1. Set-Theoretic Topology

Domingo Alcaraz Candela *Universidad Politécnica de Cartagena* Topological entropy for endommorphisms of totally bounded

abstract We analyze the relationship between the Bowen's entropy of a topological endomorphism α on a totally bounded (abelian) topological group G and the Bowen's entropy of its continuous extension to the Weil completion of G. The infinitude of Bowen's entropy for group endomorphisms of totally bounded abelian groups is studied in the following two aspects:

(i) by providing a wealth of zero entropy endomorphisms whose extension to the completion of the group has infinite entropy;

(ii) by establishing smallness of the class $QfrakG$ of compact abelian groups without endomorphisms of infinite entropy.

J. Juan Angoa Amador *Facultad de Físico Matemáticas, BUAP* Spaces of continuous functions, Σ-products and box topology

abstract For a topological space X, we will denote by X_0 the set of its isolated points and X_1 will be equal to $X \setminus X_0 . C(X)$ denotes the space of real-valued continuous functions defined on X. $\Box\mathbb{R}^\kappa$ is the Cartesian product $\mathbb{R}^\kappa$ with its box topology, and $C_\Box(X)$ is $C(X)$ with the topology inherited from $\Box\mathbb{R}^\kappa$. By $\widehat{C}(X_1)$ we denote the set $\{f \in C(X_1) : fcanbecontinuouslyextendedtoallofX\}$. A space X is almost-ω-resolvable if it can be partitioned by a countable family of subsets in such a way that every non-empty open subset of X has a non-empty intersection with the elements of an infinite subcollection of the given partition. We analyze $C_\Box(X)$ when X_0 is F_σ and prove: (1) for every T_1 topological space X, if X_0 is F_σ in X, and $\emptyset \neq X_1 \subset cl_X X_0$, then $C_\Box(X) \simeq \Box\mathbb{R}^{X_0}$; (2) for every Tychonoff space X such that X_0 is F_σ, $cl_X X_0 \cap X_1 \neq \emptyset$ and $X_1 \setminus cl_X X_0$ is almost-ω-resolvable, then $C_\Box(X)$ is homeomorphic to a free topological sum of $\geq |\widehat{C}(X_1)|$ copies of $\Box\mathbb{R}^{X_0}$, and, in this case, $C_\Box(X) \simeq \Box\mathbb{R}^{X_0}$ if and only if $|\widehat{C}(X_1)| \geq 2^{|X_0|}$. We also analyze $C_\Box(X)$ when $|X_1| = 1$ and when X is countably compact, and prove that the Σ-product $\Sigma_{\widehat{0}}\mathbb{R}^\kappa$ with the box product topology is not homeomorphic to $\Box\mathbb{R}^\delta$ for any δ when $cof(\delta) > \aleph_0$.

Liljana Babinkostova *Boise State University* Screenability and classical selection principles

abstract In this presentation we discuss the relationship between the selection principle $S_c(A,B)$ and the classical selection principles $S_{fin}(A,B)$ and $S_1(A,B)$.

Taras Banakh and Murat Tuncali *Ivan Franko Lviv National University and Nipissing University* Suslinian continua and "connected" versions of some classical topological cardinal invariants

abstract We introduce several cardinal invariants related to the Suslinian property of continua. Following A.Lelek we say that a continuum X is *Suslinian* if it contains no uncountable disjoint family of non-degenerated subcontinua. This property leads to the cardinal invariant $\bar{c}(X) = \sup\{|\mathcal{C}| : \mathcal{C}$ is a disjoint family of non-degenerate subcontinua in $X\}$ defined for any continuum X. The cardinal function $\bar{c}(\cdot)$ can be considered as a "connected" analogue of the cellularity where non-degenerate subcontinua paly the role of open sets. Following this ideology we can also introduce "connected" counterparts of other cardinal invarinats such as density, weight, π-weight. In particular, a "connected" analogue of the density is

$\bar{d}(X) = \min\{|D| : D$ is a subset of X meeting each non-degenerate subcontinuum$\}$. The definitions of $\bar{c}(X)$ and $\bar{d}(X)$ can be extended to all Tychonov spaces X letting $\bar{c}(X) = \min\{\bar{c}(Y) : Y$ is a continuum containing $X\}$ and similarly for $\bar{d}(X)$. Unlike their classical originals, the cardinal functions $\bar{c}(\cdot)$ and $\bar{d}(\cdot)$ are monotone with respect to taking subspaces.

The main our result asserts that the weight $w(X)$ of any Tychonov space is $\leq \min\{\bar{d}(X), \bar{c}(X)^+\}$. Moreover, under the generalized Suslin Hypothesis, $w(X) \leq \bar{c}(X)$. Consequently each Suslinian continuum is hereditarily decomposable, has weight $\leq \aleph_1$ (and is metrizable if the Suslin Hypothesis holds). This answers one question of D.Daniel, J.Nikiel, L.B.Treybig, M.Tuncali and E.D.Tymchatyn. Each compact space X with $w(X) > \bar{c}(X)$ is the limit of an inverse well-ordered spectrum of length $\bar{c}(X)^+$ consisting of compacta with weight $\leq \bar{c}(X)$ and monotone bonding maps.

If X is a space with $\bar{c}(X) < 2^{\aleph_0}$, then rim-weight of X is $\leq \bar{c}(X)$ and $\bar{c}(X) \leq w(X) \leq \bar{c}(X)^+$. It is clear that $\bar{c}(X) \leq \bar{d}(X)$ for any space X and $\bar{c}(L) < \bar{d}(L)$ for a Suslin line L. On the other hand, we do not know if there is a metrizable continuum with $\bar{c}(X) < \bar{d}(X)$.

