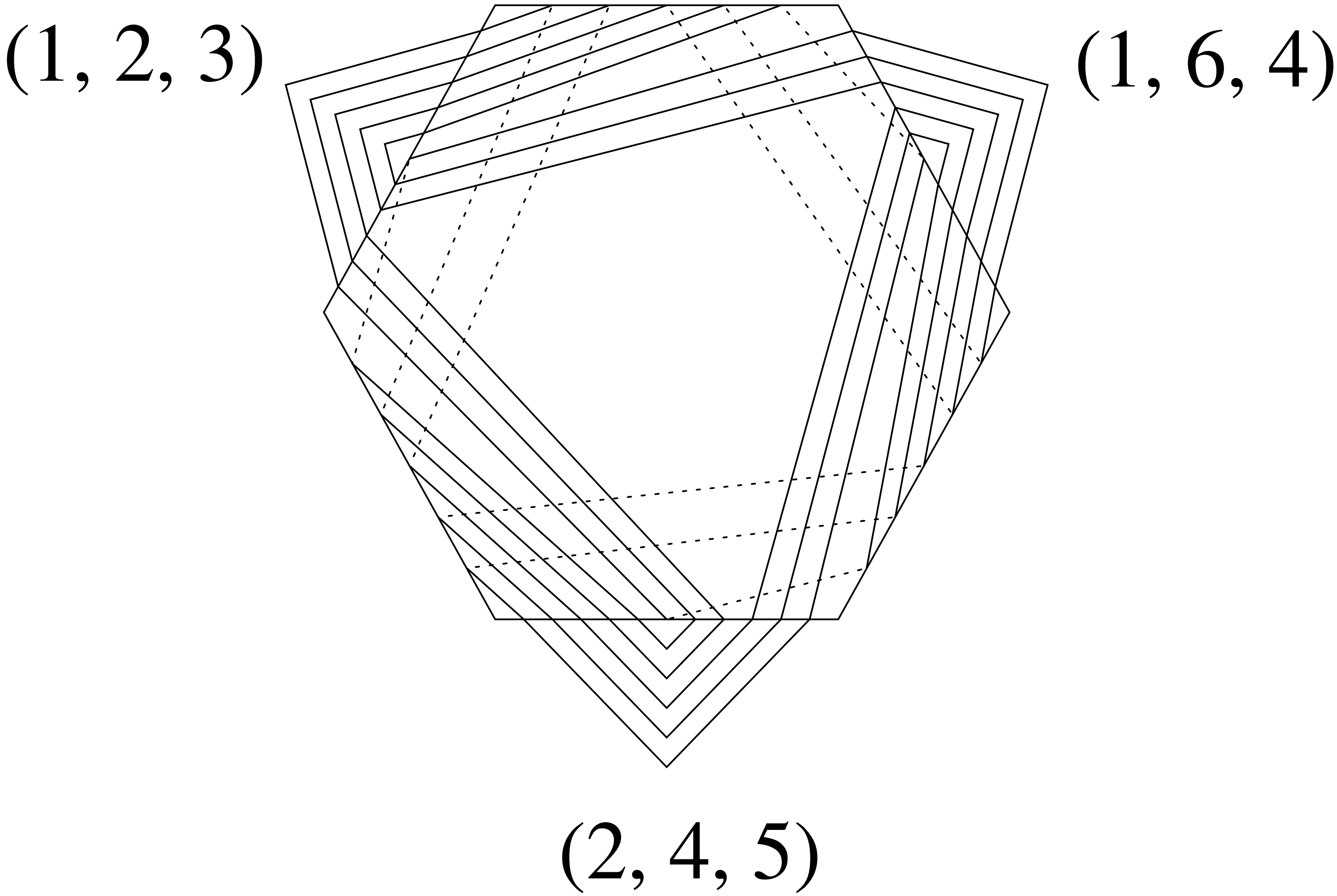


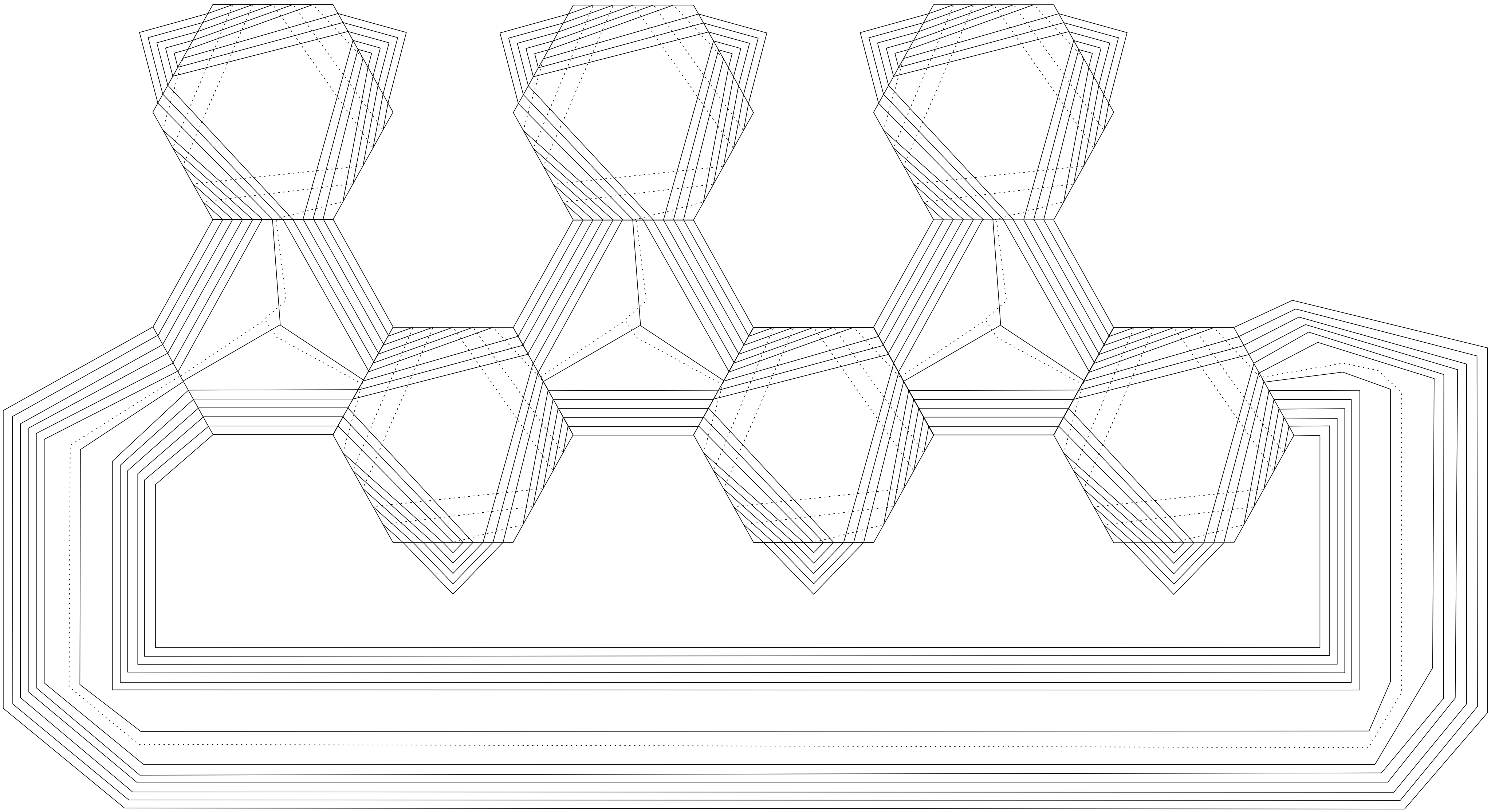
$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$

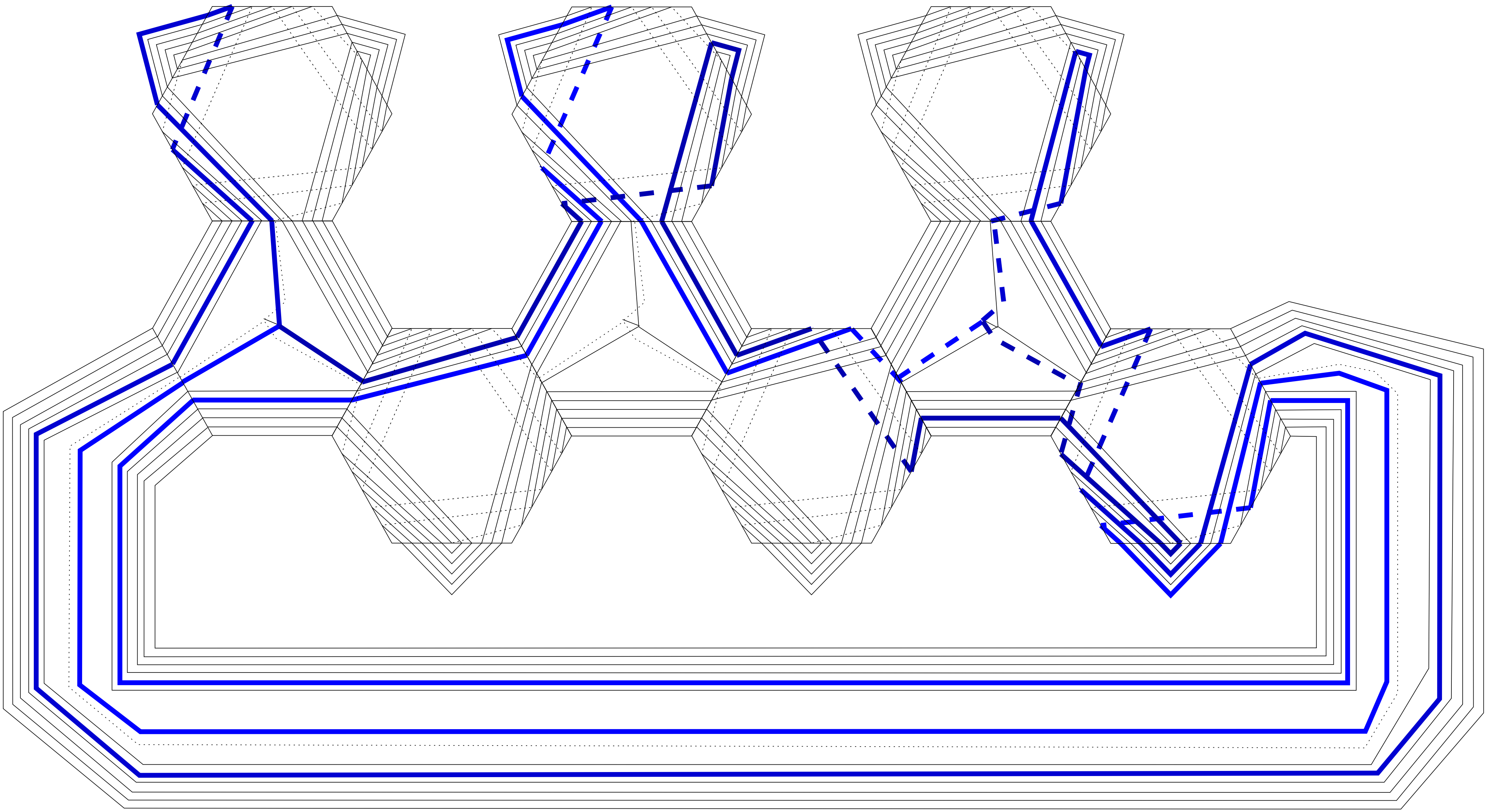


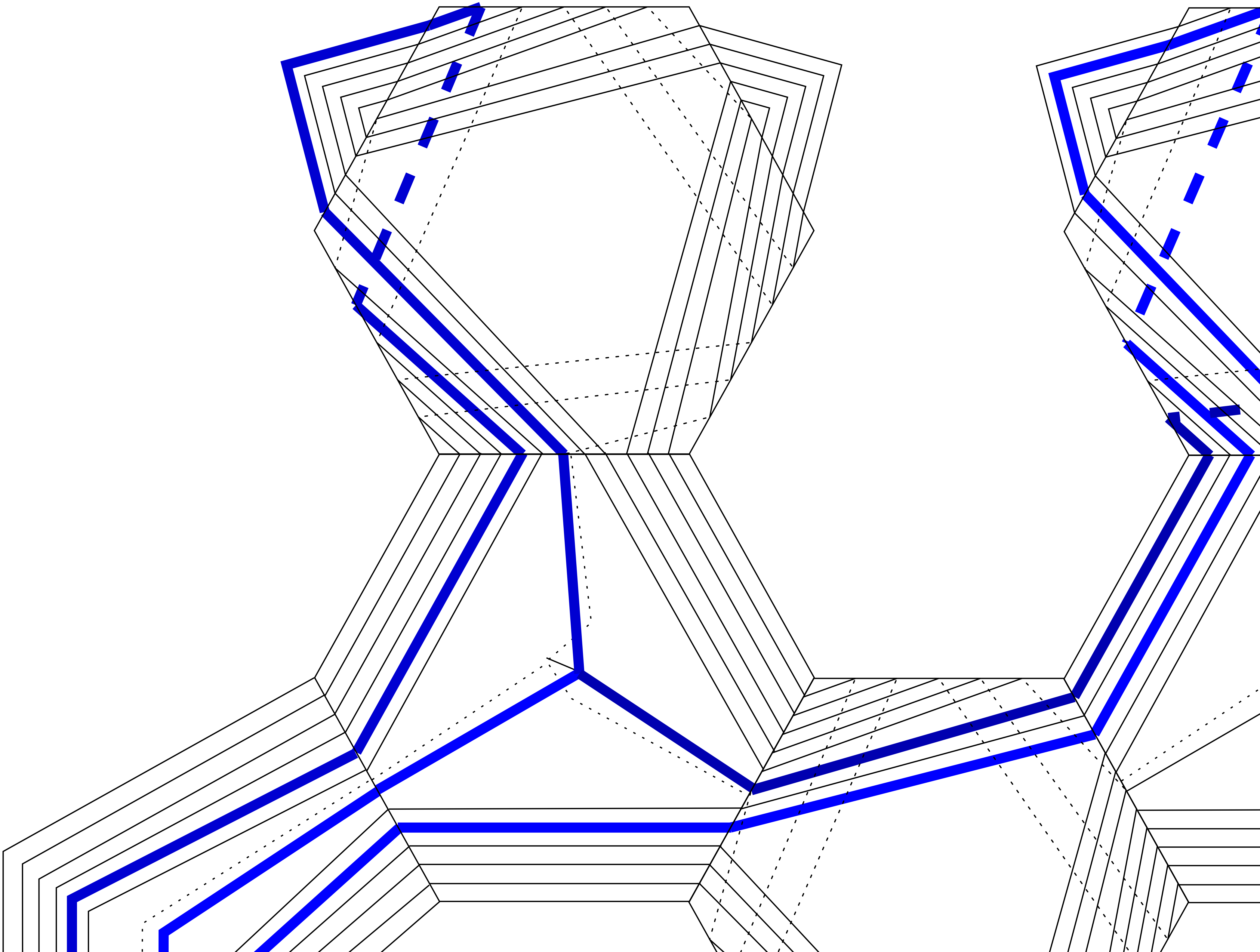
$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

Especificación del problema.