Jörg Brendle *The Graduate School of Science and Technology, Kobe University*
Measure and Category in Generalized Cantor Spaces

abstract For a cardinal λ, we consider the *generalized Cantor space* 2^λ equipped with the product topology and product measure as usual. Let M_λ denote the *meager ideal* on 2^λ and N_λ, the *null ideal* on 2^λ. Notice that every member of M_λ is contained in $A \in M_\lambda$ with countable support which means there are $\Lambda \in [\lambda]^{\aleph_0}$ and $A^* sub 2^\Lambda$ meager such that $A = \{x \in 2^\lambda : x re \Lambda \in A^*\}$. Thus M_λ and $M = M_{\aleph_0}$ are rather similar. An analogous comment applies to N_λ and $N = N_{\aleph_0}$. A number of people, including Cichoń, Fremlin, Kraszewski, and Miller, have investigated cardinal invariants related to the M_λ and the N_λ. For example, $add(M_\lambda) = \aleph_1$ and $cof(M_\lambda) = \max\{cof(M), cof([\lambda]^{\aleph_0})\}$ for $\lambda \geq \aleph_1$, $cov(M_\lambda)$ is decreasing in λ and stabilizes from some $\lambda < cc$ onwards. Similarly for N_λ.

Given a stationary set $S sub \omega_1$, $\clubsuit_S$ is the combinatorial principle asserting the existence of a sequence $la A_\alpha : \alpha \in S$ is a limit ordinal and A_α is cofinal in αra such that for all $A \in [\omega_1]^{\aleph_1}$ there is α with $A_\alpha sub A$. $\clubsuit$ abbreviates $\clubsuit_{\omega_1}$. $\clubsuit$ easily entails $cov(M_{\aleph_1}) = cov(N_{\aleph_1}) = \aleph_1$. Fuchino, Shelah, and Soukup proved the consistency of $\clubsuit$ with $cov(M) = cc \geq \aleph_2$. A fortiori, their model satisfies $cov(M) > cov(M_{\aleph_1})$. We subsequently obtained the analogous consistency result for the null ideal.

Recently we proved:

Theorem. It is consistent that $cov(M) = cc \geq \aleph_2$ and $\clubsuit_S$ holds for every stationary set S.

This extends the result of Fuchino, Shelah, and Soukup and answers a question of theirs.

Theorem The $cov(M_\lambda)$ may simultaneously assume any finite number of distinct values.

This answers a question of Kraszewski.

The purpose of our talk is

- to give an overview of results on cardinal invariants of the meager and null ideals on generalized Cantor spaces,
- to sketch proofs of Theorems 1 and 2 above, and
- to survey open problems in the area.

Dennis Burke *Miami University, Oxford, Ohio* Spaces with a sharp base
*abstract*Joint work with Zoltan Balogh.

The property of having a sharp base is not preserved under a perfect map. **Example.** There exists a space X with a sharp base and a perfect mapping $f : X \to Y$ onto a space Y which does not have a sharp base. It is known that a spaces with a sharp base have a point-countable sharp base. This can be sharpened to "point-finite" on the set of isolated points. **Theorem.** If X has a sharp base then X has a point-countable sharp base which is point-finite on the set H of isolated points. (Hence H is an F_σ set.) This theorem follows from a more general combinatorial argument about certain $\{0,1\}$ matrices on $\kappa \times \kappa$.

Agustín Contreras Carreto *Facultad de Ciencias Físico-Matemáticas de la BUAP* Some properties of cardinal functions wl, qwl, aql, ac, lc and ql

abstract One of the known equalities is the Bell-Ginsburg-Woods's inequality: if X is a T_4-space, then $|X| \leq 2^{wl(X)\chi(X)}$. In this talk we are going to introduce the cardinal function qwl, wich satisfices $qwl(X) \leq wl(X)$, for every topological space; and we will to establish the following most general result:if X is a T_4-space, then $|X| \leq 2^{qwl(X)\chi(X)}$. Later we give an example to shows that the our result can give better estiamtion than one of Bell-Ginsburg-Woods's inequality. Moreover we will present some reflection properties for cardinal functions wl, qwl, aql, ac, lc and ql.

Dikran Dikranjan *Udine University* Separation via sequential limit laws in topological groups

abstract Let (u_n) be a sequence of integers. We say that an element x of a topological group G satisfies the *sequential limit law* (SLL, in brief) (u_n), if the powers x^{u_n} tend to e_G.

Motivated by the definition of T_0 and T_1 separation axioms for topological spaces and replacing points x by cyclic subgroups $\langle x\rangle$, we propse separation axioms $\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$ for a topological group X as follows:

(a) X is $\mathcal{G}_0$, if for distinct $\langle x\rangle$ and $\langle y\rangle$ in X there exists a SLL (u_n) such that either x satisfies (u_n) while y does not satisfy (u_n), or y satisfies (u_n) while x does not satisfy (u_n);

(b) X is $\mathcal{G}_1$, if for every y in X that does not belong to $\langle x\rangle$ there exists a SLL (u_n) such that x satisfies (u_n) while y does not satisfy (u_n);

(c) X is $\mathcal{G}_2$, if for every x in X there exists a SLL (u_n) such that x satisfies (u_n), while no y that does not belong to $\langle x\rangle$ satisfies (u_n).

Replacing the sequence of integers (u_n) by a sequence of characters of X and asking convergence of $u_n(x)$ to 1 in the circle group $\mathbf{T}$ instead of the above defined SLL (u_n), one can define similarly separation axioms $\mathcal{S}_0$, $\mathcal{S}_1$ and $\mathcal{S}_2$ for a topological group X. Then:

1. $\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$ coincide for any non-discrete locally compact group X and are equivalent to X being isomorphic to $\mathbf{T}$.
2. $\mathcal{S}_0$ and $\mathcal{S}_1$ coincide for any topological abelian group X and are equivalent to X being maximally almost periodic.
3. A topological abelian group X satisfies $\mathcal{S}_2$ iff its Bohr topology has countable pseudocharacter.

Szymon Dolecki *Mathematical Institue of Burgundy* Combinatorics in convergence theory

abstract Examples of combinatorial problems arising in topology and convergence theory will be presented. Estimates of sequential order of finite products of sequential topologies in terms of nodalities lead to certain transfinite combinatorial problems [1]. Study of irregularity numbers of pretopologies leads to combinatorics of subintervals of some trees [2].

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Stefano Ferri *Universidad de los Andes, Bogotá, Colombia.* Continuity in Topological Groups

abstract A topological group is a group equipped with a (Hausdorff) topology such that both multiplication and inversion are continuous mappings. However, in certain cases one can deduce that a group G with a topology is a topological group under less restrictive assumptions. For example if G is a Baire metrizable space one can deduce that G is a topological group under the only assuption that multiplication is a separately continuous mapping. In this talk we consider a group G equipped with a Baire metrizable topology and prove that, under these assumptions, if right translations are continuous and left translations are almost-continuous, then G is a topological group. This is joint work with Salvador Hernández Muñoz and Ta-Sun Wu.

Adalberto García Maynez *Instituto de Matemáticas, UNAM, México* Upper bounds for uniform weights

abstract Consider a Tychonoff space $(X,\mathcal{T})$ and a compatible uniformity $\mathcal{U}$ on X. We denote by $\omega(X,\mathcal{U})$ the minimum possible cardinality of a basis of $\mathcal{U}$. If $(X,\mathcal{T})$ is not a discrete space, we know $\aleph_0 \leq \omega(X,\mathcal{U})$ and $\aleph_0 = \omega(X,\mathcal{U})$ implies that X is metrizable. We prove that always $\omega(X,\mathcal{U}) \leq z(X \times X)$, where for every space Y, $z(Y)$ denotes the cardinality of the family of zero subsets of Y. If $\mathcal{U}$ is totally bounded, we have a better upper bound, namely $\omega(X,\mathcal{U}) \leq z(X)$. If $\mathcal{U}_n$ is the fine uniformity of $(X,\mathcal{T})$, we know that $\omega(X,\mathcal{U}_n) \leq \aleph_0$ if and only if the space X is metrizable and the set of limit points X^a is compact. Is it true that $\omega(X,\mathcal{U}) \leq z(X)$? this may be a forcing problem. If δX is the density of X, we know $\delta X \leq z(X) \leq z(X \times X) \leq 2^{\delta X}$, so if we assume the GCH, δX and $2^{\delta X}$ are the only possible values of $z(X)$ and $z(X \times X)$. The task is very clear although it might be very difficult) : Using the example in ZFC of a Tychonoff space X such that $z(X) < z(X \times X)$, calculate $\omega(X,\mathcal{U})$. The talk will concentrate on the methods to obtain upper bounds for uniform weights.

Chris Good *University of Birmingham, UK* Inhomogeneities in inverse limits of tent maps.

Gary Gruenhage *Auburn University* Paracompact ordered spaces are base-paracompact

abstract J.E. Porter defined a space X to be base-paracompact if X has a base B of cardinality the weight of X such that every open cover of X has a locally finite refinement by members of B. He proved that paracompact ordered spaces of weight $\aleph_1$ (or less) are base-paracompact, and asked if all paracompact ordered spaces are

base-paracompact. We show that they are. We should remark that Porter also asked whether every paracompact space is base-paracompact; this is still unsolved.

Yasunao Hattori *Shimane University, Japan* On representations of spaces by unions of locally compact subspaces

abstract Vitalij A.Chatyrko, Yasunao Hattori and Haruto Ohta In this talk, we shall discuss the possibility of different presentations of (locally compact) spaces as unions or disjoint unions of locally compact subspaces. We begin with an observation of locally closed sets. A subset A of a space X is called a *locally closed* (we say here a G_{co}*-set in* X) if there are a closed subset F and an open subset O of X such that $A = F \cap O$. Furthermore, we put $G_{co}(X) = \{A : A \text{ is a } G_{co}\text{-set in X }\}$.

Proposition. *For every space* X*,* $G_{co}(X)$ *is a semiring.*

Corollary. *Let* $A = \cup_{i=1}^{n} A_i$*, where* $A_i \in G_{co}(X)$ *for every* $i = 1, 2, \ldots, n$*.*

(1) *There are finitely many disjoint sets* $B_1, ..., B_t \in G_{co}(X)$ *such that* $A = \cup_{i=1}^{t} B_i$ *and for every* i*, there is* $M_i \subset \{1, \ldots, t\}$ *such that* $A_i = \cup\{B_s : s \in M_i\}$*.*

(2) *There are finitely many disjoint sets* $C_1, ..., C_l \in G_{co}(X)$ *such that* $X \setminus A = \cup_{i=1}^{l} C_i$*.*

Now, we define cardinal numbers $lc(X)$ and $lcd(X)$ for a space X as follows: $lc(d)(X) = \min\{\tau : X$ has a cover (partition) $\{L_t : t \in B\}$ of locally compact subspaces of X such that card $(B) = \tau\}$. Evidently, the inequalities $lc(X) \leq lcd(X) \leq \text{card}(X)$ hold for every space X. Then we have the following.

Example. For every natural number $i = 1, 2, \ldots$ there is a countable discrete subspace (and hence locally compact subspace) X_i of the closed interval $\mathbb{I} = [0, 1]$ such that

(1) $X_i \cap X_j = \emptyset$ if $i \neq j$,

(2) $lc(A_n) = lcd(A_n) = n$, for each $n \geq 2$, where $A_n = \cup_{i=1}^{n} X_i$,

(3) $lc(\mathbb{I} - A_n) = lcd(\mathbb{I} - A_n) = n$, for each $n \geq 2$.

Concerning the relationship between lc and lcd, we have

Theorem 1. *Let* X *be a Hausdorff space with* $lc(X) \leq \aleph_0$*, then* $lc(X) = lcd(X)$ *holds.*

As an application of the corollary above to dimension theory, we have the following.

Theorem 2. *Let* X *be a perfectly normal space.*

(1) *If* $X = (\cup_{i=1}^{n} A_i) \cup B$*, where* $A_i \in G_{co}(X)$ *(in particular, if* A_i *is a locally compact subspace of* X*) for every* i*, then* $\dim X = \max\{\dim A_i, i = 1, ..., n, \dim B\}$.