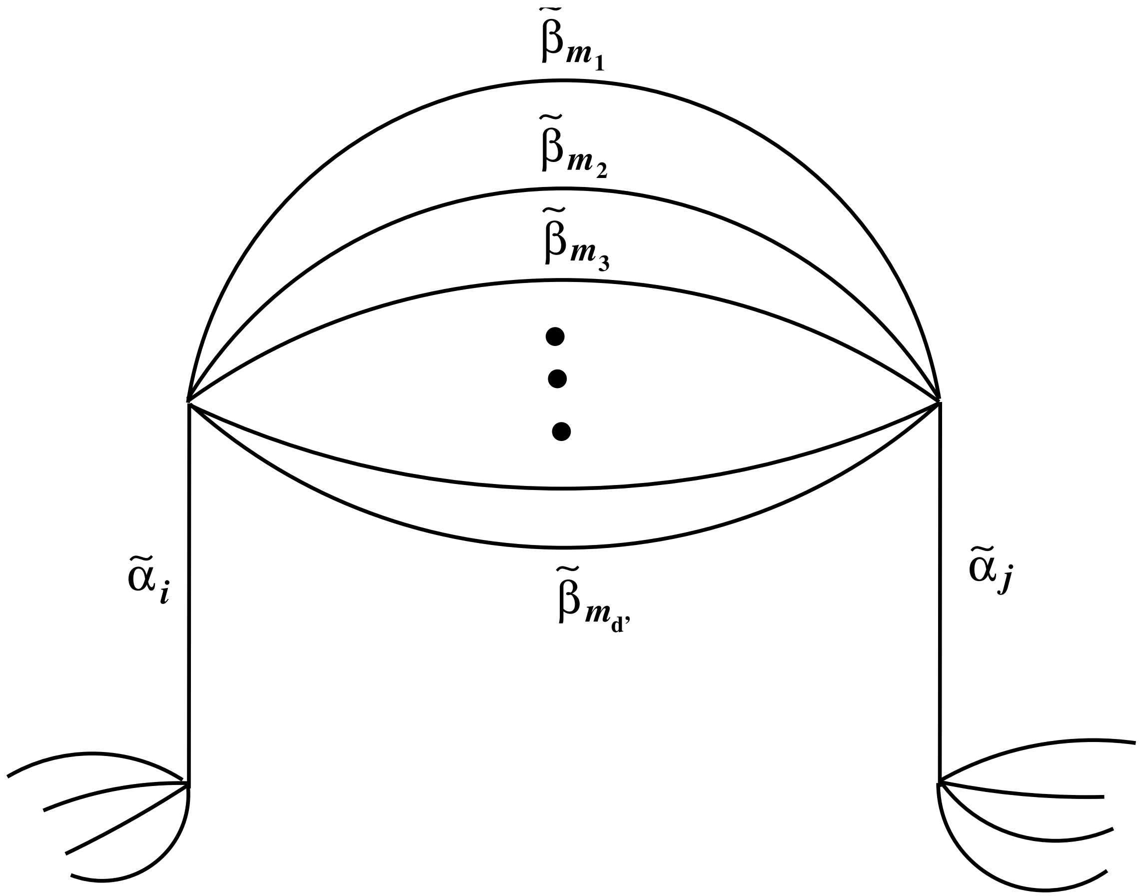

(1, 2, 3)
(1, 6, 4)
(2, 4, 5)





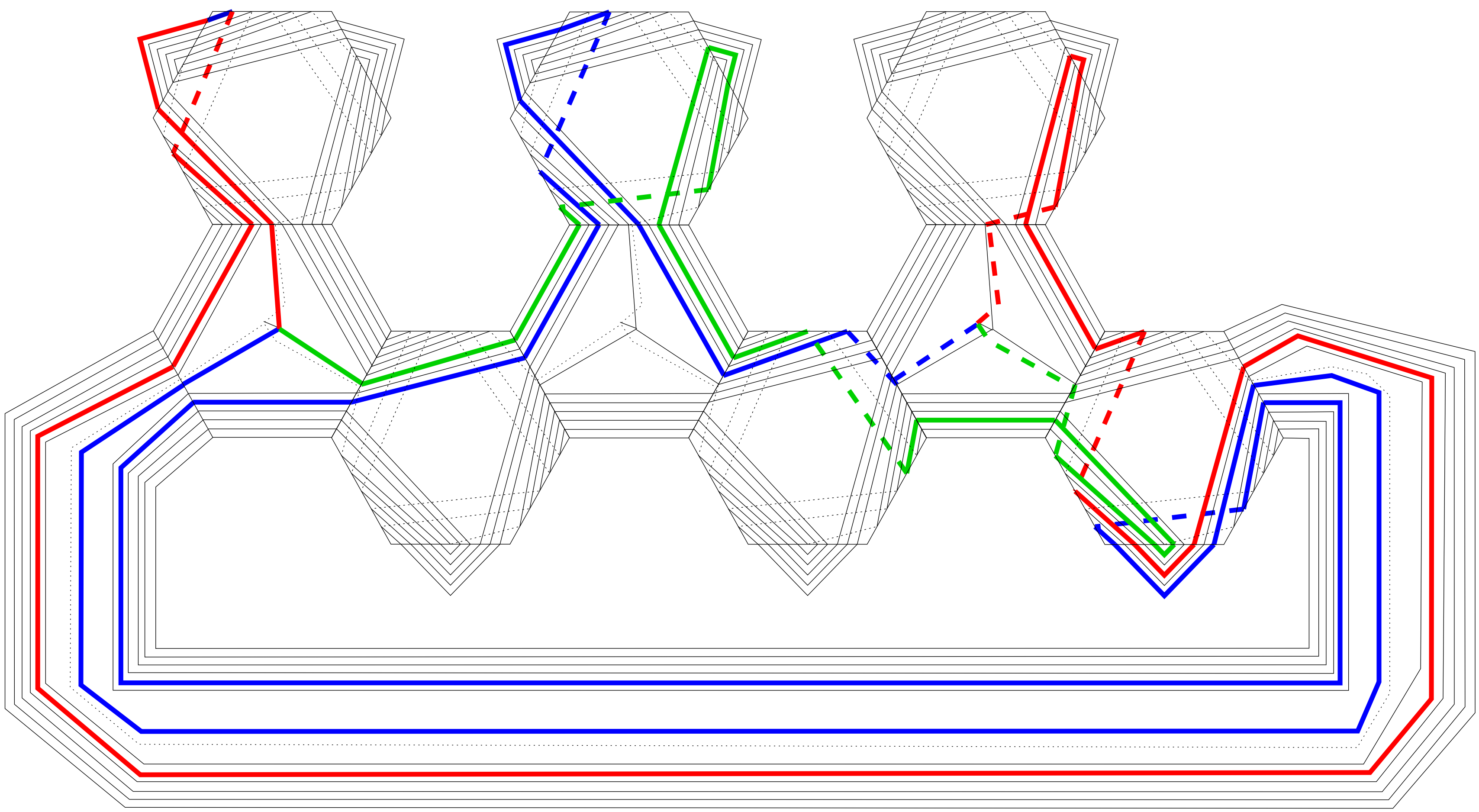


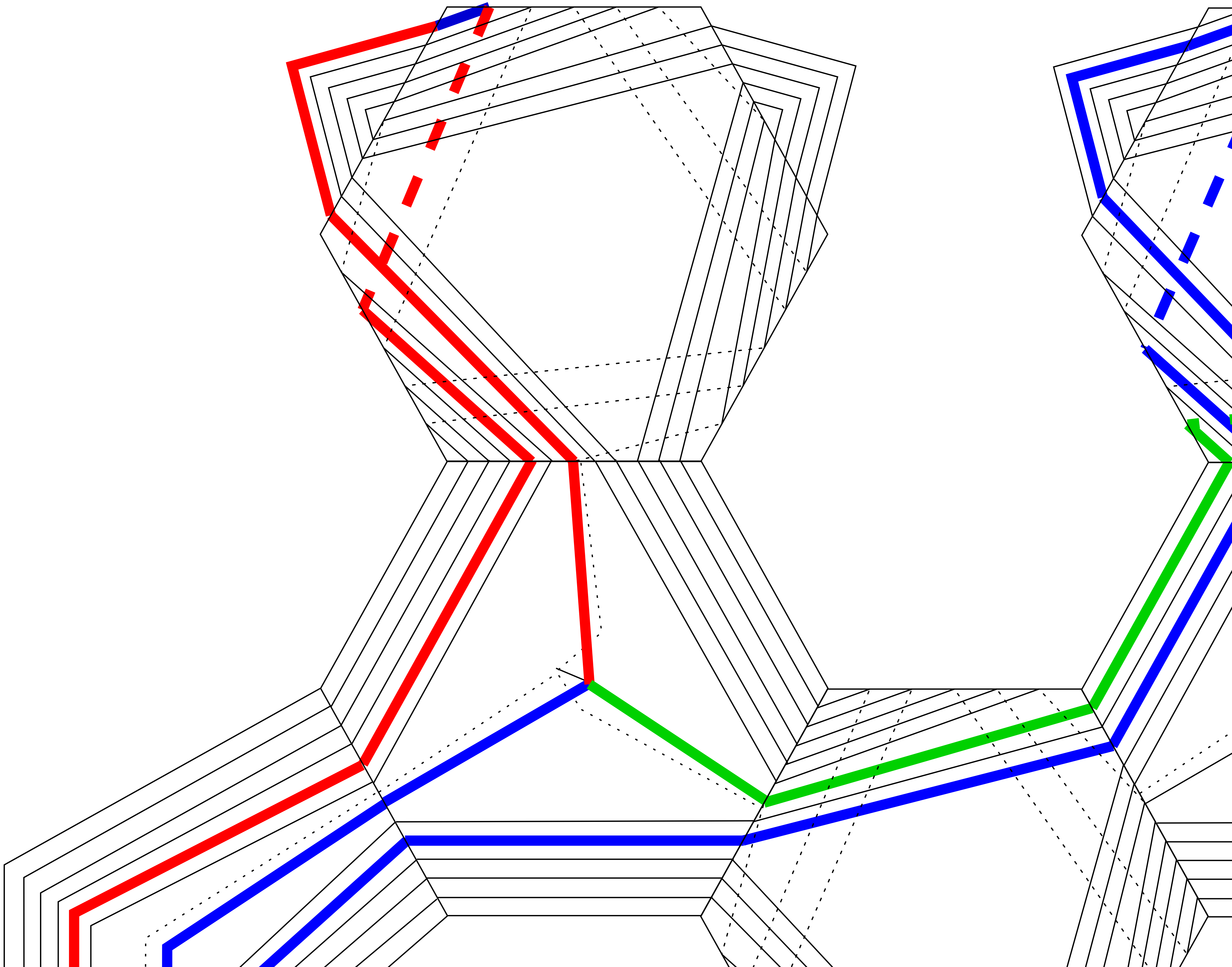
En general



$\varphi^{-1}(\beta_m)$ es una unión de gráficas θ

Dado un arco $\beta_m \subset \partial B$, una pareja de arcos consecutivos en $\varphi^{-1}(\beta_m)$ se llama un ciclo de ramificación





Borremos todos los arcos, excepto uno, en cada uno de los ciclos de ramificacón de $\varphi^{-1}(k)$.

Lo que resulta se llama

una <u>poda</u> de $\varphi^{-1}(k)$ sobre $\varphi^{-1}(B) \cong B_\omega$

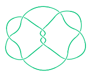
Teorema. (M. Jordán y V.)
Sea $k \subset S^3$ un enlace en posición de n puentes y sea (B, ℓ) una almohada $2n$-gonal para k. Sea $\omega : \pi_1(S^3 - k) \to S_d$ una representación transitiva y sean $\varphi : M \to (S^3, k)$ y $\psi : B_\omega \to (B, B \cap k)$ las cubiertas ramificadas de d hojas inducidas por ω.

Si existe un encaje $\varepsilon : B_\omega \hookrightarrow M$ tal que los ciclos de ramificación sobre $\varepsilon(\partial B_\omega)$ son frontera de 2-células ajenas en $\overline{M - \varepsilon(B_\omega)}$, entonces cualquier homeomorfismo $\varepsilon(B_\omega) \cong \varphi^{-1}(B)$ se puede extender a un homeomorfismo de parejas $(M, \tilde{\ell}) \cong (M, \varphi^{-1}(k))$ donde $\tilde{\ell}$ es alguna poda de $\varepsilon(\psi^{-1}(\ell))$.

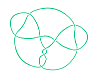
Nótese que la pareja $(\partial B_\omega, \text{ciclos de ramificación})$ induce un diagrama de Heegaard para M. [1]

[1] Esto ayuda a identificar qué variedad es M

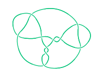
Indecisos.