(2) *If* X *is paracompact and* $X = (\cup \nu) \cup B$*, where* $\nu = \{A_s : s \in S\}$ *is a locally finite system of* G_{co}*-sets in* X *(locally compact subspaces of* X*) then* $\dim X = \max\{\dim A_s, s \in S, \dim B\}$.

Melvin Henriksen *Harvey Mudd College* One point metric completions

abstract If a metrizable space X is a dense in a metrizable space Y, then Y is called a metric extension of X. If T1 and T2 are metric extensions of X and there is a continuous map of T2 into T1 that keeping X pointwise fixed, we write T1<=T2. If X is noncompact and metrizable, then (M(X),<=) denotes the set of metric extensions of X, where T1 and T2are identified if T1<=T2 and T2<=T1, i.e., if there is a homeomorphism of T1 onto T2 keeping X pointwise fixed. (M(X),<=) is a large complicated poset studied extensively by V. Bel'nov [The structure of the set of metric extensions of a noncompact metrizable space, Trans. Moscow Math. Soc. 32 (1975), 1-30]. We study the poset (E(X),<=) of one-point metric extensions of a locally compact metrizable space X. Each such extension is a (Cauchy) completion of X with respect to a compatible metric. This poset resembles the lattice of compactifications of locally compact space if X is also separable. For Tychonoff X , let X*= X X, and let Z(X) be the poset of zerosets of X partially ordered by set inclusion. Theorem If X and Y are locally compact separable metrizable spaces, then (E(X),<=) and (E(Y),<=) are order-isomorphic iff Z(X*) and Z(Y*) are order isomorphic, and iff X* and Y* are homeomorphic. We construct an order preserving bijection lamda E(X)−− >Z(X*) such that a one-point completion in E(X) is locally compact iff its image under lamda is clopen. We extend some results to the nonseparable case, but leave problems open. This is part of joint research with L. Janos and R.G. Woods that will appear in C.M.U.C.

Fernando Hernández-Hernández *IM-UNAM (Morelia)* Realcompactness on Psi-spaces

abstract I will discuss some conditions on the almost disjoint family $CalA$ to get realcompactness of $\Psi(CalA)$ and some realted problems.

Heikki Junnila *University of Helsinki* Hereditary covering properties of weak*-topologies

abstract We characterize several well-established properties (such as having an equivalent *uniformly Gateaux smooth* norm or being *weakly countably determined*) of Banach spaces in terms of hereditary covering properties (such as hereditary *bounded σ-metacompactness* or hereditary *σ- distributive metacompacteness*) of the weak*-topologies of the dual spaces.

Masaru Kada *Advanced Research Institute for Science and Engineering, Waseda University* How many miles to βX?

abstract Joint work with Kazuo Tomoyasu and Yasuo Yoshinobu. It is known that the Stone–Čech compactification βX of a metrizable space X is approximated by the collection of Smirnov compactifications of X for all compatible metrics on X [Woods, 1995]. If we confine ourselves to locally compact separable metrizable spaces, the corresponding theorem holds for Higson compactifications [Kawamura-Tomoyasu, 2001]. So how many compatible metrics do we actually need to approximate βX by Smirnov or Higson compactifications of X? Let $sa(X)$ denote the smallest cardinality of a set D of compatible metrics on X such that βX is approximated by Smirnov compactifications for all metrics in D, and $ha(X)$ the corresponding cardinal for Higson compactifications. We present the following results. (1) For a locally compact separable metrizable space X, $sa(X) = ha(X) = d$ (the dominating number) if the set of nonisolated points of X is noncompact, and otherwise $sa(X) = ha(X) = 1$. In particular, $ha(X)$ is either d or 1 while it is defined. (2) There is a metrizable space X for which $sa(X) > d$. This theorem leads $sa(\omega) = ha(\omega) = 1$. To investigate the case of ω further, we consider the following cardinals. Let sp be the the smallest cardinality of a set D of compatible metrics on ω such that, $\beta\omega$ is approximated by Smirnov compactifications for all metrics in D but any finite subset of D does not suffice, and hp the corresponding cardinal for Higson compactifications. We will present ZFC-results and consistency results on the relationship among sp, hp and other known cardinal invariants of the reals.

Martin Maria Kovar *University of Technology, Brno* Maximal compact topologies in the light of the de Groot dual

abstract Using some advanced properties of the de Groot dual and a modified Hofmann-Mislove theorem, we will present a positive solution of an old question of D. E. Cameron (1977): Is every compact topology contained in some maximal compact topology? The solution is based on a construction of a maximal ring of sets, containing the closed sets of the given compact space, which is contained in the family of the compact sets. If the given space is sober T_1, then one can obtain directly the requested maximal compact topology which is generated by the ring (Kunzi + Zyphen, 2003). However, for a general space we need the de Groot dual of the ring, too. The topology generated by the maximal ring of sets need not be compact in general, but luckily, its dual is always compact. And even more luckily, between this topology and its de Groot dual there always exists a maximal compact topology The only remaining problem is, how one can get the original topology below that maximal compact topology. To do it, we will use a modified Hofmann-Milsove theorem with some additional tricks, which allow us to finish and solve the puzzle.

Justin Tatch Moore *Boise State University* Counterexamples to basis problems in set theory and topology

abstract I will present the following ZFC result.

Theorem: There is a hereditarily Lindelöf, non-separable space.

One immediate consequence is that the uncountable regular topological spaces do not have a three element basis. The combinatorial object which makes the construction work also gives a number of other examples. In particular it produces a binary relation R which is neither below $\omega \cdot \omega_1$ nor above $[\omega_1]^{<\omega}$ in the Tukey order. It also gives an example of a function c from $\omega_1 \times \omega_1$ to ω_1 which takes all values on any product of uncountable sets.