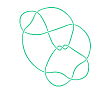
$9_{35} = \mathrm{m}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$



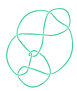
$9_{48} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3})$



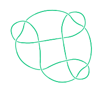
$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



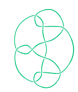
$10_{68} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{1}{3})$



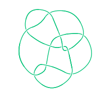
$10_{69} = \mathrm{m}(\frac{3}{5}, \frac{2}{3}, \frac{2}{3})$



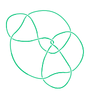
$10_{75} = \mathrm{m}(\frac{2}{3}, \frac{2}{3}, \frac{5}{3})$



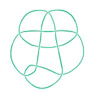
$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$



$10_{145} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{-2}{3})$

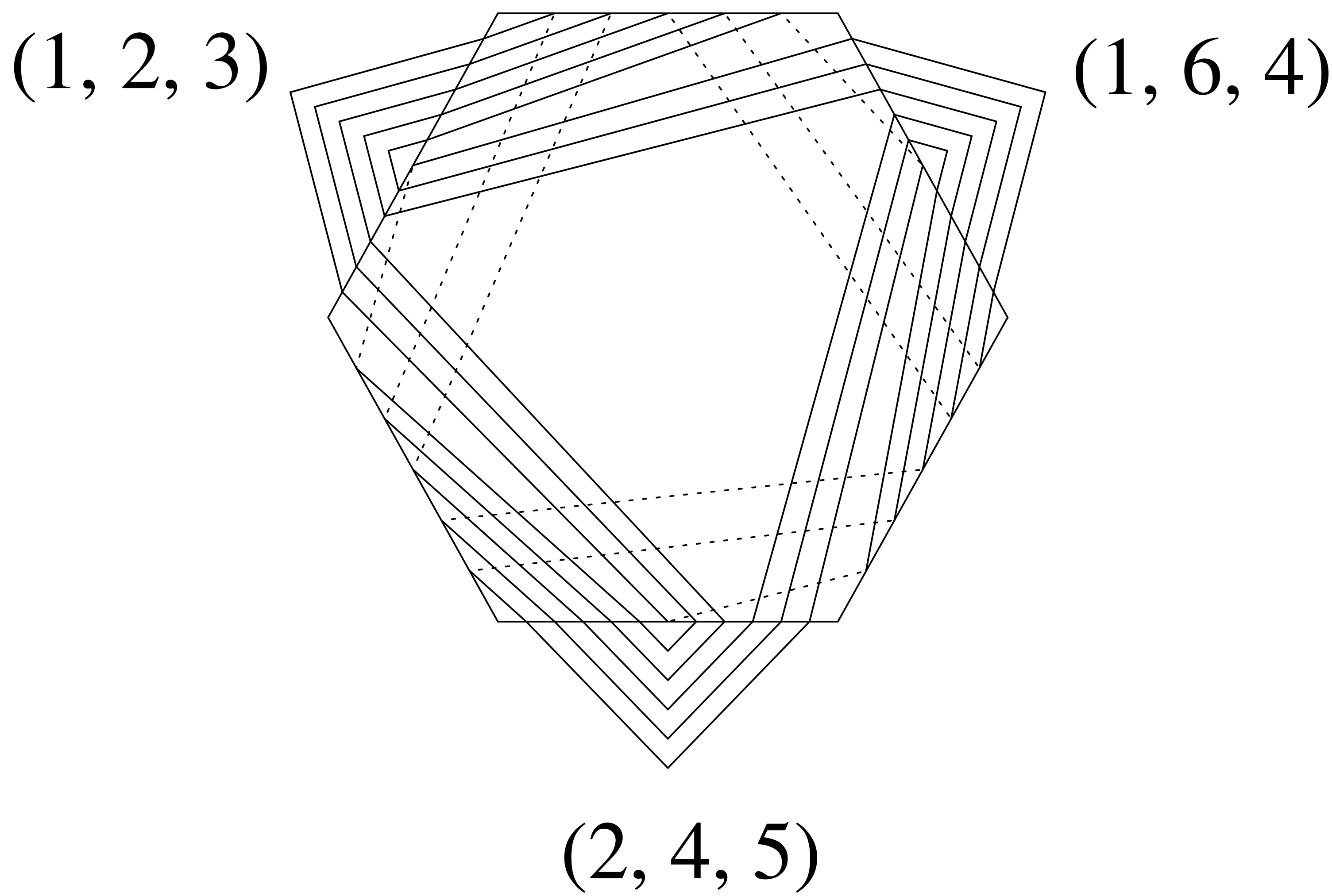


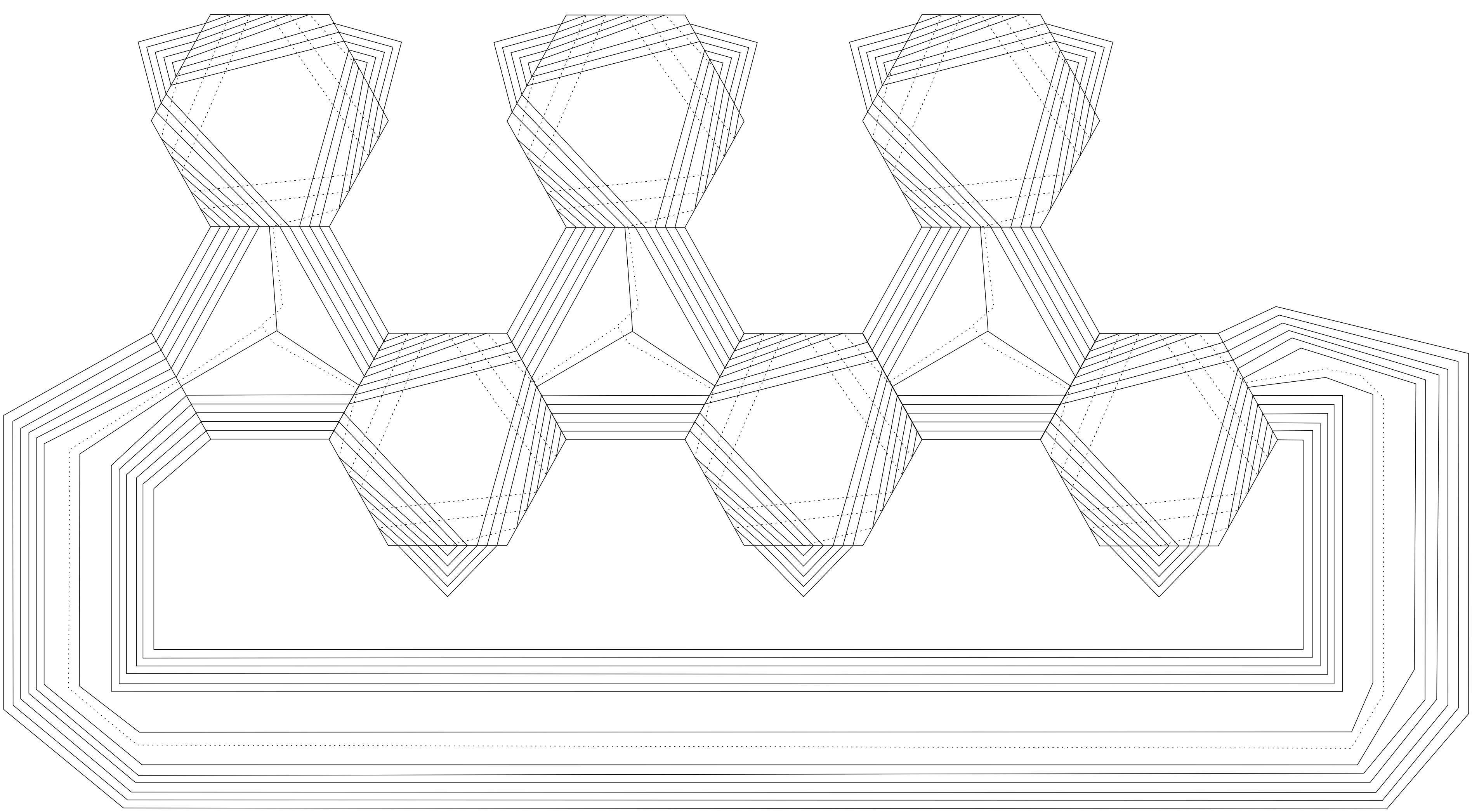
$10_{146} = \mathrm{m}(\frac{2}{5}, \frac{2}{3}, \frac{-1}{3})$

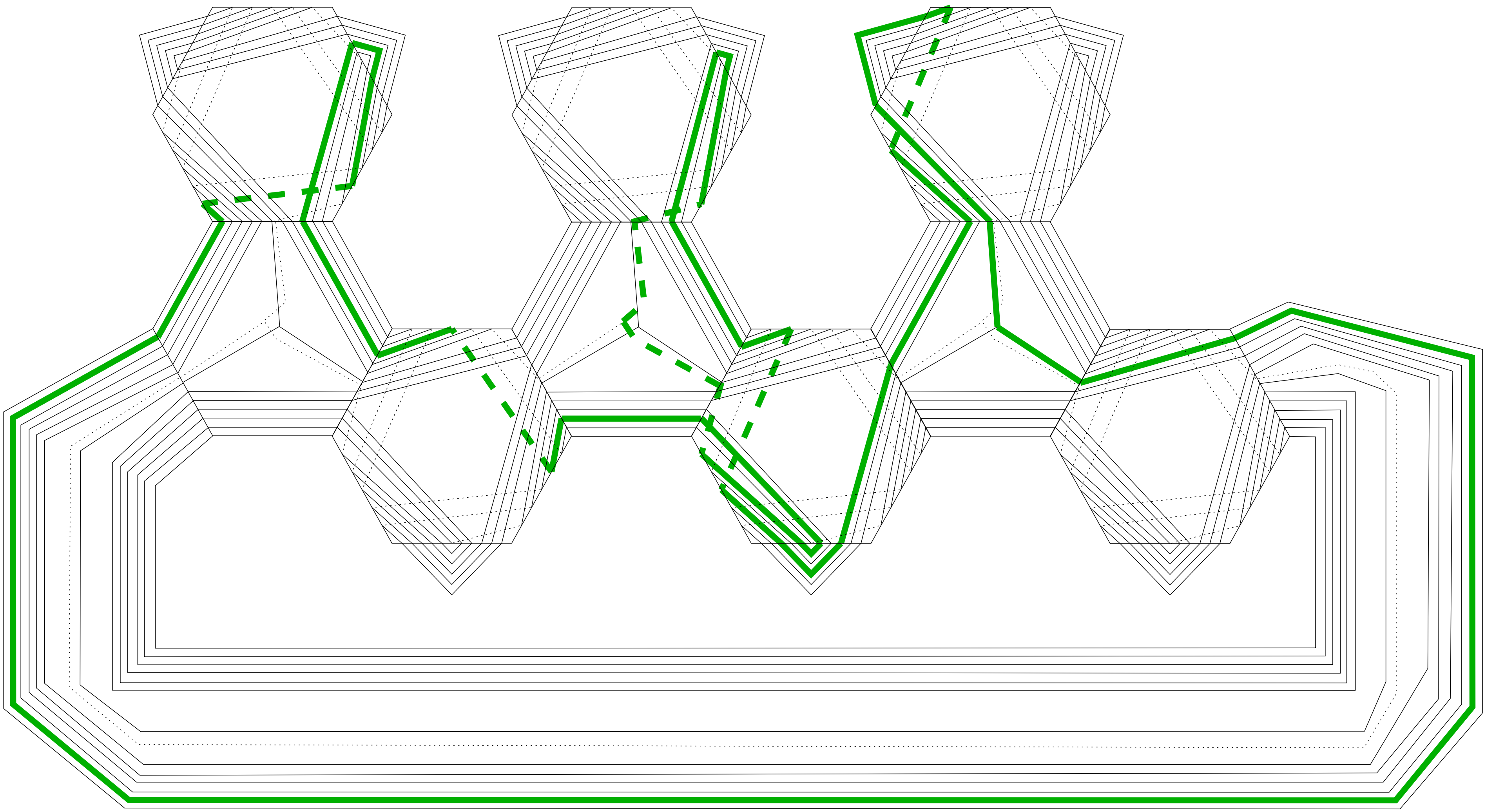


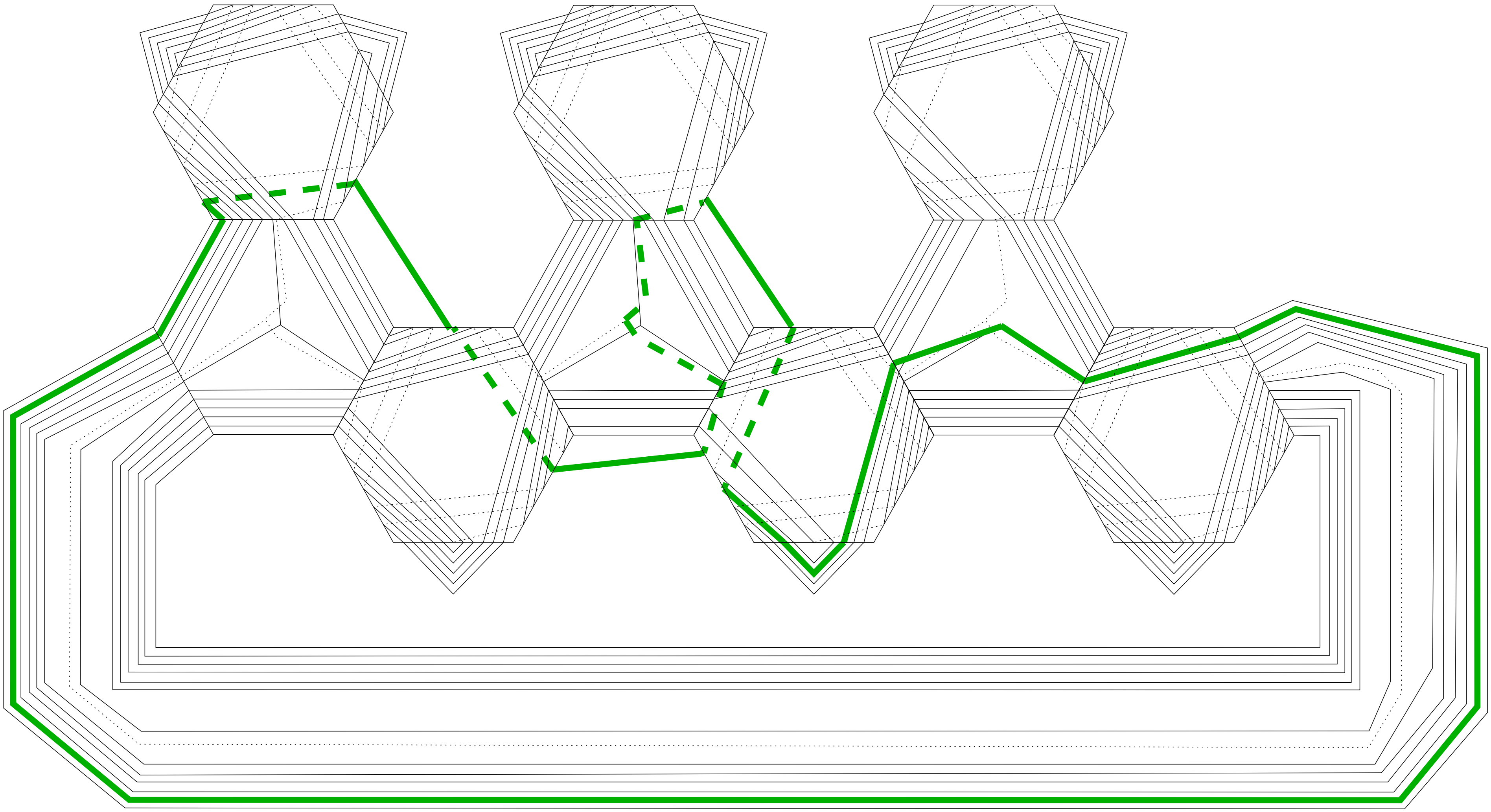
$10_{147} = \mathrm{m}(\frac{3}{5}, \frac{1}{3}, \frac{-1}{3})$

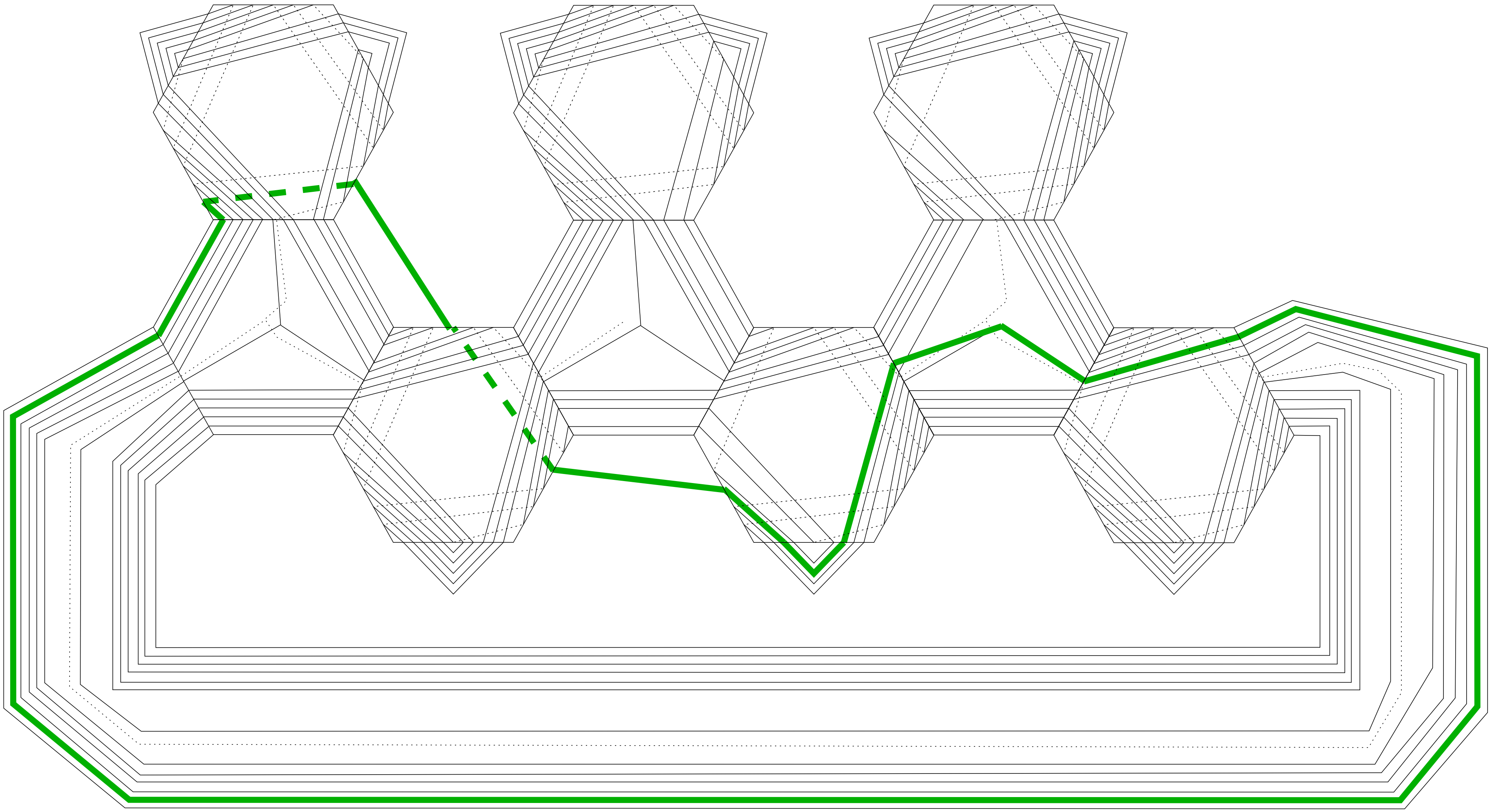
$$9_{35} = m(1/3, 1/3, 1/3)$$

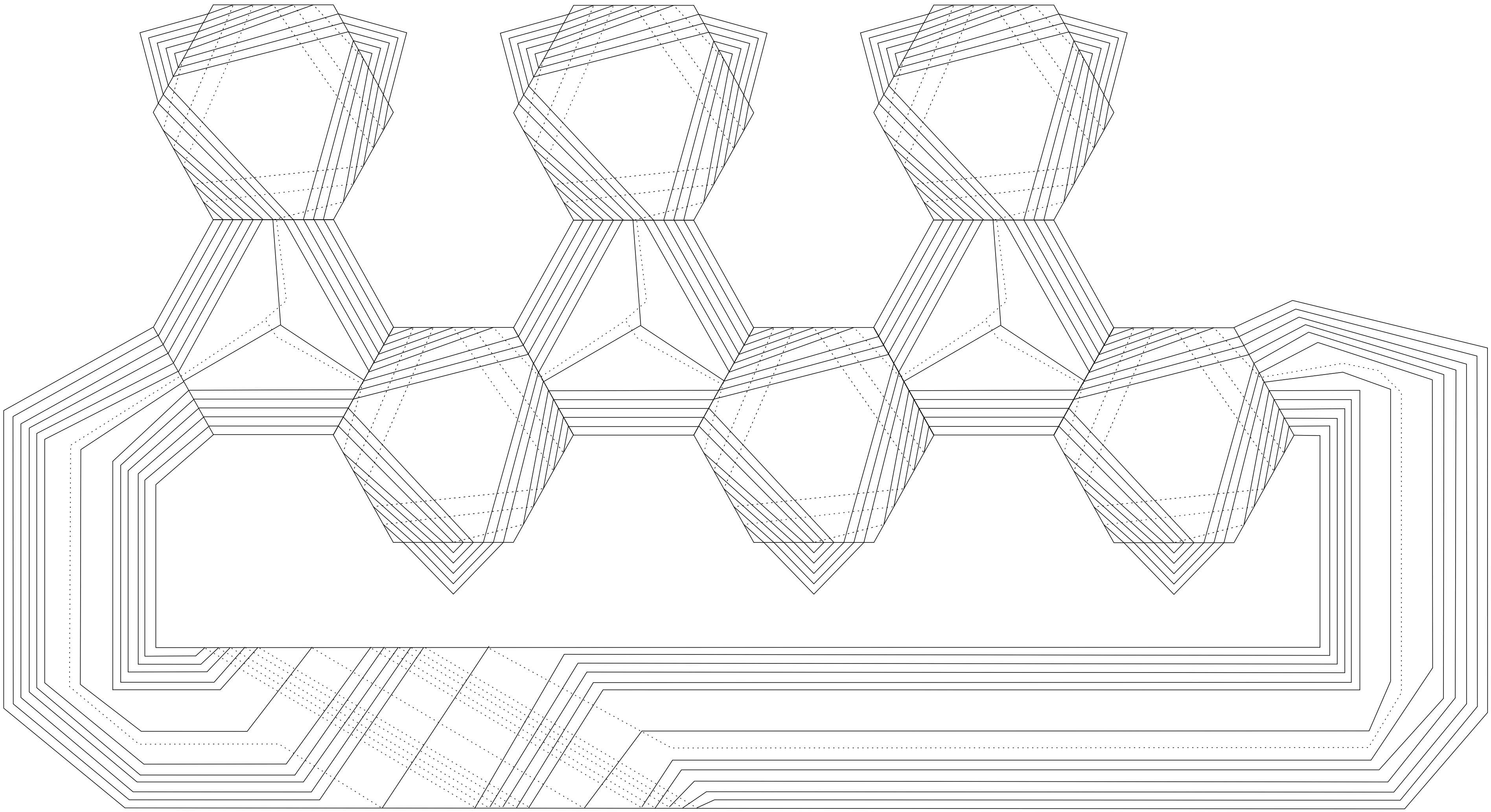


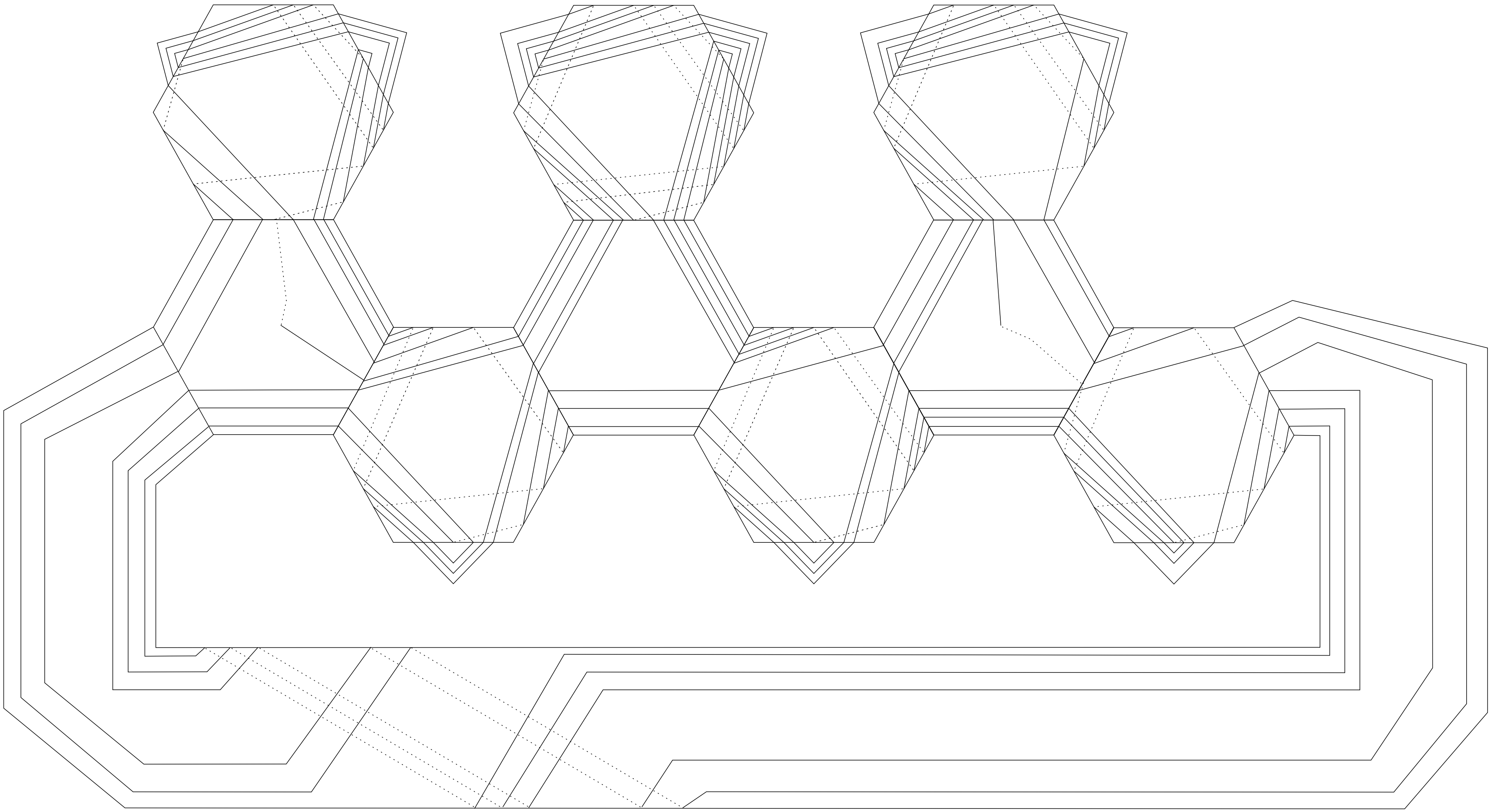


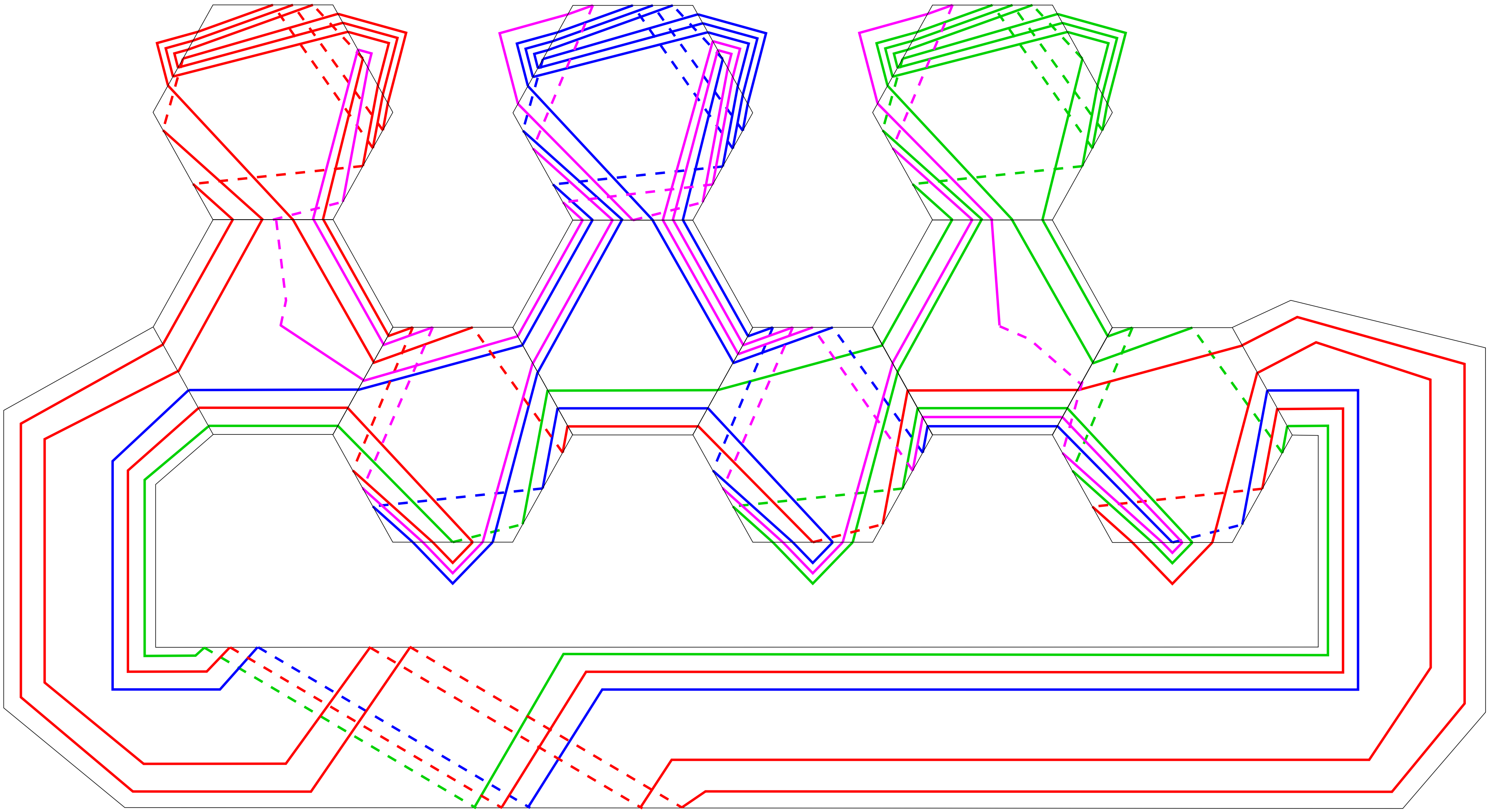


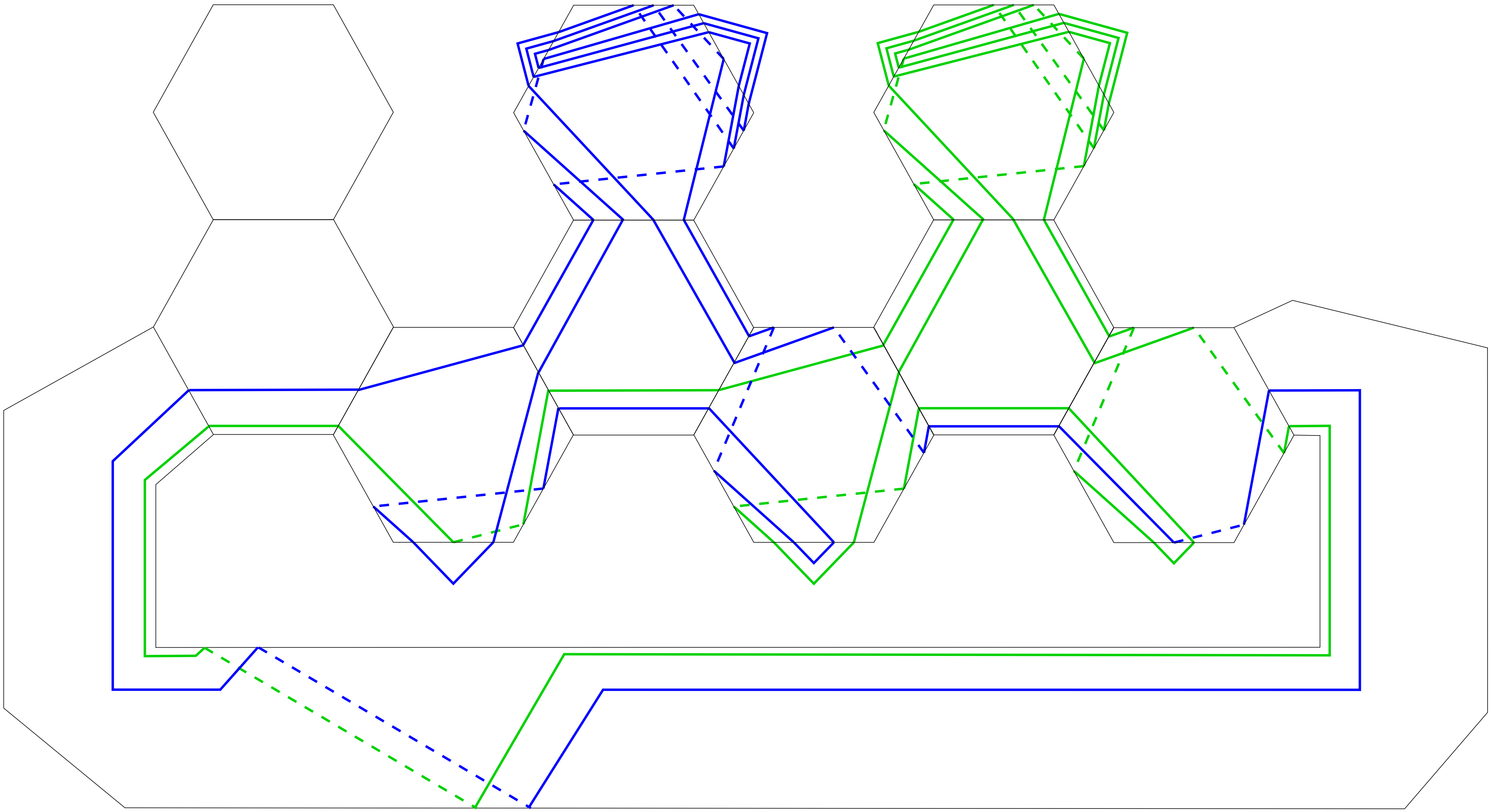