Peter Nyikos *University of South Carolina* Recent research on the compact-open topology and modifications

abstract Let $C_k(X)$ stand for the space of continuous functions from X to $\mathbb{R}$ with the compact-open topology. For compact K, $C_k(K)$ is simply the Banach space given by the sup norm, but when X is not locally compact, $C_k(X)$ is very complicated. Gartside and Reznichenko showed that $C_k(X)$ is stratifiable whenever X is a Polish space; as a result, $C_k(\mathbb{P})$ has emerged as a prime candidate for a negative solution to the 43-year-old problem of whether every stratifiable space is M_1. The following problem is also of interest: Problem 1 Let X be separable metrizable. If $C_k(X)$ is stratifiable, must X be completely metrizable?

The converse is true. Problem 1 easily reduces to the 0-dimensional case. Since every scattered metrizable space is completely metrizable, the only restriction on the following partial solution to Problem 1 is in the last clause in the hypothesis.

Theorem 1 Let X be a 0-dimensional separable metrizable space which is not scattered, and has the property that every compact subset is countable. Then $C_k(X)$ is not stratifiable.

This result is new even in the special case $X = \mathbb{Q}$, answering a question posed by Gary Gruenhage at the 2003 Lubbock conference. Theorem 1 made use of the following elegant criterion in.

Theorem Let X be a 0-dimensional separable metrizable space. Then $C_k(X)$ is stratifiable if, and only if, it is possible to assign to each clopen subset W of X a compact $F(W) \subset W$, and to each compact $K \subset X$ a compact $\phi(K) \supset K$ in such a way that, whenever $W \cap K \neq \emptyset$, it follows that $F(W) \cap \phi(K) \neq \emptyset$ also.

This theorem also figures in the proof of Theorem 2 below, which represents the first progress towards the solution of the following problem. Problem 2 Let $C_s(\mathbb{P}, \omega)$ stand for the set of continuous natural-number-valued functions on $\mathbb{P}$ with the sequential modification of the compact-open topology. Is $C_s(\mathbb{P}, \omega)$ 0-dimensional? The modification in question is the one in which a set is open iff it is sequentially open in $C_k(\mathbb{P}, \omega)$. Sequential convergence in $C_k(\mathbb{P}, \omega)$ has the following appealing characterization:

$$f_n \to f \iff f_n(x_n) \to f(x) \text{ whenever } x_i \to x.$$

A positive solution to Problem 2 would be enough to solve a problem in theoretical computer science. This problem is whether two competing approaches to higher-type real-number computability actually coincide on level 3. References , and explain these concepts, and shows how analogues of Problem 2, obtained by iterating the functor $C_s(\cdot, \omega)$, would establish the coincidence at all levels. Definition A space X is semiregular if it has a base of regular open sets, and countably 0-dimensional if whenever $x \in X$ and F is a countable closed subset of X, then there is a clopen set containing x and missing F.

Theorem 2 $C_s(\mathbb{P}, \omega)$ is semiregular and countably 0-dimensional. In fact, if $x \in C_s(\mathbb{P}, \omega) \setminus F$ and F is a countable closed subset of $C_s(\mathbb{P}, \omega)$, then there is a set U that is open in $C_p(\mathbb{P}, \omega)$ and closed in $C_s(\mathbb{P}, \omega)$, contains F, and misses x.

Here C_p refers to the product topology, which is much coarser than the compact-open topology in this context.

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Oleg Pavlov *Towson University* A zero-dimensional homogenous space with the fixed-point property

abstract Jan van Mill asked at the 2004 Spring Topology Conference whether there exists a nontrivial zero-dimensional homogenous space with the fixed-point property for homeomorphisms. We answer this question affirmatively.

Robert Raphael *Concordia University* The Epimorphic problem for C(X).

abstract We recall that epimorphisms need not be surjective in the category of commutative rings. Assume all spaces Tychonoff. Call a space X absolutely CR-epic if whenever X is a subspace of Y the induced ring homomorphism from C(Y) to C(X) is a ring epimorphism.

Our goal is characterize absolutely CR-epic spaces. There is an easy description of these spaces in the Lindelof case. There are complete results in the first countable case, and there are nuanced countable examples in the presence of the continuum hypothesis.

The work is joint. The most recent paper is with Barr and Kennison.

Ivan L Reilly *University of Auckland* Generalized closed sets

abstract This talk will discuss generalized closed sets (in the sense of Norman Levine) and their developments. It will consider their use in characterizing low separation properties, extremally disconnected spaces and variations of submaximal spaces. It represents joint work with J. Cao, M. Ganster, S. Greenwood and Ch. Konstadilaki.

Ennis Rosas *Universidad de Oriente-Nucleo de Sucre- Venezuela* (α, β) Contra Continuous Functions and (α, β) Contra Irresolute Functions

abstract

Masami Sakai *Kanagawa University* On κ-Fréchet Urysohn property in $C_p(X)$

abstract A space X is said to be κ-Fréchet Urysohn if for every open U of X and any point x in the closure of U, there exists a sequence in U which converges to x. This notion was introduced by Arhangel'skii. We give a characterization of κ-Fréchet Urysohn property in $C_p(X)$. The characterization introduce a special subset of the real line.

A.A. Salama *egypt* FUZZY -CONTINUITY AND

abstract In this paper, we study the concepts of fuzzy -open sets, fuzzy -closed sets. By using these concepts, we introduce and study the concept of fuzzy -continuity and -compactness in fuzzy topological spaces. In section 1, we study the concepts of fuzzy -open sets and fuzzy -closed sets in the light of quasi-coincident notion. In section 2 we study some properties of -continuous function and Urysohn space in fuzzy setting. In secton3, we introduce the concept of -compactness in fuzzy setting. Also, we give a characterizations and properties of -compactness in the light of the concept of -shading. A characterization of -compactness is given by using the concept of -finite intersection property due to [3]. Also, we show that in a fuzzy extremally disconnected space the concepts of -compactness, -compactness and -near compactness are equivalent. We investigate the image and the inverse image of -compactness under some types of functions. Finally, we define a locally -compactness in fuzzy setting and give some results on it.