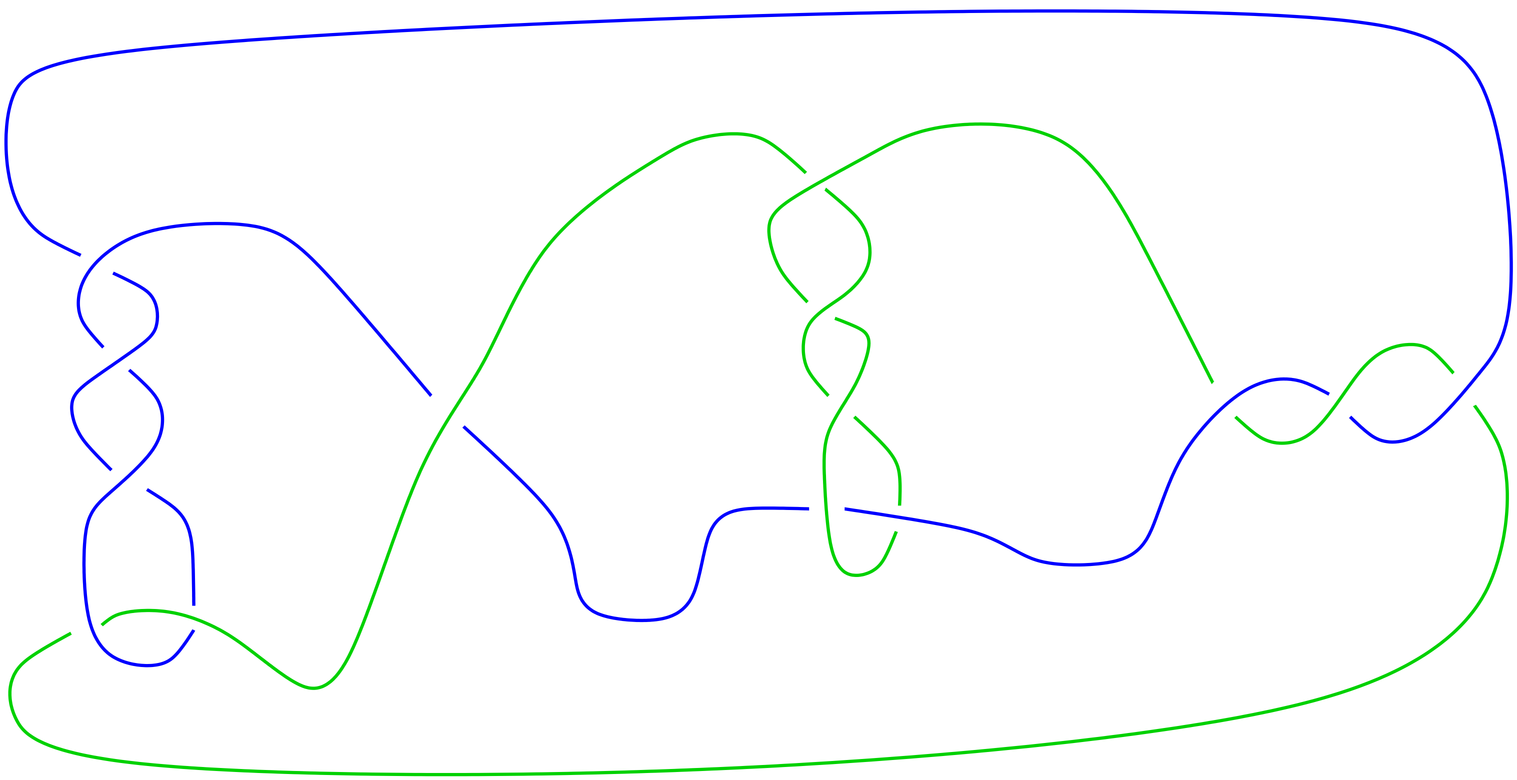








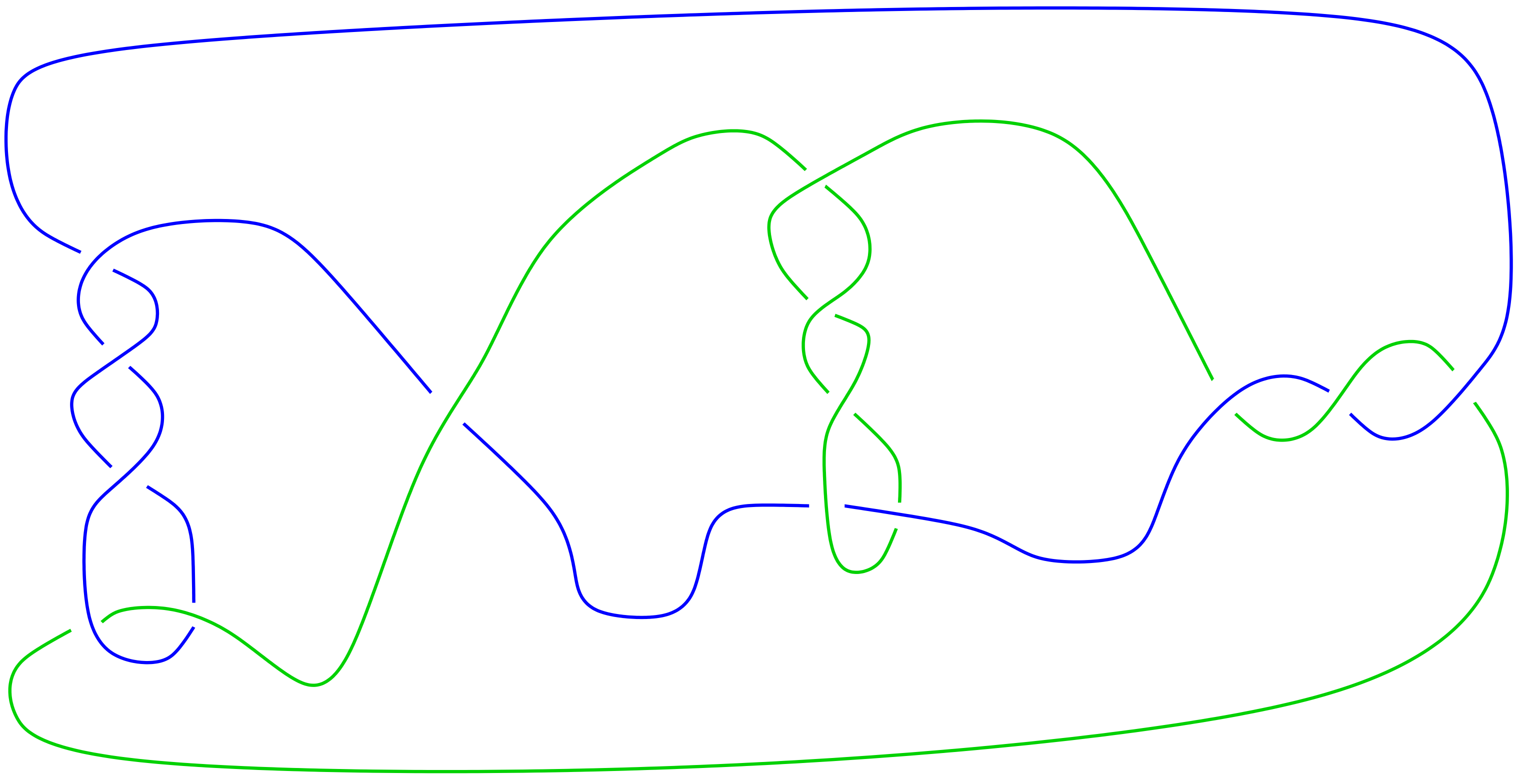




$$m(2/7, 1, 2/7, 3) \sim m(9/7, 23/7)$$

$$m(\frac{\beta_1}{\alpha_1}, \frac{\beta_2}{\alpha_2}) \sim m(-\frac{\alpha_1\beta_2 + \alpha_2\beta_1}{\alpha_2 r_1 + \beta_2 s_1})$$

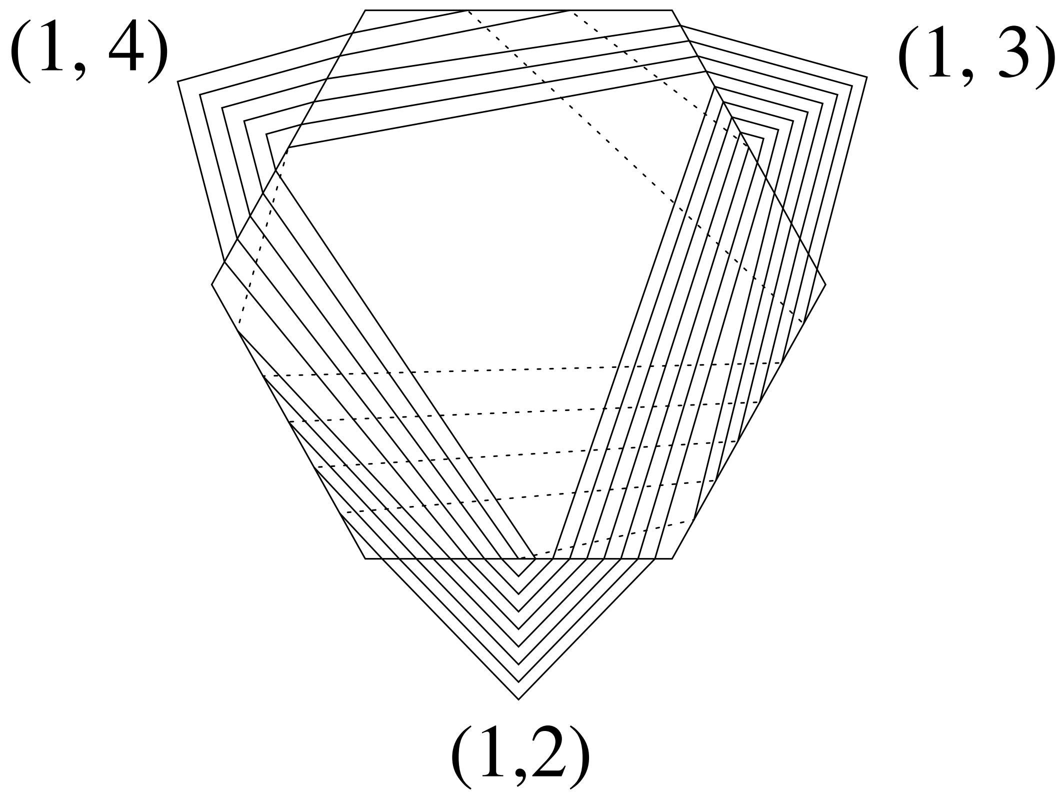
donde $\alpha_1 r_1 - \beta_1 s_1 = 1.$

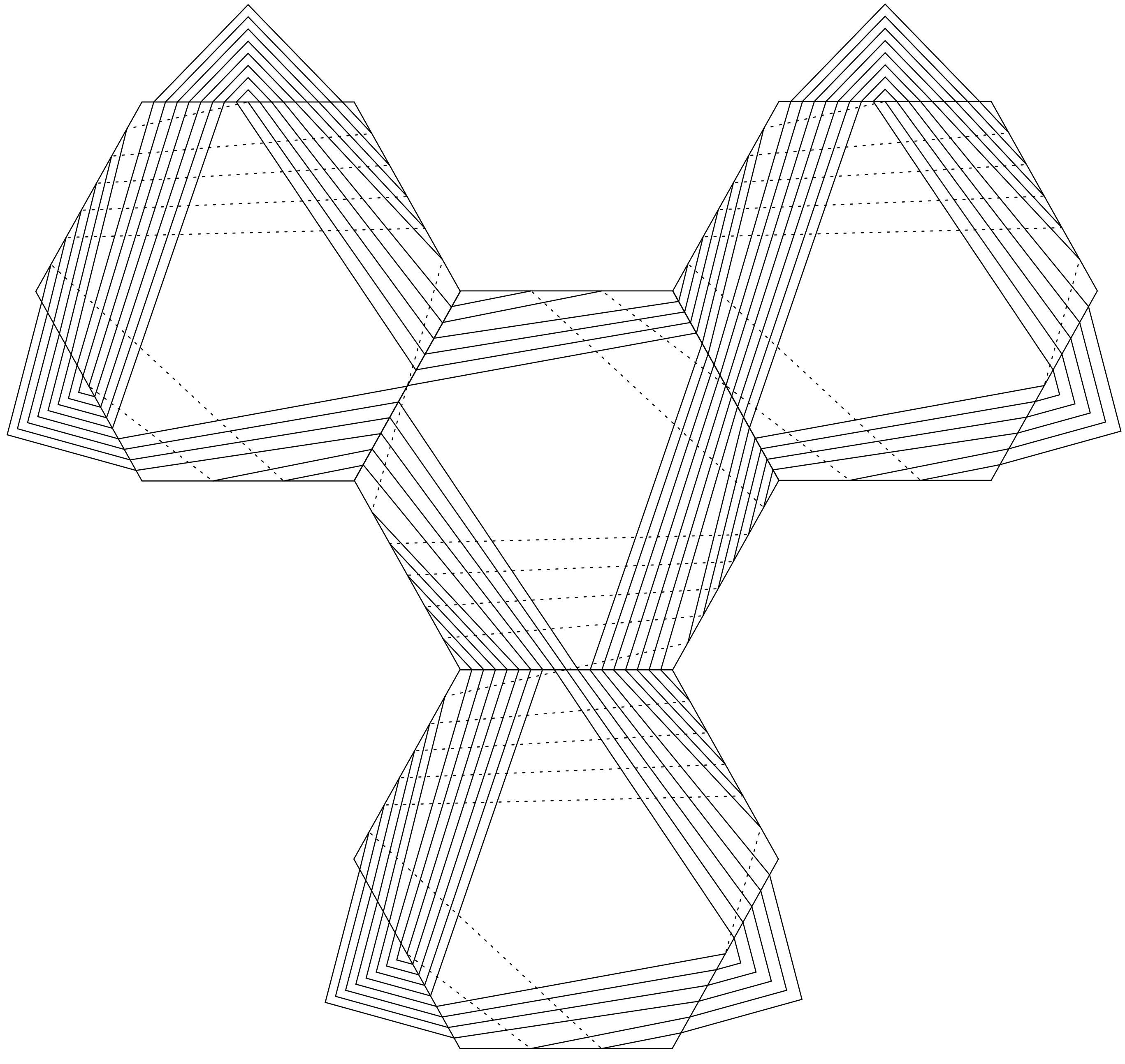


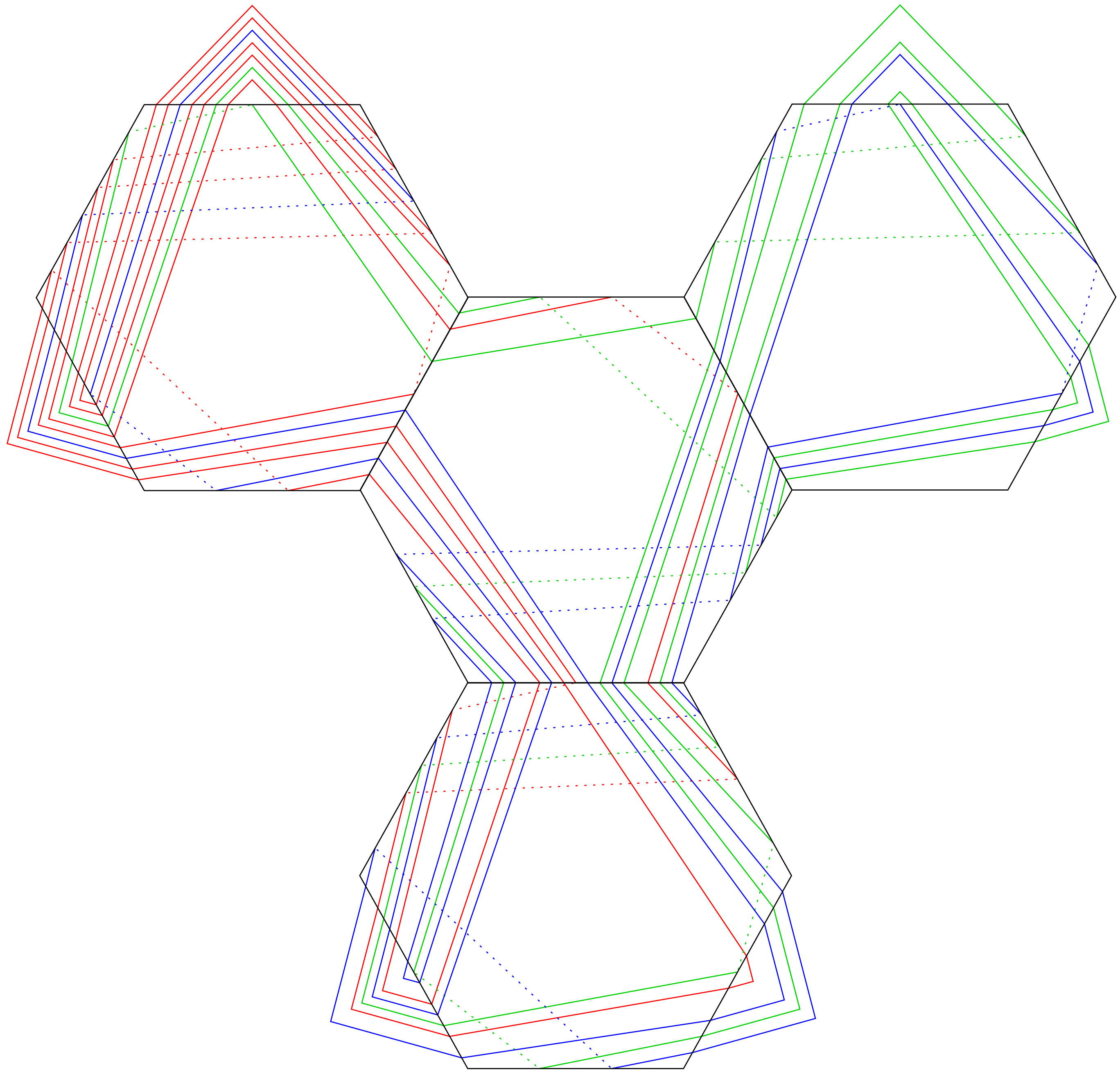
$$m(2/7, 1, 2/7, 3) \sim m(9/7, 23/7) \sim m(-224/97)$$