Manuel Sanchis *Universitat Jaume I de Castell (Spain)* Some Solved and some Unsolved Problems in Linearly Ordered Dynamical Systems

abstract A (discrete) dynamical system (X, f) is said to be *linearly ordered* (in short, *LODS*) if the phase space X is a topological linearly ordered space. The main purpose of this note is to presente what is known and what is unknown (so far the author knows) in the realm of this kind of dynamical systems. The study of *LODS* was starting by Schimer in [5] who showed that every connected *LODS* satisfies the *right part* of the classical Šarkovskiĭ's Theorem about the structure of the periodic points and asking if the *left part* is also valid. This question was answered in the negative by Baldwin [2] by proving that there is three class of connected linearly ordered spaces related with Šarkovskiĭ's Theorem (the so-called Baldwin's classification): (1) spaces satisfying Šarkovskiĭ's Theorem, (2) spaces with no continuous functions having periodic points of period not a power of 2, and (3) spaces with no continuous functions having periodic points of period not a power of 2, or any power of two higher than n for some $n \geq 0$. Baldwin's classification arises the question of finding dynamical properties characterizing spaces (1), (2) and (3). For spaces (1), a characterization is obtained in [1] by means of the concept of turbulent functions. For spaces (2) and (3), the question remains open. Another way of research in $LODS$ was begun in [1]: the study of minimal sets. For a dynamical system (X, f), recall that a subset $M \subseteq X$ is said to be a *minimal set for* f if it is nonempty, closed and invariant, and if no proper subset of M has these three properties. It is apparent that a finite subset is minimal if and only if it is a periodic orbit. However, identifying infinite minimal sets may be an arduous work (and in many occurrences, it is an open question). A well-known result in one dimensional dynamics characterizes infinite minimal sets (see e.g. [3]) by means of Cantor sets. Actually, every infinite minimal set for a function on the interval $\mathbb{I}$ is a Cantor set and, conversely, given a Cantor subset C of $\mathbb{I}$, there exists $f \in C(\mathbb{I}, \mathbb{I})$ such that C is a minimal set for f. This elegant and powerful result arises question of characterizing minimal sets in a continuum $LODS$, that is in a $LODS$ whose phase space is both compact and connected (recall that a separable continuum linearly ordered space is linearly isomorphic to the interval). The first result in this field was obtained in [1]: every infinite minimal set in a continuum $LODS$ enjoys the same properties as a Cantor set except that it can fail to be metrizable. Moreover, it is also shown that it is not possible to obtain a result similar to the one in the case of the interval: there exist linearly ordered continua where the minimal set are exactly the periodic orbits. In spite of the previous results, no examples of such subsets have been known. To finish we present the construction (in ZFC) presented in [4] of $2^{\mathfrak{c}}$ non-homeomorphic, non-metrizable infinite minimal sets on a continuum $LODS$ of cardinality $2^{\mathfrak{c}}$. Some open question related to minimality on $LODS$ are also commented.

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[4] Husek, M., Sanchis, M. and Tamariz–Mascarúa, A., *Non Cantor Minimal Sets*, submitted.

[5] Schimer, H., *A topologist's view of Šarkovskiĭ's Theorem*, Houston J. Math., **11**, no. 3 (1985), 385–395.

Marion Scheepers *Boise State University* Selection principles in topological groups

abstract We discuss classical selection principles in the context of topological groups.

Paul J. Szeptycki *York University* Products of ordinals and $sigma$-products

abstract I will give a short survey of properties of products of ordinals and some natural subspaces, focussing on σ-products. It was proved by the author and N. Kemoto that if X is a $sigma$-product or Σ-product of ordinals at base point 0, then X is strongly zero-dimensional, countably paracompact et al. If the base point is different from 0, these properties may fail. In particular, countable paracompactness of a σ-product depends on the choice of the base point.

Angel Tamariz Mascarúa *Facultad de Ciencias, UNAM* Spaces of Continuous Functions Defined on Mrwka Spaces

abstract Joint work with M. Hruzăk and P.J. Szeptycki.

We prove that for a maximal almost disjoint family $\mathcal{A}$ on ω, the space $C_p(\Psi(\mathcal{A}), 2^\omega)$ of continuous Cantor-valued functions with the pointwise convergence topology defined on the Mrówka space $\Psi(\mathcal{A})$ is not normal. Using CH we construct a maximal almost disjoint family $\mathcal{A}$ for which the space $C_p(\Psi(\mathcal{A}), 2)$ of continuous $\{0,1\}$-valued functions defined on $\Psi(\mathcal{A})$ is Lindelöf. These theorems improve some results due to A. Dow and P. Simon in [DS]. We also prove that this space $C_p(\Psi(\mathcal{A}), 2) = X$ is a Michael space; that is, X^n is Lindelöf for every $n \in \mathbb{N}$ and neither X^ω nor $X \times \omega^\omega$ are normal.

Moreover, we prove that for every uncountable almost disjoint family $\mathcal{A}$ on ω and every compactification $b\Psi(\mathcal{A})$ of $\Psi(\mathcal{A})$, the space $C_p(b\Psi(\mathcal{A}), 2^\omega)$ is not normal.

Mikhail Tkachenko *Universidad Autónoma Metropolitana* Independent, transversal, and T_1-complementary topologies