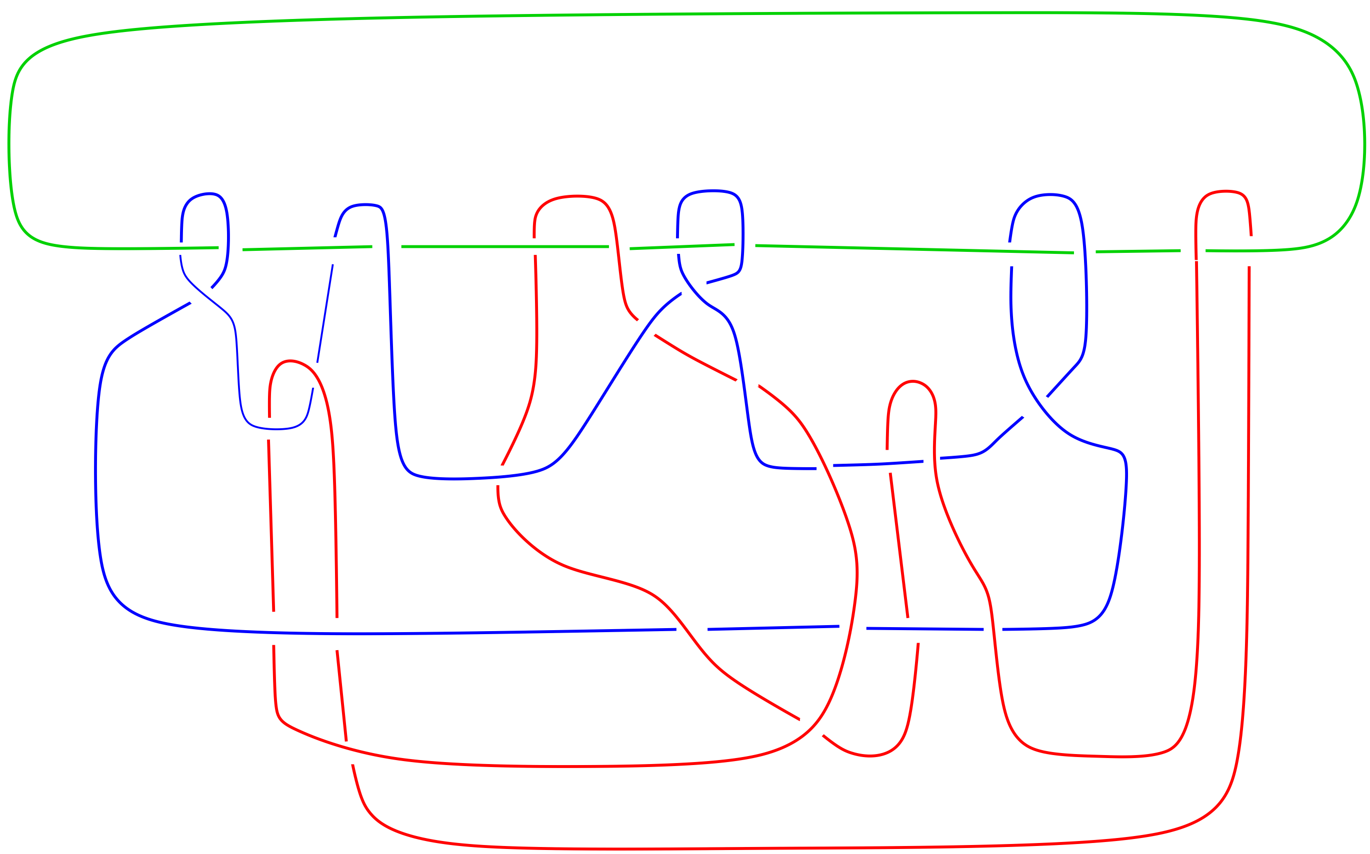
El nudo 9_{35} *es universal.*

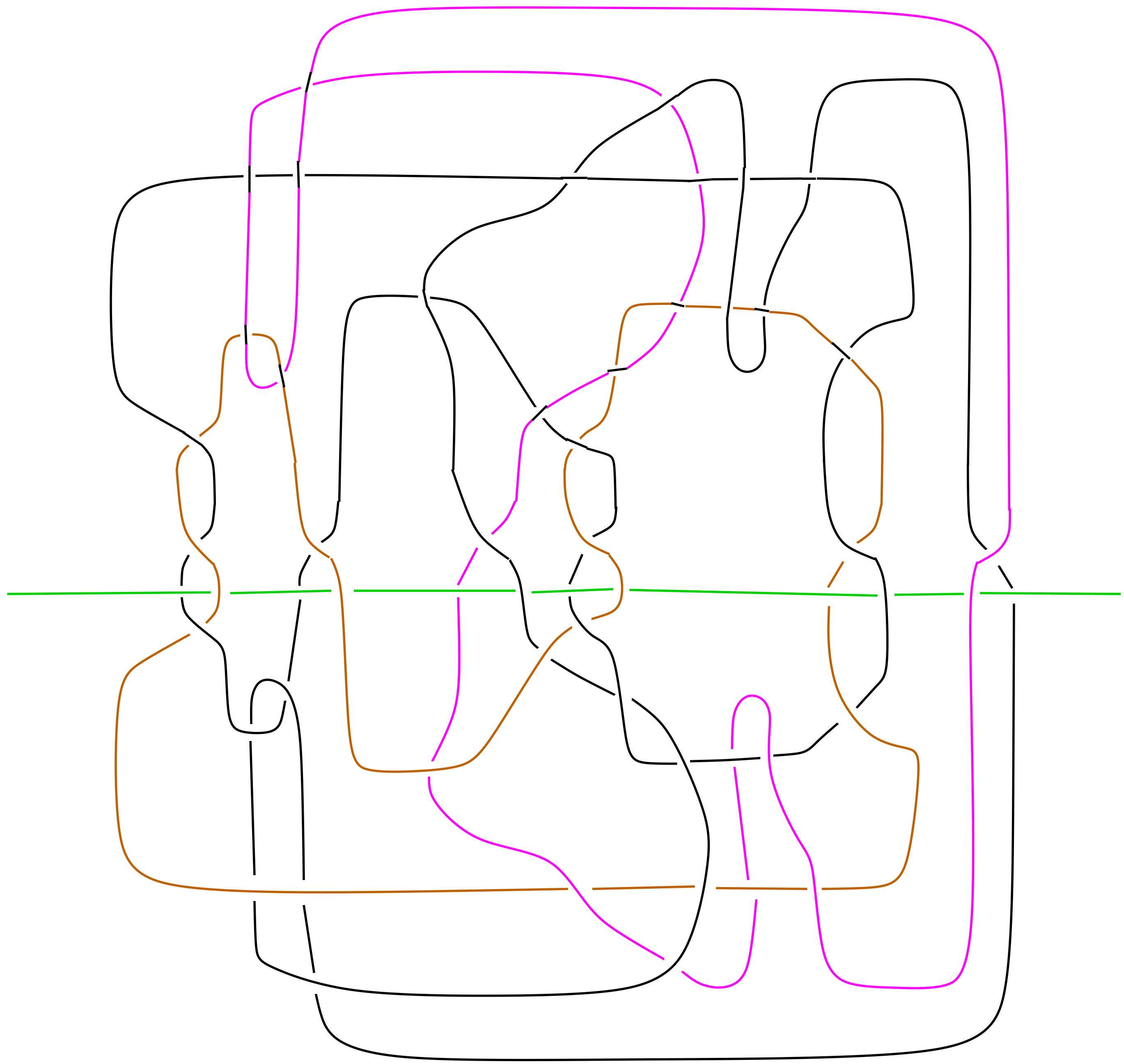
$$9_{48} = m(2/3, 2/3, -1/3)$$

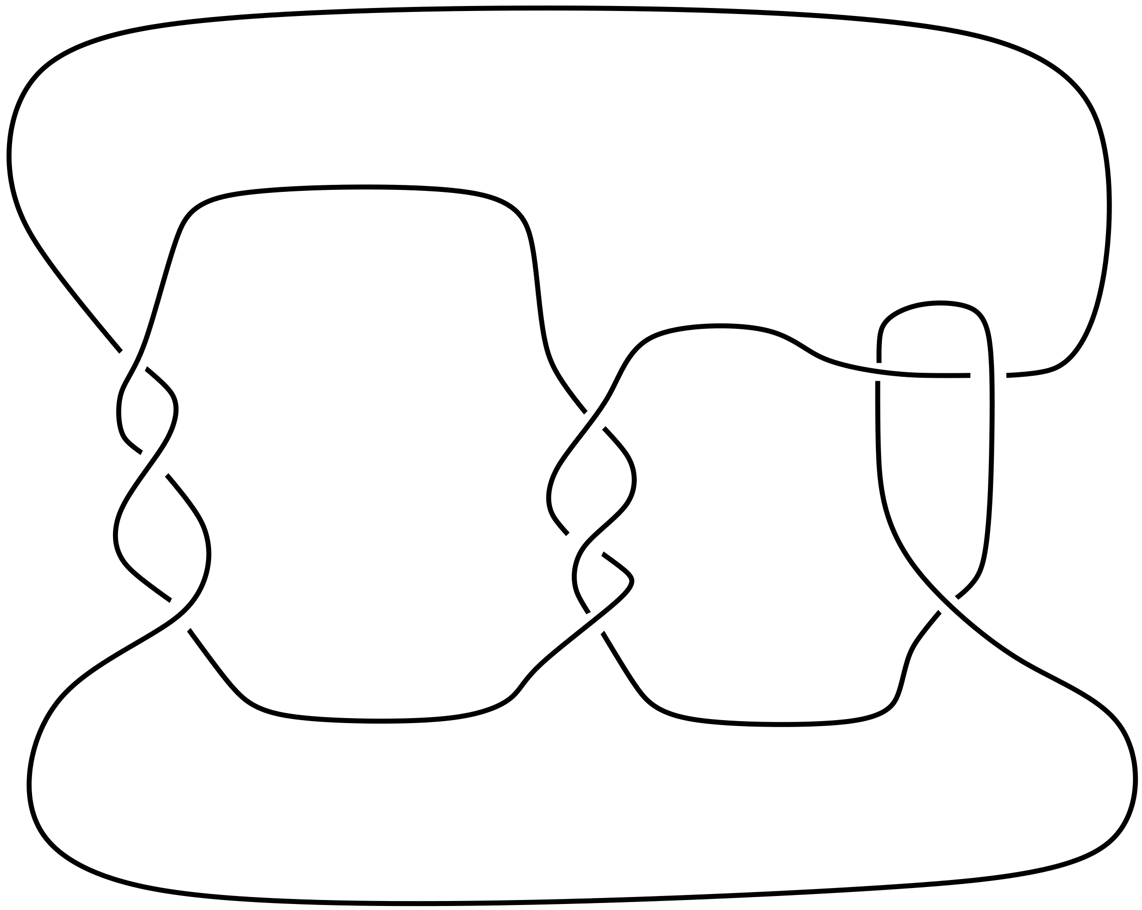












$$m(1/3, 1/3, 2/3) \sim m(4/3, 4/3, -4/3) \leftarrow m(1/3, 1/3, -1/3)$$

El nudo 9_{48} es universal

- $10_{68} = m(3/5, 1/3, 1/3) \sim m(-(19 \cdot 3/5, 19/3, 19/3) \leftarrow m(-3/5, 1/3, 1/3) \sim 10_{145}$

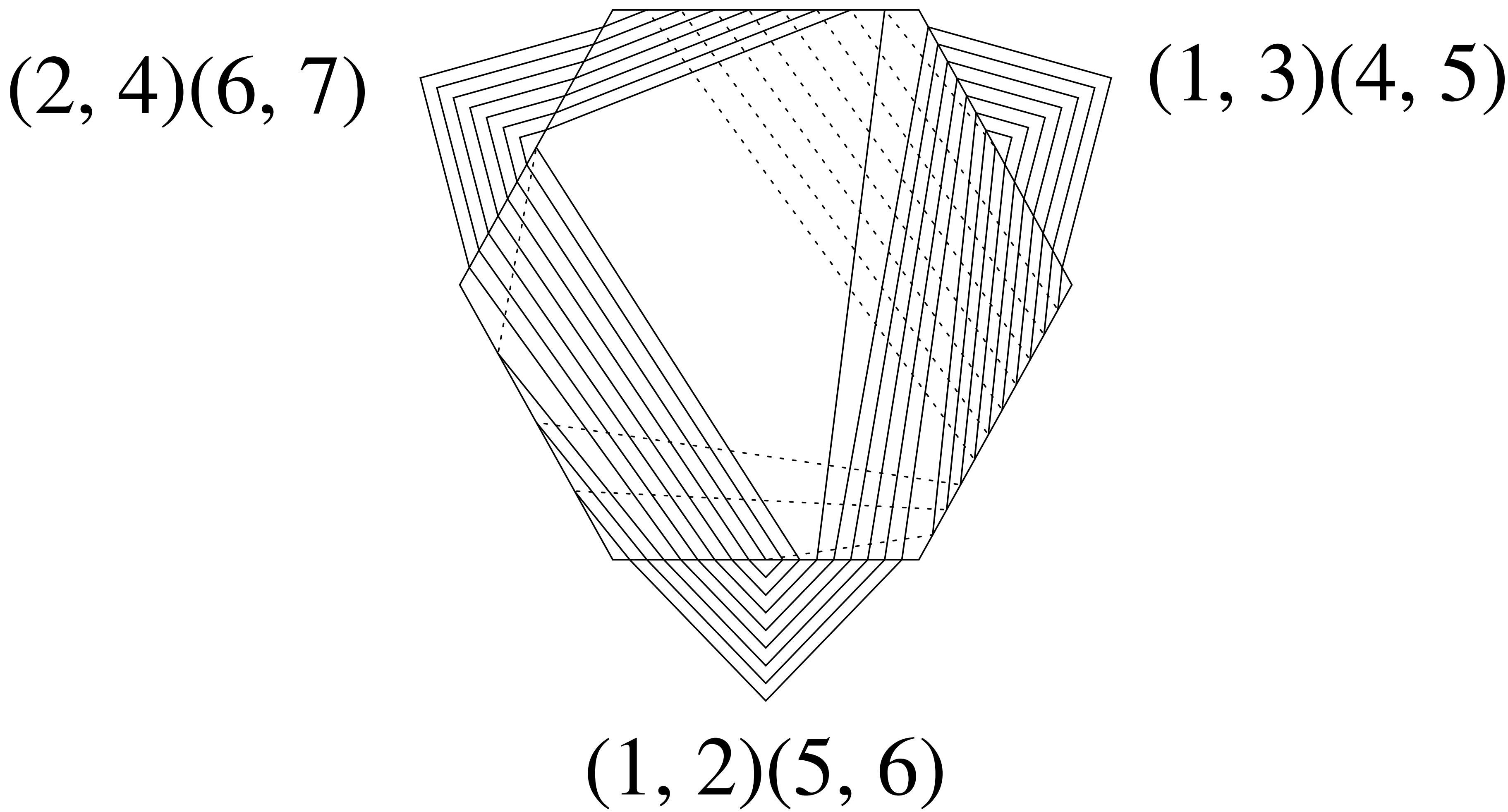
- $10_{69} = m(3/5, 2/3, 3/3) \sim m(-(29 \cdot 3)/5, 29/3, 29/3) \leftarrow m(-3/5, 1/3, 1/3) \sim 10_{145}$

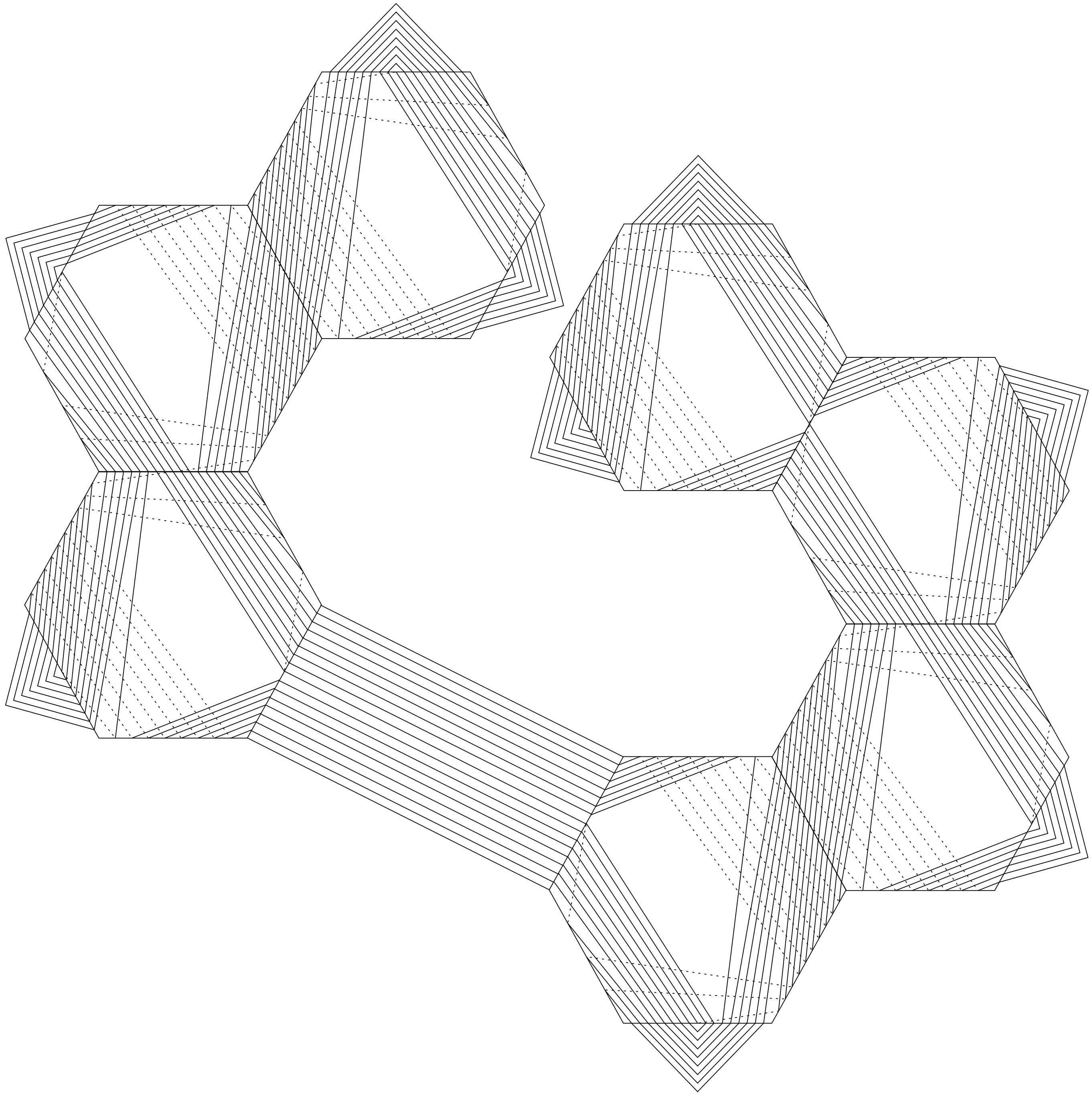
- $10_{146} = m(2/5, 2/3, -1/3) \sim m(-(11 \cdot 3)/5, 11/3, 11/3) \leftarrow m(-3/5, 1/3, 1/3) \sim 10_{145}$

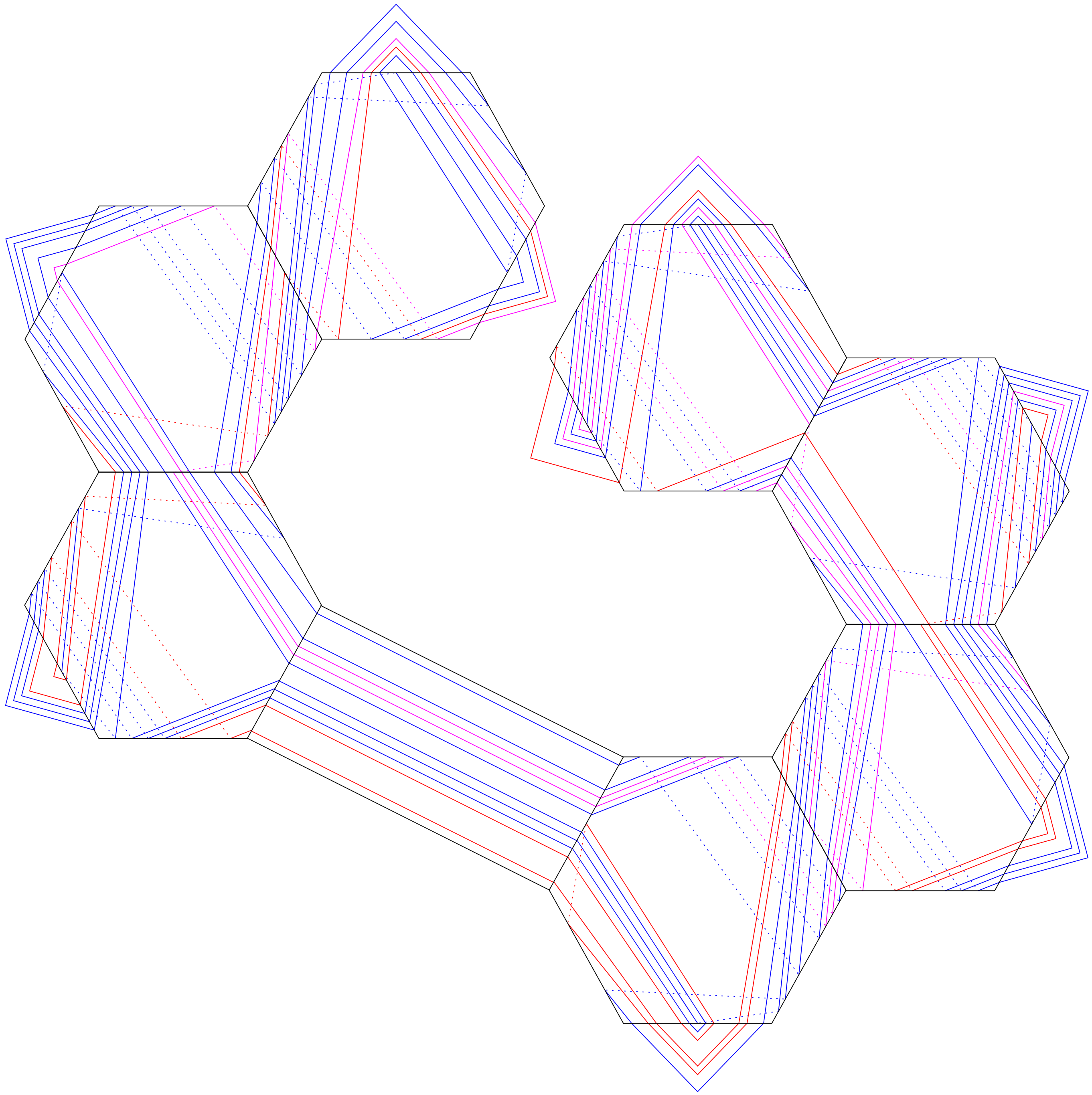
- $10_{75} = m(2/3, 2/3, 5/3) \leftarrow 10_{145}$

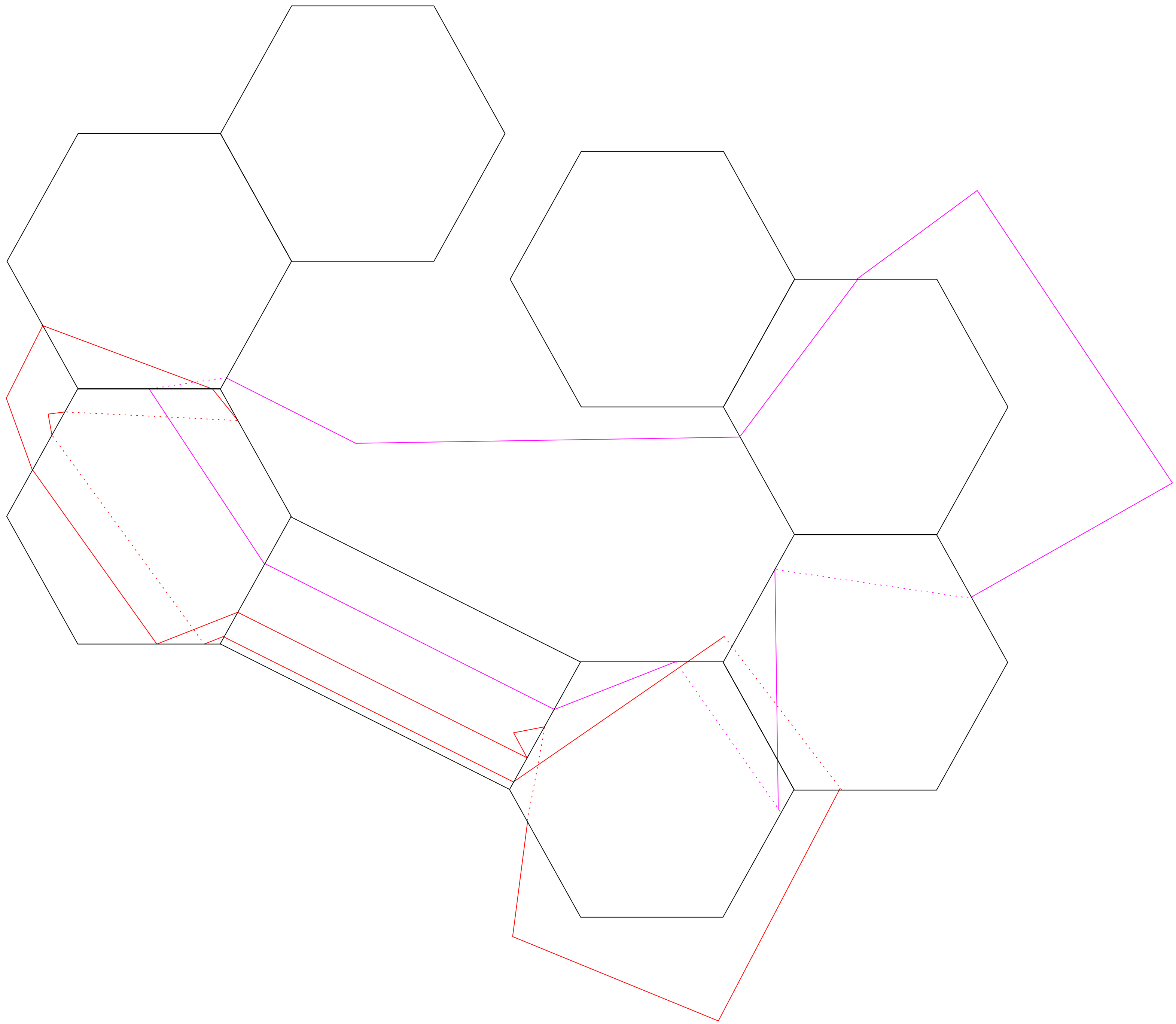
- $10_{147} = m(3/5, 1/3, -1/3) \leftarrow 10_{145}$

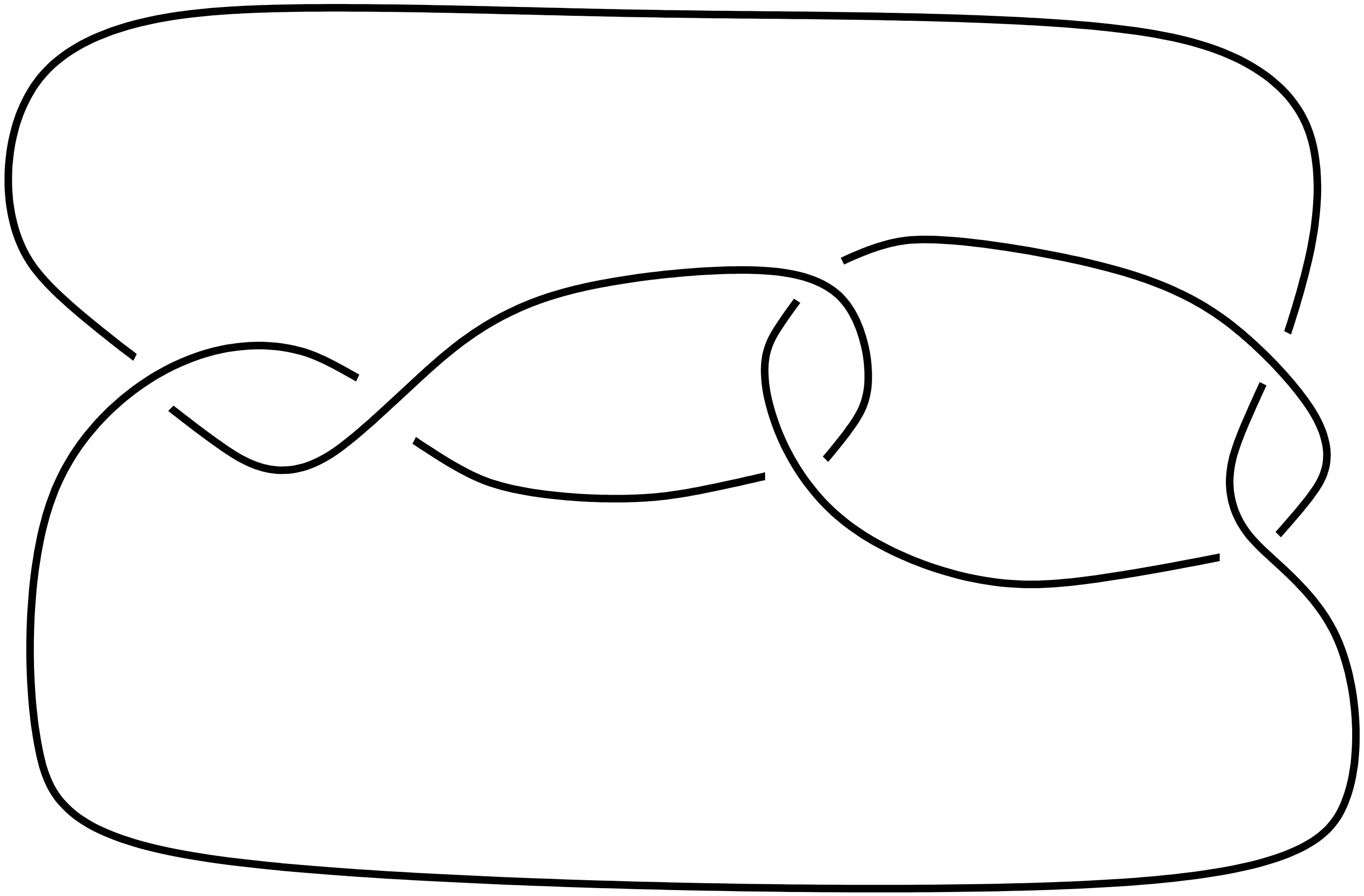
$$10_{145} = m(2/5, 2/3, -1/3)$$







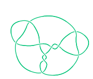




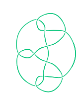
$$m(2, -\frac{1}{2}, \frac{1}{2}) \sim m(-\frac{1}{2}, \frac{5}{2}) \sim m(\frac{8}{3})$$

El nudo 10_{145} *es universal*

Teorema. *Todos los nudos de Montesinos hiperbólicos y con menos de once cruces son universales, excepto*



$10_{67} = \mathrm{m}(\frac{2}{5}, \frac{1}{3}, \frac{2}{3})$



$10_{137} = \mathrm{m}(\frac{2}{5}, \frac{3}{5}, \frac{-1}{2})$

(Todavía no sabemos).