*abstract*All topologies we consider are assumed to satisfy the T_1 separation axiom. Two topologies τ_1 and τ_2 on a set X are said to be *T_1-independent* if the intersection $\tau_1 \cap \tau_2$ is the cofinite topology on X. A pair τ_1, τ_2 of T_1-independent topologies on an infinite set has to have very special properties. For example, if both τ_1 and τ_2 are Hausdorff and the first topology is sequential, then the second one is countably compact and does not contain non-trivial convergent sequences. One can construct, under Martin's Axiom, a Hausdorff topological group topology $\mathcal{T}$ on the group of reals R such that $\mathcal{T}$ is T_1-independent of the usual interval topology τ on R (see [3]). Clearly, the topology $\mathcal{T}$ is countably compact and all compact subsets of the group $(R, \mathcal{T}))$ are finite. Topologies τ_1 and τ_2 on a set X are called *transversal* provided that the join $\tau_1 \vee \tau_2$ of τ_1 and τ_2 is the discrete topology of X. In other words, for every point $x \in X$ there are sets $U_1 \in \tau_1$ and $U_2 \in \tau_2$ such that $U_1 \cap U_2 = \{x\}$. Transversal topologies are abundant: if a topology τ_1 contains two disjoint non-empty open sets, then it admits a transversal compact Hausdorff topology [2]. In particular, every Hausdorff topology admits a transversal compact Hausdorff topology. If topologies τ_1 and τ_2 on a set X are T_1-independent and transversal, they are called *T_1-complementary.* Furthermore, if there exists a bijection f of X onto itself such that $\tau_2 = \{f(U) : U \in \tau_1\}$, then the topology τ_1 is said to be *self T_1-complementary.* It is not easy at all to construct a completely regular self T_1-complementary topology on an infinite set, the first construction of such a topology was given by S. Watson [4]. Quite recently, Shakhmatov and the author succeded in constructing a compact Hausdorff space which is a T_1-complement of itself [1]. In the talk we will present a survey of results concerning T_1-independent, transversal and T_1-complementary topologies and formulate a number of open problems in this area.

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Vladimir Tkachuk *Universidad Autonoma Metropolitana de Mexico* Domination by the irrationals and K-analyticity.

abstract We consider the irrationals to be the space ω^ω; given $p, q \in \omega^\omega$ let $p \leq q$ if $p(n) \leq q(n)$ for all $n \in \omega$. A space X is *dominated by the irrationals* if there exists a compact cover $\{K_p : p \in \omega^\omega\}$ of the space X such that $p \leq q$ implies $K_p \subset K_q$. The space X is said to be *strongly dominated by the irrationals* if if there exists a compact cover $\{K_p : p \in \omega^\omega\}$ of the space X such that $p \leq q$ implies $K_p \subset K_q$ and, for any compact $K \subset X$ there is $p \in \omega^\omega$ such that $K \subset K_p$. Every K-analytic space is dominated by the irrationals; since the converse of this statement does not hold, it is a natural question when the domination with the irrationals coincides with K-analyticity. We prove that, for any space X, the space $C_p(X)$ is dominated by the irrationals if and only if it is K-analytic. The importance of strong domination by the irrationals stems from the fact that this concept generalizes hemicompactness; besides, a second countable space is strongly dominated by the irrationals if and only if it is completely metrizable. We show that it is independent of ZFC whether ω_1 is strongly dominated by the irrationals. We prove, among other things, that a space $C_p(X)$ is strongly dominated by the irrationals if and only if X is countable and discrete.

Artur Hideyuki Tomita *Sau Paublo University* A solution to Comforts question on the countable compactness of powers of a topological group.

abstract Comfort and Ross (Pacific J., 1966) showed that pseudocompactness is productive in the class of topological groups. E. van Douwen (Trans. AMS, 1980)showed that Martins Axiom imply that there exists two countably compact groups whose square is not countably compact. Hart and van Mill (Trans. AMS, 1991)showed that under Martins Axiom for countable posets, there exists a countably compact group whose square is not countably compact. Garcia-Ferreira, Tomita and Watson (Proc. AMS, to appear) showed that the existence of a selective ultrafilter implies the existence of two countably compact groups whose product is not countably compact. Recently, Tomita showed that from the existence of a selective ultrafilter, there exists a countably compact group whose square is not countably compact. Scarborough and Stone (Trans. AMS, 1966) showed that the product of a family of countably compact spaces is

countably compact if every product of a subfamily of size at most $2^{2^{\mathfrak{c}}}$ is countably compact. Ginsburg and Saks (Pacific J., 1975) showed that it suffices to consider subfamilies of size at most 2^{2^ω}. The example of Hart and van Mill and the result of Ginsburg and Saks motivated Comfort (Open Problems is Topology, Question 477) to ask the following;

Is there, for every (not necessarily infinite) cardinal $\alpha \leq 2^{\mathfrak{c}}$, a topological group G such that G^γ is countably compact for all cardinals $\gamma < \alpha$ but G^α is not countably compact?

Using Martin's Axiom for countable posets, it was shown that 2 (Hart and van Mill, op. cit.), some $k \in [n+1, 2^n]$ for each $n \in \omega$ (1996, Tomita, CMUC), 3 (Tomita, Topology Appl., 1999) and every positive integer (Tomita, Topology Appl., to appear) are such cardinals. However, Comfort's Question remained open for infinite cardinals.

Garcia-Ferreira and Tomita (Bol. Soc. Mex. Mat., 2003) showed via forcing there exists a family of topological groups $\{G_\xi : \xi < 2^{\mathfrak{c}}\}$ such that for each $I \subseteq 2^{\mathfrak{c}}$, $\prod_{\xi \in I} G_\xi$ is countably compact if and only if $|I| < 2^{\mathfrak{c}}$. However, the method did not allow to take powers nor could be used in infinite cardinals smaller than $2^{\mathfrak{c}}$.

In this talk we will sketch the construction of consistent examples to answer Comfort's Question in the affirmative for every cardinal $\alpha \leq 2^{\mathfrak{c}}$. Our examples do not require forcing nor some form of Martin's Axiom, only selective ultrafilters and the regularity of $2^{\mathfrak{c}}$. We improve a technique that appears in Tomita and Watson (Topology Appl., 2004) that showed that incomparable selective ultrafilters are Comfort-group incomparable.

Yolanda Torres Falcón *Universidad Autónoma Metropolitana - Iztapalapa* An example of a σ-compact monothetic group which is not compactly generated

abstract We construct a countable (hence σ-compact) monothetic topological group G which is not compactly generated, thus answering in the negative a question posed by Fujita and Shakhmatov. In addition, our group G is precompact and sequentially complete.

Hideki Tsuiki and Shuji Yamada *Kyoto University and Kyoto Sangyo University* Every Dense-in-itself Compact Metric Space has a Full $\{0,1,\perp\}$-Representation

abstract

We consider a representation of a space X as infinite $\{0,1,\perp\}$-sequences. More precisely,

1. we consider an embedding φ of X in $\mathbb{T}^\omega$, which is the space of infinite $\{0,1,\perp\}$-sequences with the Scott topology,
2. let $S_n^0 = \{x \in X \mid \varphi(x)[n] = 0\}$ and $S_n^1 = \{x \in X \mid \varphi(x)[n] = 1\}$. When $\varphi(x)[n] = \perp$, every neighbourhood of x intersects with both S_n^0 and S_n^1.

For such a representation, S_n^0 and S_n^1 are regular opens which are exterior of each other, and S_n^0, S_n^1 $(n = 0, 1, 2, \ldots)$ forms a subbase of X, which we call a dyadic subbase [Tsuiki04, in Topology Proceedings].

We say that such a representation is full if all the $\{0,1\}$-sequences are obtained by filling the bottoms of the images of φ with 0 and 1. Fullness corresponds to the no-redundancy of the representation.

In this talk, we show that every dense-in-itself compact metric space has a full $\{0,1,\perp\}$-representation. This construction also entails that when a compact space X is embedable in I^n (it holds, when $n >= 2dimX + 1$), we can form a surjective continuous map from X to I^n, which is just like the space-filling Peano curve from I to I^n.

Jerry E. Vaughan *University of North Carolina at Greensboro.* On τ-pseudocompact spaces *abstract* Here we consider $T_{3\frac{1}{2}}$-spaces, and infinite cardinals (denoted τ or κ). We present some new results concerning τ-pseudocompact spaces. Recall two definitions of J. F. Kennison: a space X is τ-pseudocompact provided $f(X)$ is a closed subset of $\mathbb{R}^\tau$ for every continuous function $f : X \to \mathbb{R}^\tau$, and a set $H \subset X$ is called a Z_τ-set provided H is the intersection of at most τ zero-sets. It is easy to see that τ-pseudocompactness is weaker than the classical property of Alexandroff-Urysohn *initially τ-compactness* (i.e., every open cover of cardinality at most τ has a finite subcover). A. V. Arhangel′slii defined a space to be Z_τ-normal provided for every pair H, K of disjoint sets with H a Z_τ-set and K a closed set, there exists a Z_τ-set G such that $K \subset G$ and $G \cap H = \emptyset$. Theorem 1: If X is Z_τ-normal, τ-pseudocompact and $< \tau$-bounded, then X is initially τ-compact. Theorem 2. If X is Z_τ-normal and τ-pseudocompact, then X is initially κ-compact for all κ such that $2^{<\kappa} \leq \tau$. These theorems generalize several known results.

Richard G. Wilson *Universidad Autónoma Metropolitana, Iztapalapa* Minimal properties between T_1 and T_2

abstract A space is a US-space if every convergent sequence has a unique limit; it is an SC-space if each convergent sequence together with its limit is closed and is a KC-space if every compact subset is closed. We study the existence of spaces which are minimal with respect to these properties. We obtain a number of results regarding minimal SC-spaces, we show that the class of infinite minimal US-spaces is empty and we give a consistent example of a Tychonoff topology which contains no minimal KC-topology. The only previously known example of such a space is not Hausdorff (see [F]).

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Kohzo Yamada *Department of Mathematics, Faculty of Education, Shizuoka University* Products of straight spaces with compact spaces

abstract A metric space X is called straight if any continuous function which is uniformly continuous on each set of a finite cover of X by closed sets, is itself uniformly continuous. In this talk, we show that for a straight space X, $X \times C$ is straight if and only if $X \times K$ is straight for eny compact metric space K. Furthermore, we show that for a straight space X, if $X \times C$ is straight, then X is precompact, where C is the convergent sequence $\{1/n : n \in N\}$ with its limit 0 in the real line with the usual metric. Note that the notion of straightness depends on the metric on X. Indeed, the above result yields that $\mathbb{R} \times C$ is not straight, where $\mathbb{R}$ means the real line with the usual metric. On the other hand, we show that $(0,\, 1) \times C$ is straight, where $(0,\, 1)$ means the unit open interval with the usual metric.

Kaori Yamazaki *University of Tsukuba* Base-normality and product spaces

abstract We introduce the notion of base-normality, which is a natural generalization of base-paracompactness introduced by J. E. Porter (2003). A space X is said to be *base-normal* if there is a base $\mathcal{B}$ for X with $|\mathcal{B}| = w(X)$, where $w(X)$ is the weight of X, such that every binary open cover $\{U_0, U_1\}$ of X admits a locally finite cover $\mathcal{B}'$ of X by members of $\mathcal{B}$ such that $\overline{\mathcal{B}'}$ refines $\{U_0, U_1\}$. Every base-normal space is normal. Note that a Hausdorff space X is base-paracompact if and only if X is base-normal and paracompact.

We prove the following:

Theorem 1. For a base-normal space X and a metrizable space Y, the product space $X \times Y$ is normal if and only if $X \times Y$ is base-normal.

Theorem 2. For the countable product $X = \Pi_{i \in N} X_i$ of spaces X_i such that finite subproducts $\Pi_{i \leq n} X_i$, $n \in N$, are base-normal, X is normal if and only if X is base-normal.

Theorem 3. Every Σ-product of metric spaces is base-normal.

Beatriz Zamora Aviles *York university* Countable Dense Homogeneity with Lipschitz functions.

abstract A topological space X is CDH if given A, B countable dense subsets of X, there exists $f : X \to X$ an homeomorphism such that $F[A] = B$. Two metric variants (iso-CDH and LCDH) of countable dense homogeneity are considered here. We show that every separable Banach space is LCDH, that is: Given A, B two countable dense subsets of a separable Banach space X and $\varepsilon \in (0, 1)$ there is an $f : X \to X$ such that $f[A] = B$ and $1 - \varepsilon \leq \frac{\rho(f(x), f(y))}{d(x,y)} \leq 1 + \varepsilon$ for every $x, y \in X$.